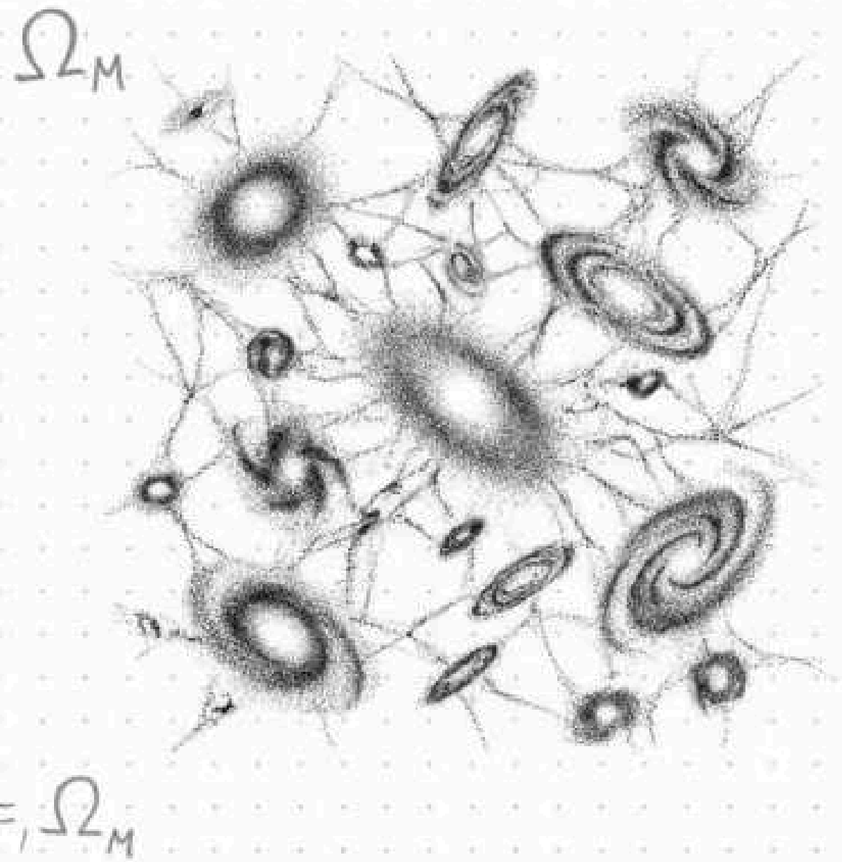
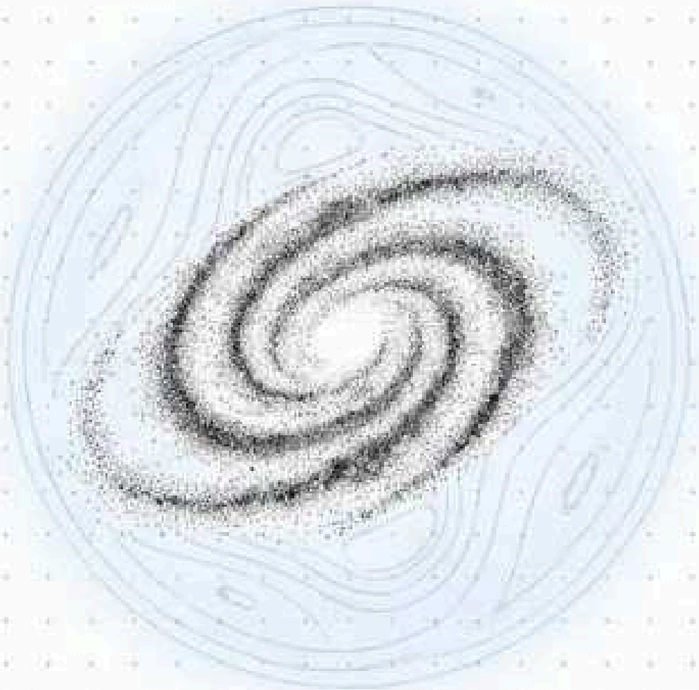
The Three Cosmic Questions

¶ 4.1



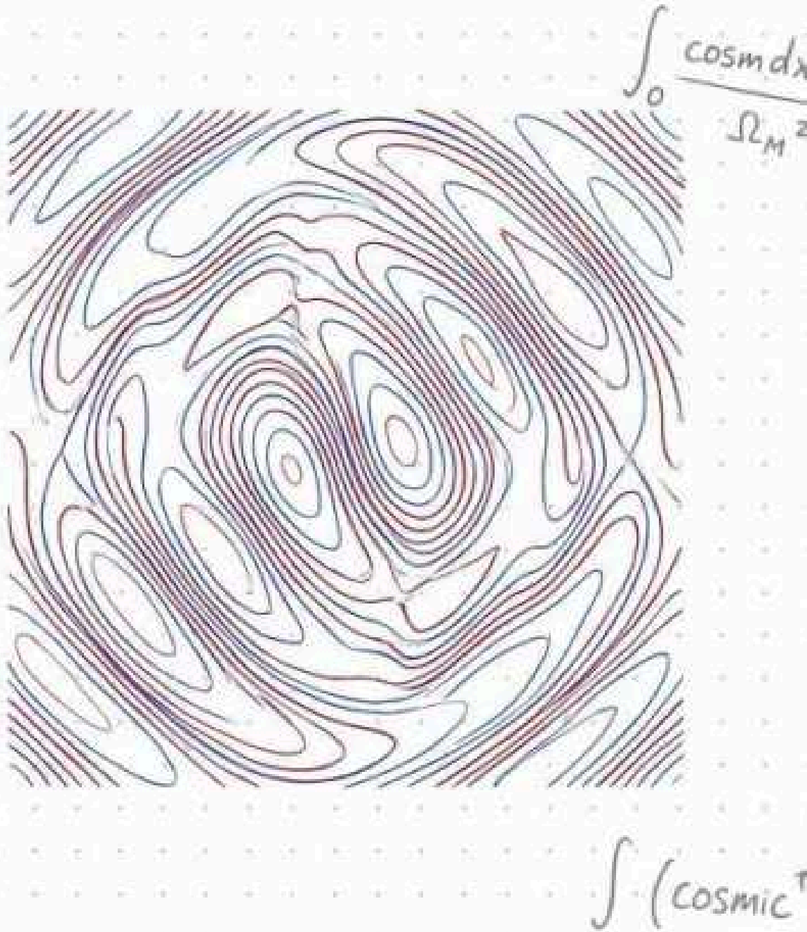
1. Arrangement

How is matter arranged across the universe?



2. Invisible Matter

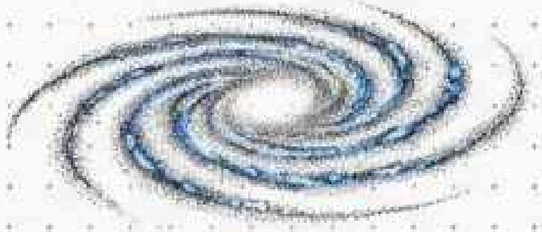
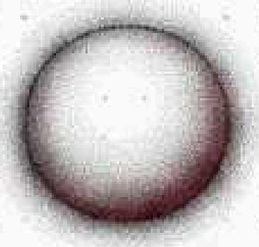
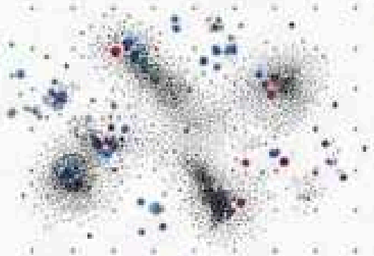
Why do astronomers believe most matter is invisible?



3. Formation

How did galaxies form from the tiny ripples in the early universe?

A Taxonomy of Galaxies

Type	Shape	Star Composition	Scale
Spiral	 Flat discs with spiral arms	Young stars	Trillions of stars (giants)
Elliptical	 Smooth, rounded collections	Mostly old stars	Varies from dwarf to giant
Irregular	 No clear shape	Mixed composition	Often small dwarfs (millions of stars)

$$v^2 = \frac{GM}{r}$$

$$L \propto M^{\alpha}$$

$$v^2 = \frac{\mathcal{N}\bar{v}_s'}{I}$$

$$\tau_{\text{evol}} \approx \frac{M}{L}$$

$$\tau_{\text{evol}} \approx \frac{M}{L}$$

$$\tau_{\text{evol}} \approx \frac{M}{L}$$

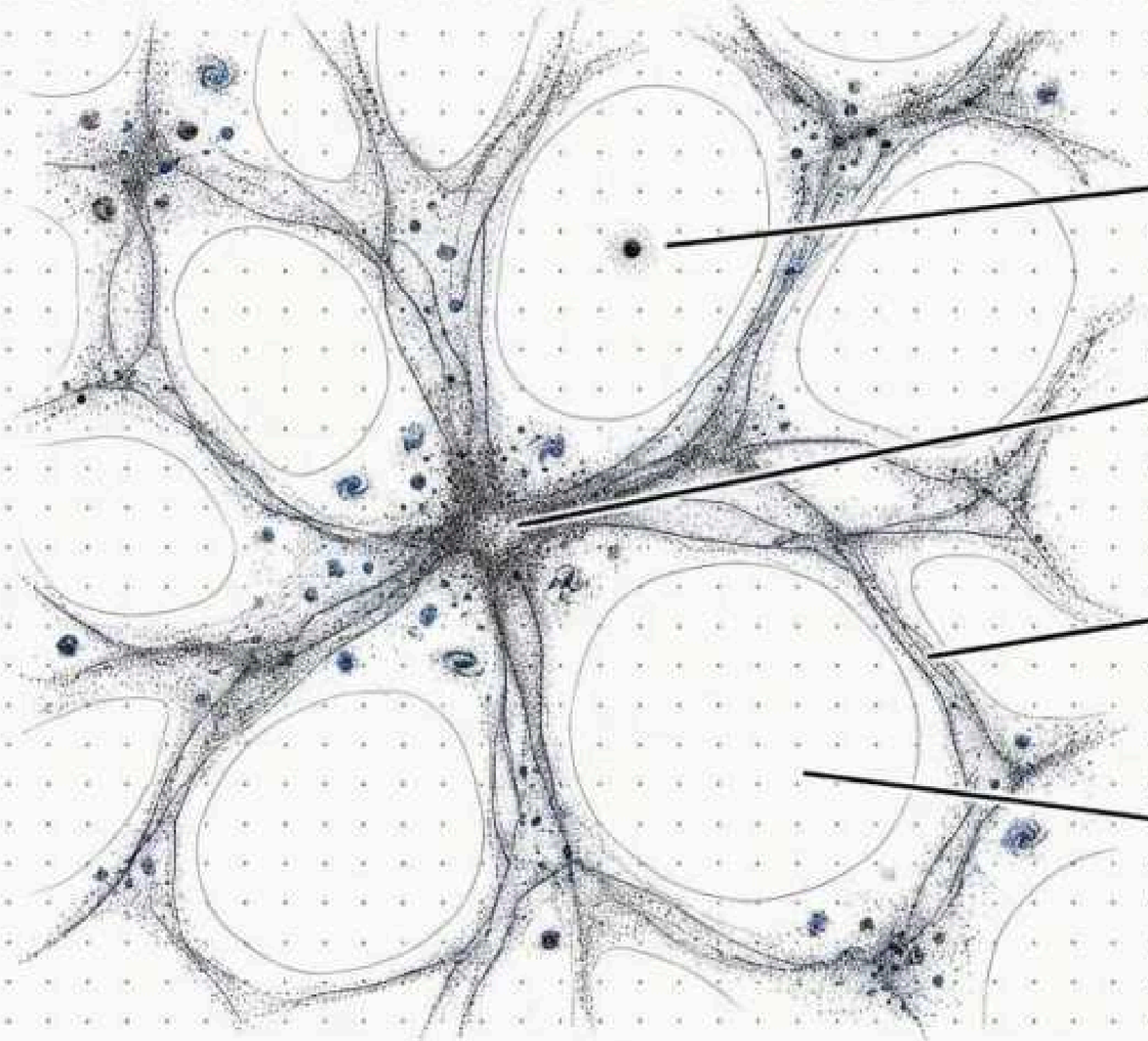
$$L \propto M^{\alpha}$$

$$\Phi(r) = -G\int \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}d^3\mathbf{r}'$$

$$\delta(\mathbf{x},t) = \frac{\rho(\mathbf{x},t) - \bar{\rho}(t)}{\bar{\rho}(t)}$$

The Cosmic Web Architecture

¶ 4.3



Galaxies gather into Groups and Clusters.

Clusters map onto vast Superclusters.

Filaments & Sheets: The threads where matter lives.

Voids: Enormous empty regions between the threads.

$$\delta(\mathbf{x},t) = \frac{\rho(\mathbf{x},t) - \bar{\rho}(t)}{\bar{\rho}(t)}$$

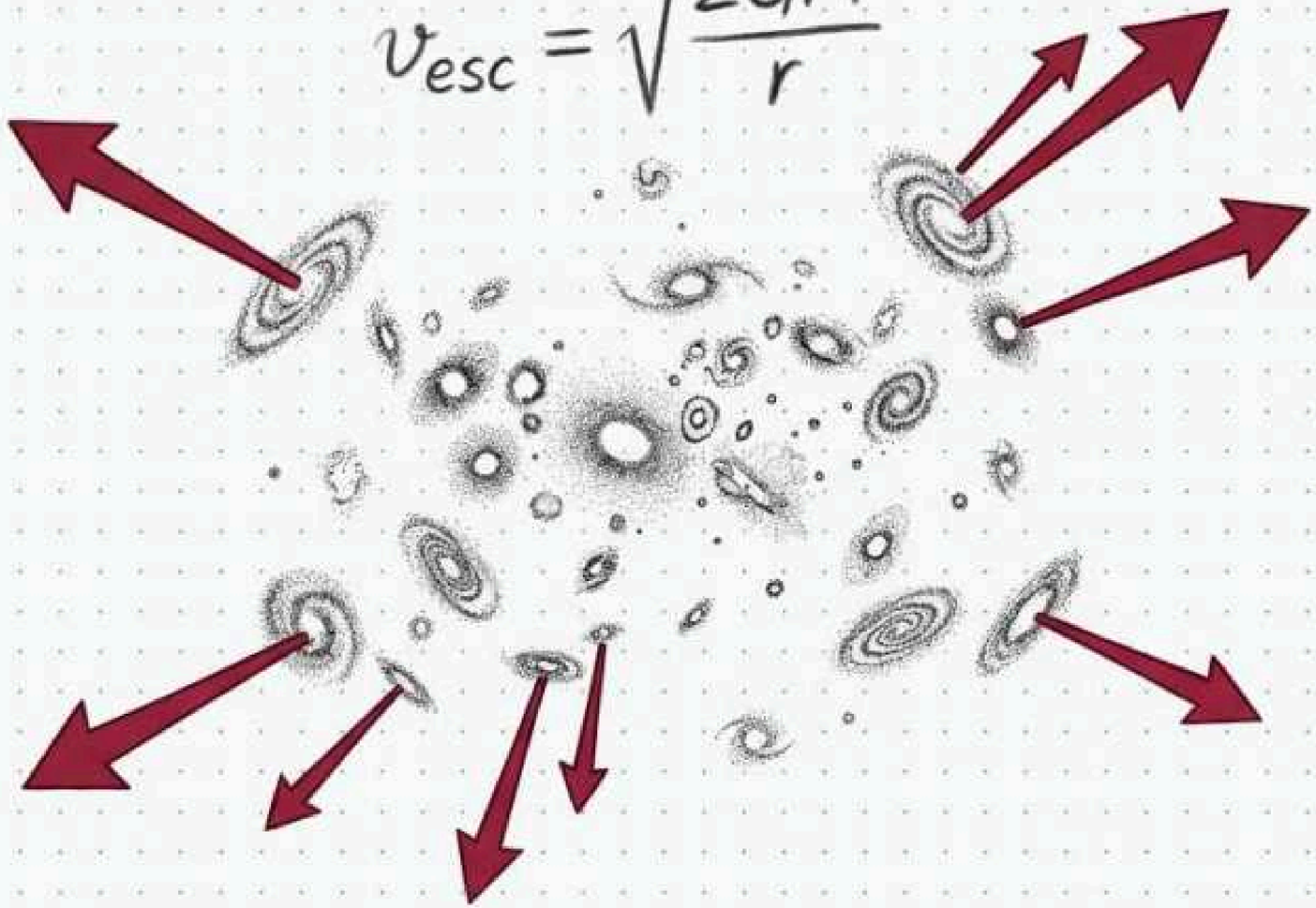
$$\nabla^2\Phi = 4\pi G\rho$$

$$\nabla^2\Phi = 4\pi G\rho$$

Context Note: Revealed by large surveys, such as the Sloan Digital Sky Survey (begun in 2000).

Zwicky's Anomaly (1933)

$$v_{esc} = \sqrt{\frac{2GM}{r}}$$



The Observation: Fritz Zwicky studies the Coma Cluster.

The Problem: Galaxies are moving **far too fast** to be held together by the gravity of their visible stars.

Conclusion: There must be a great deal of unseen mass—first termed "Dark Matter."

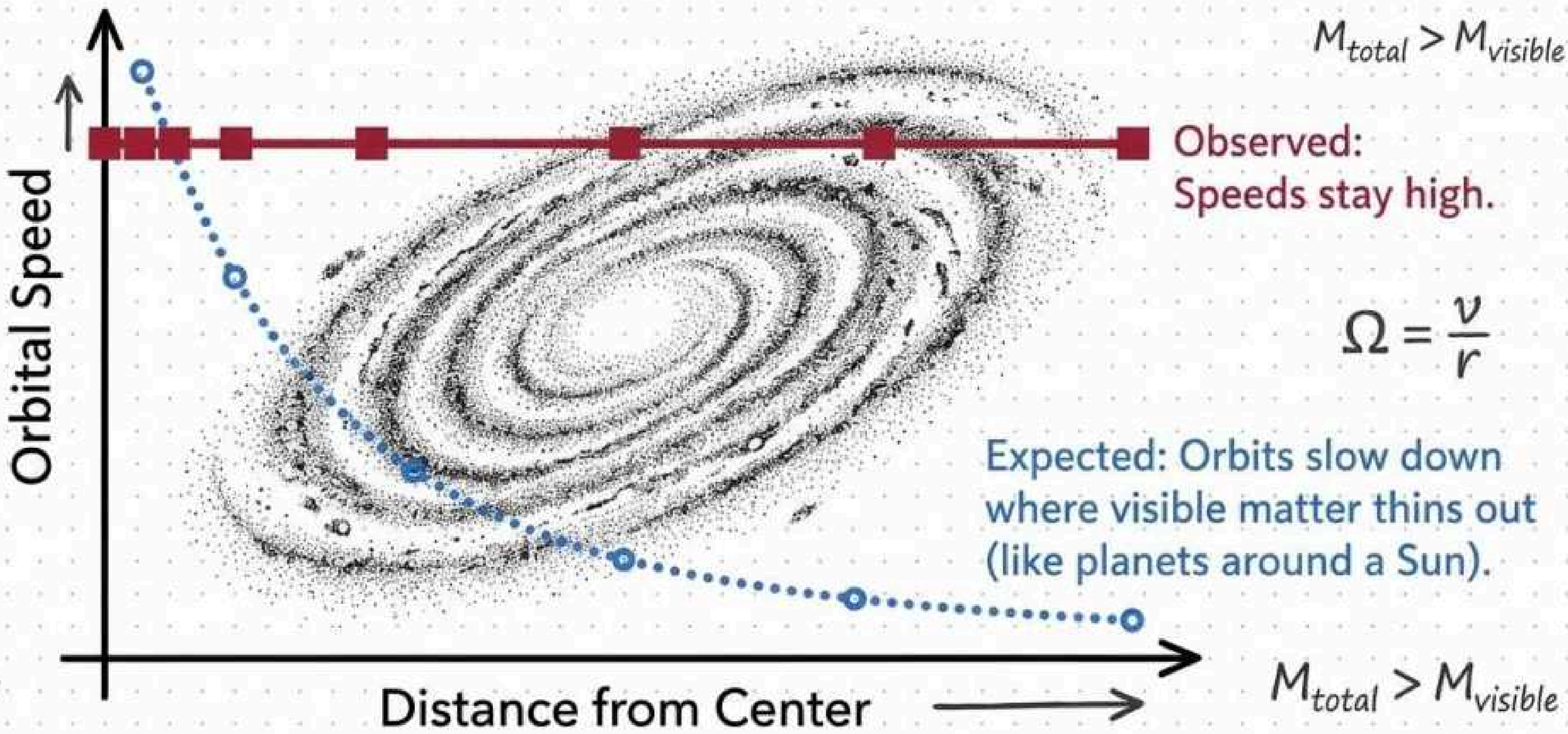
Rubin & Ford's Discovery (1970s)

¶ 4.5

$$v(r) = \sqrt{\frac{GM(r)}{r}}$$

$$\Omega = \frac{v}{r}$$

$M_{total} > M_{visible}$



$M_{total} > M_{visible}$

$$\Omega = \frac{v}{r}$$

$M_{total} > M_{visible}$

$M_{total} > M_{visible}$

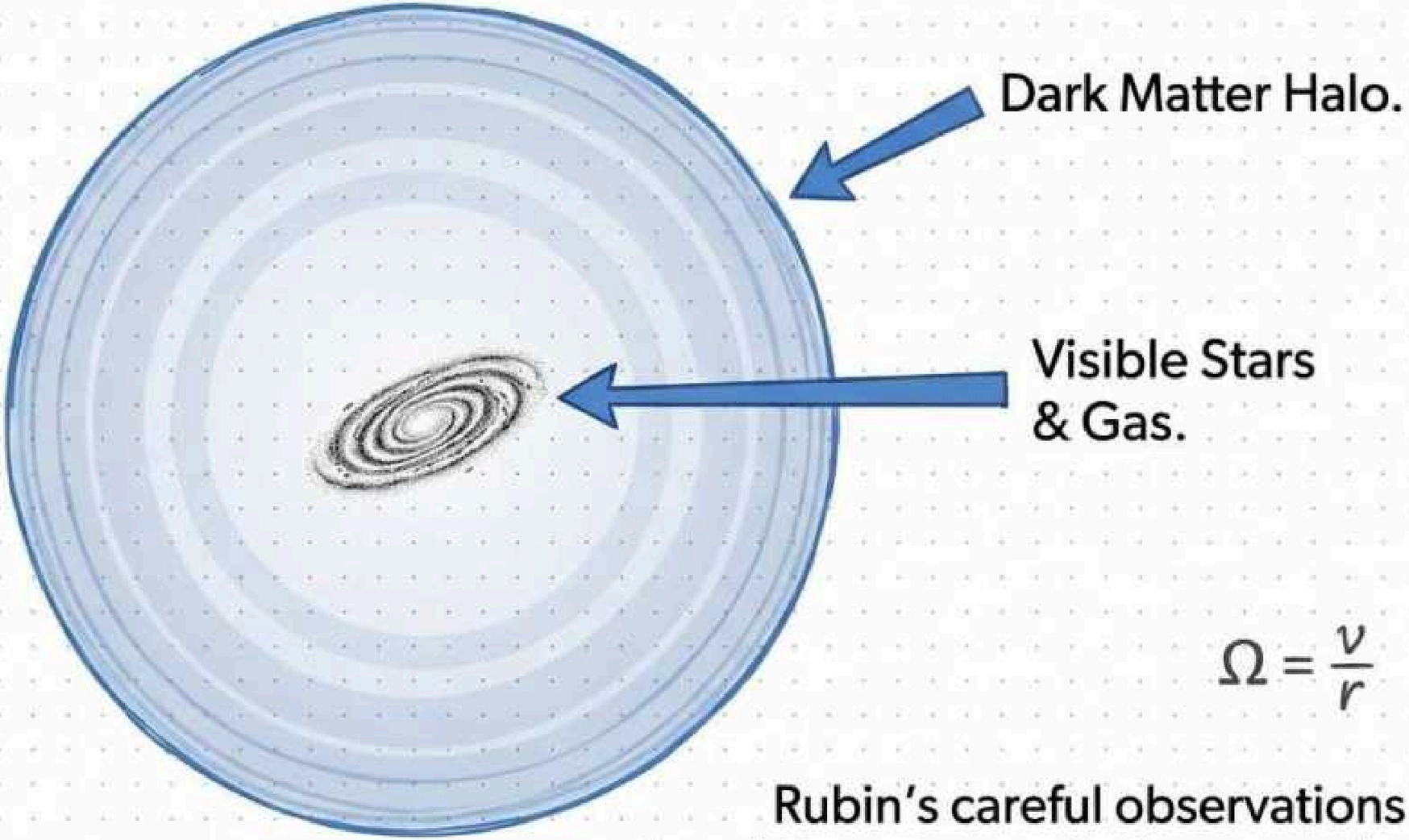
Vera Rubin and Kent Ford measured how fast stars and gas orbit in spiral galaxies. The expected drop-off in velocity never happens.

$M_{halo} \gg M_{visible}$

The Invisible Halo

¶ 4.6

$$v_c = \sqrt{\frac{GM(r)}{r}} \qquad \Omega = \frac{v}{r}$$



To produce flat rotation curves, every galaxy must sit inside a large, invisible halo of matter.

This unseen mass is several times more massive than all the visible stars and gas combined.

$$\Omega = \frac{v}{r}$$

Rubin's careful observations convinced the astronomical community that dark matter was real.

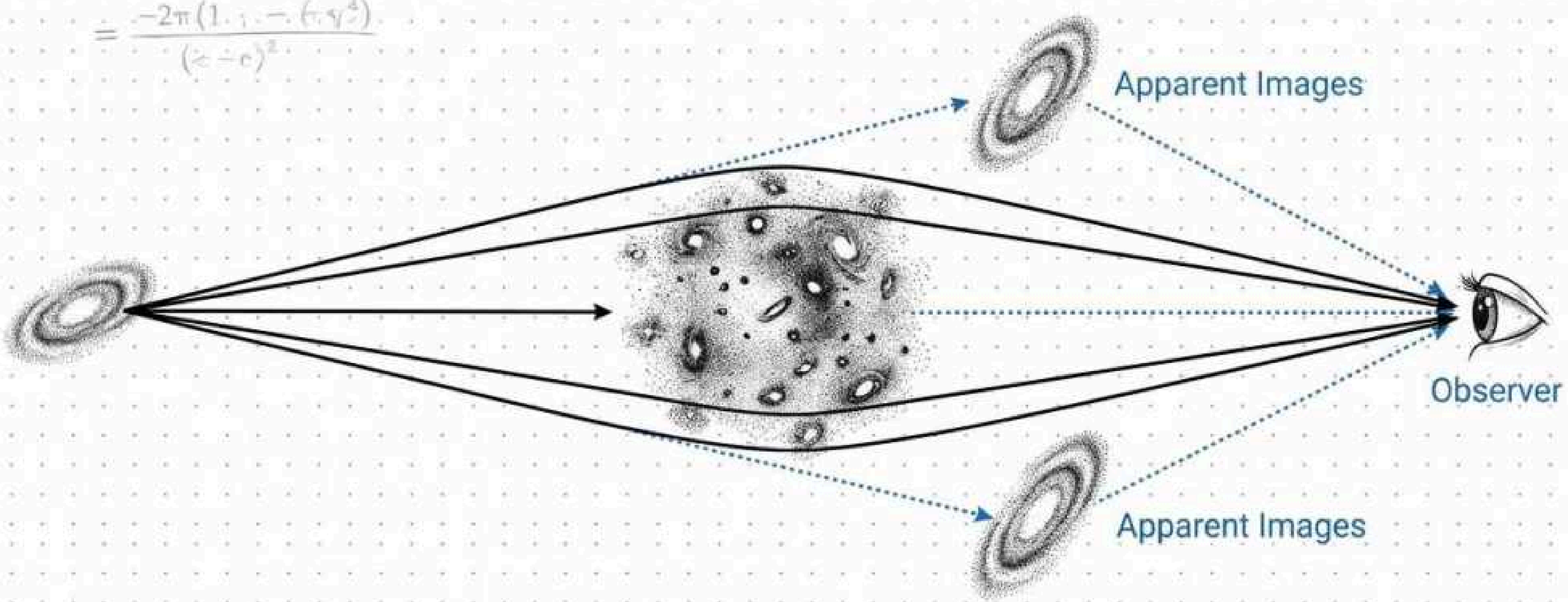
$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$= \frac{-2\pi(1. \; ; -. \; (\cdot \gamma^4)}{(\lesssim \div c)^2}$$

Gravitational Lensing

¶ 4.7



Step 1: General relativity dictates that mass bends light.

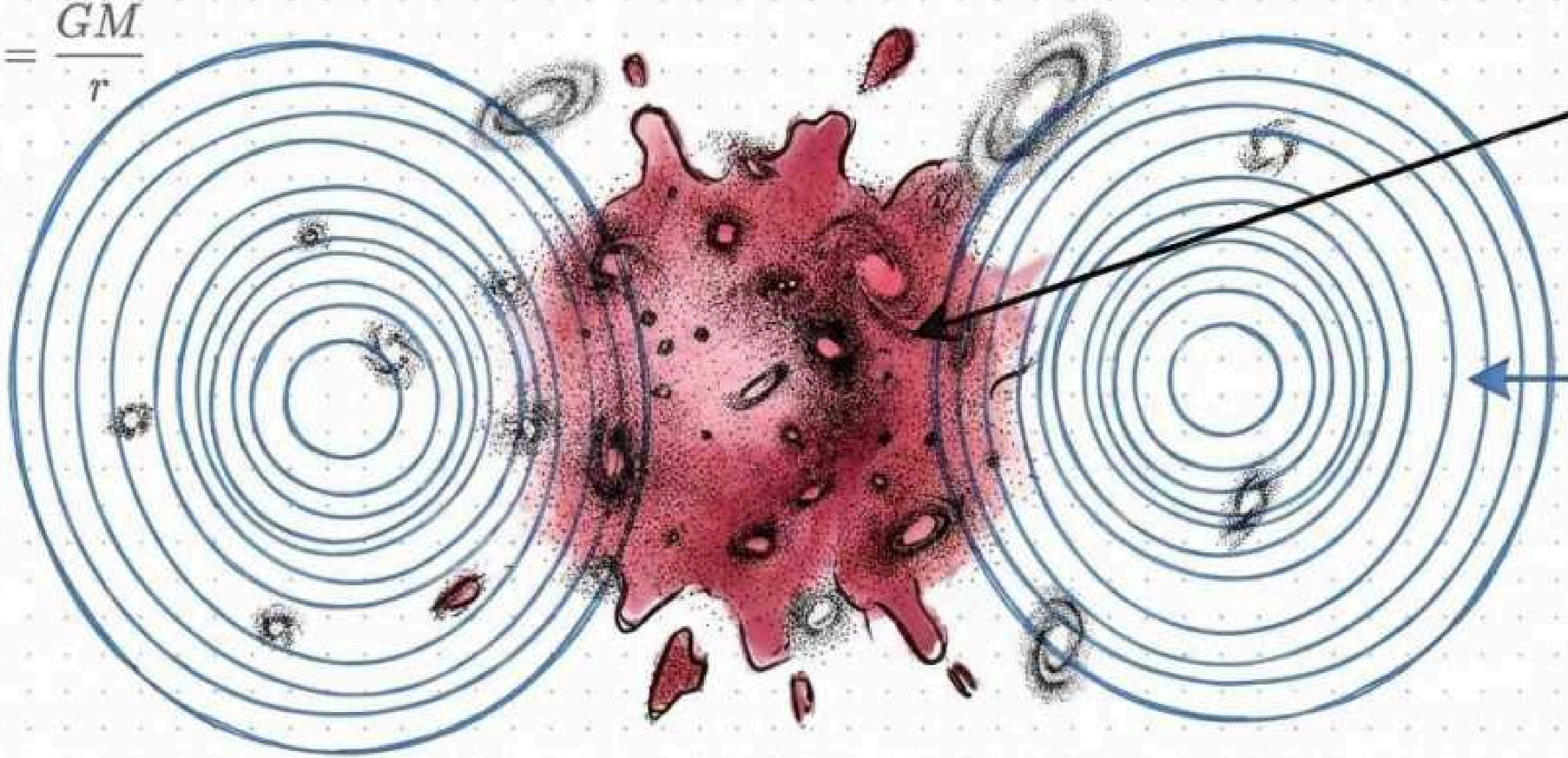
Step 2: A massive galaxy cluster acts like a lens, distorting and magnifying galaxies far behind it.

Step 3: Measuring this distortion reveals the cluster's true mass—proving again it contains far more than the visible matter.

The Bullet Cluster Collision (2006) ¶ 4.8

$$G = 6.674 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$$

$$v^2 = \frac{GM}{r}$$



Ordinary matter (hot gas) slows down and clumps together during the collision.

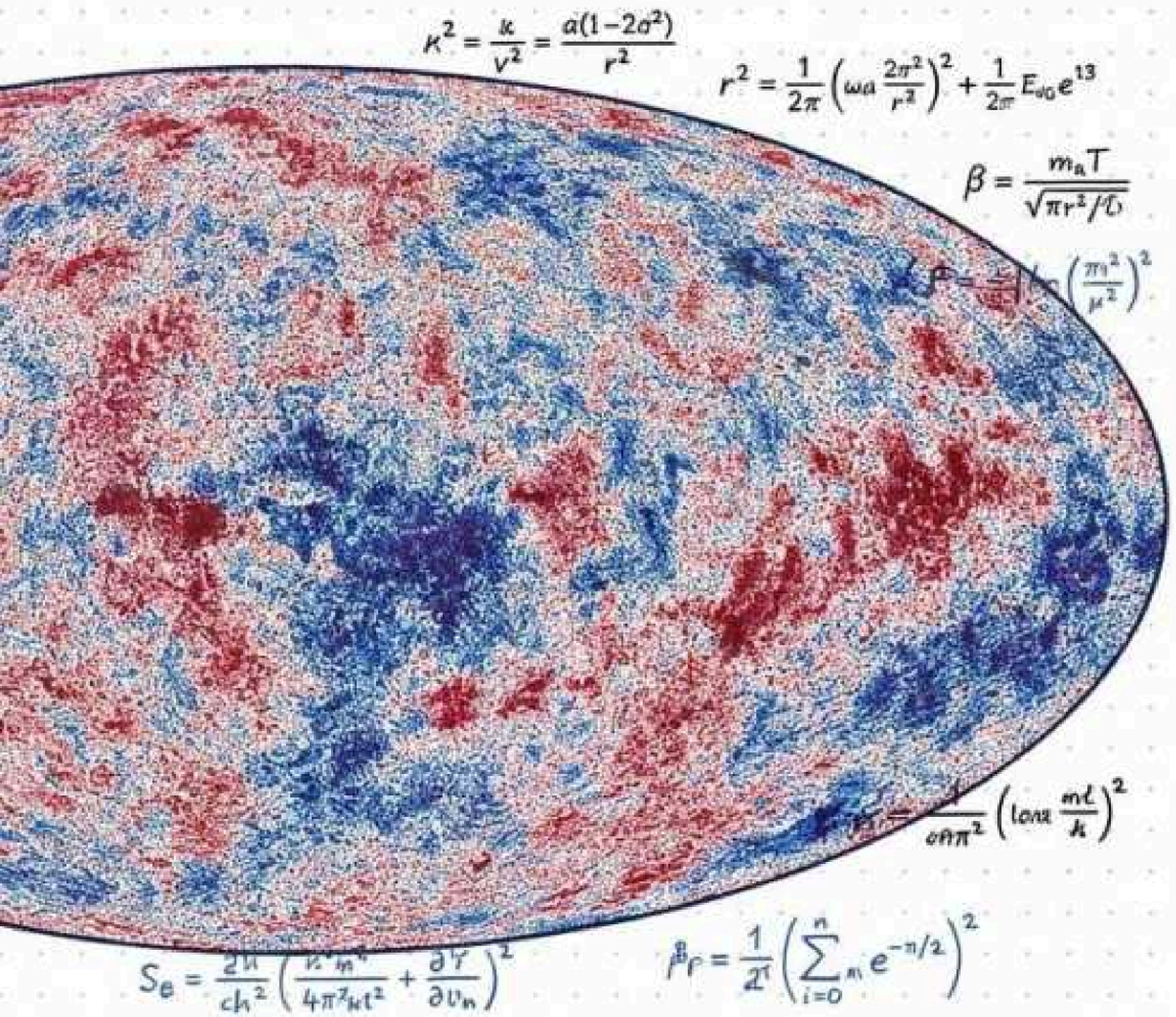
The vast majority of the mass (mapped by lensing) passes straight through unimpeded.

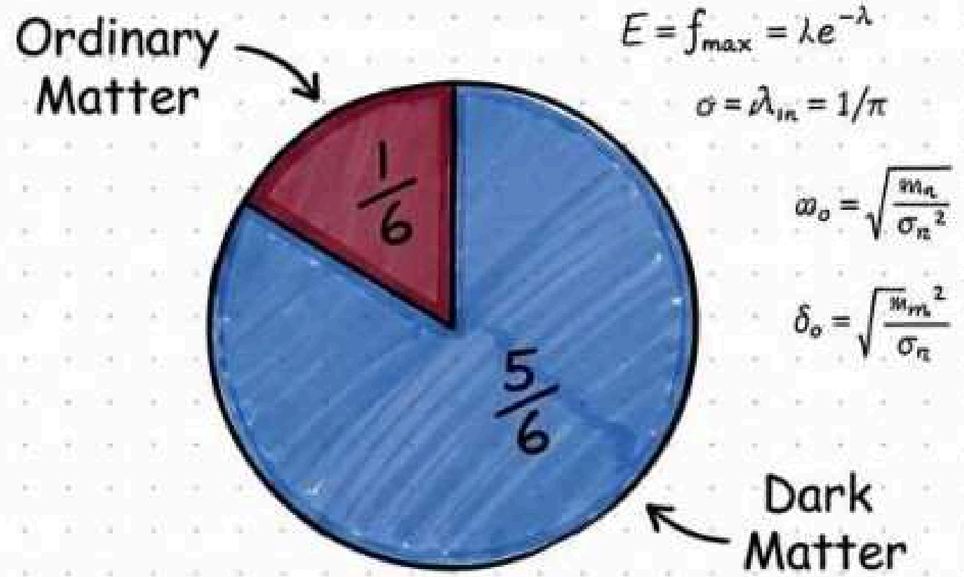
$$v^2 = \frac{GM}{r}$$

Key Takeaway: The ordinary matter and the underlying mass have physically separated. This is direct observational evidence of Dark Matter.

The Cosmic Microwave Background

¶ 4.9

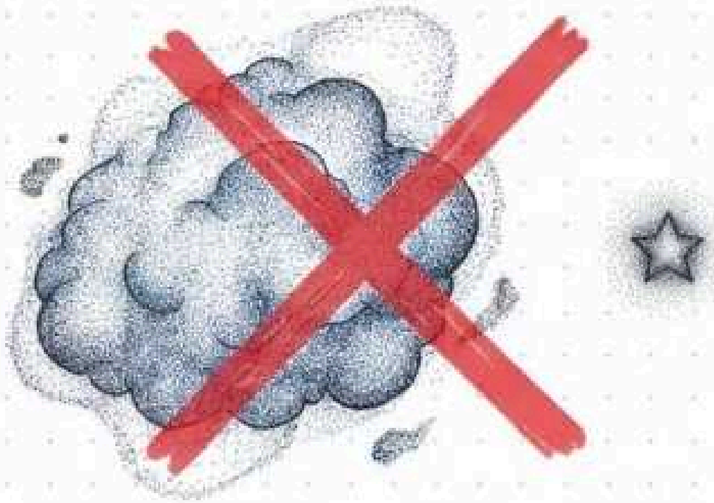




- The pattern of ripples in the early universe depends strictly on the ratio of ordinary matter to dark matter.
- Planck satellite results confirm a 5:1 ratio of Dark Matter to Ordinary Matter.
- Synthesis: This perfectly matches the independent evidence from galaxy rotation and cluster lensing.

The Suspects: What is Dark Matter?

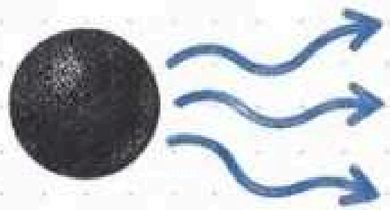
Ruled Out



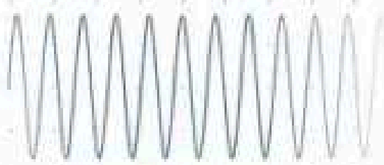
Not ordinary gas, dust, or dim stars.
(They would leave observable traces).

The Hypothesis

A new kind of particle that interacts almost exclusively through gravity.



Candidate 1 (WIMPs):
Weakly Interacting Massive Particles.



Candidate 2 (Axions):
Extremely light theoretical particles.

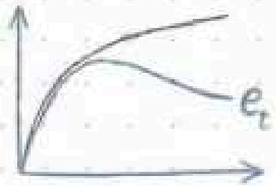
Current Status: Undetected. Despite decades of laboratory searches, no particle has been caught.

The Alternative: Modified Gravity (MOND)

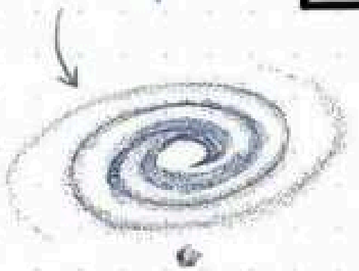
Concept: Modified Newtonian Dynamics (Milgrom, 1983) proposes gravity behaves differently at extremely low accelerations, removing the need for invisible mass.

$\vec{F} = m\vec{a}$

$\vec{F} = \frac{GMm}{r^2}$



$\vec{F} = \frac{GMm}{r^2}$

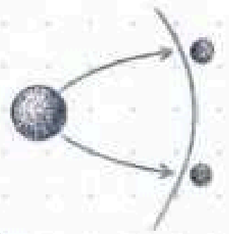


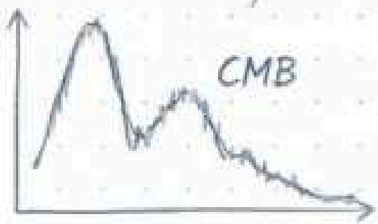
Synthesis Matrix		
Observational Test	**MOND**	**Dark Matter**
Galaxy Rotation Curves	✓	✓
Galaxy Clusters	✗	✓
Gravitational Lensing	✗	✓
Cosmic Microwave Background	✗	✓

$\vec{F} = \frac{GMmm}{r^2}$









Conclusion: Most cosmologists favor Dark Matter because it fits the universe at all scales.

¶ 4.12

The Growth of Structure

$\frac{\delta\rho}{\rho} \propto a$

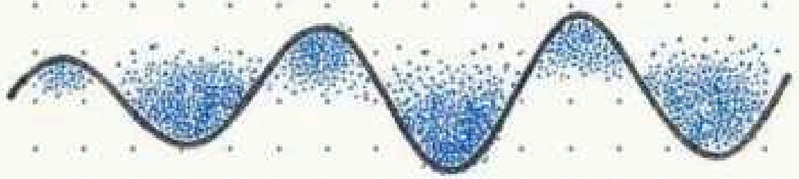
$\Omega_m > \Omega_\Lambda$



1. The Ripples: Tiny density variations in the early universe.

$\mathfrak{F}_m = Mi^2$

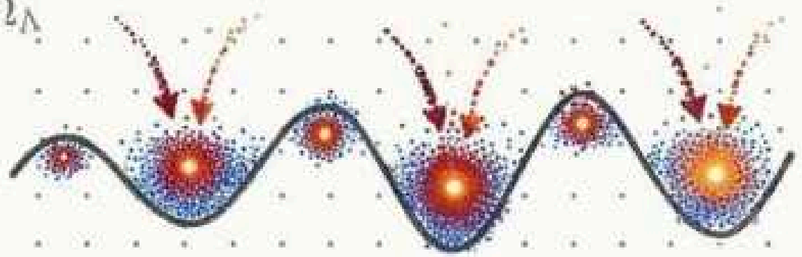
$J_0 = \sqrt{2\pi \left(\frac{1}{\rho}\right)^2}$



2. The Scaffolding: Unaffected by light pressure, Dark Matter clumps first, establishing deep gravity wells.

$\Omega_m > \Omega_\Lambda$

$\text{Jeans mass} = \sqrt{\frac{m\ell}{m^2}}$



3. Ignition: Ordinary matter falls into these clumps, igniting the first stars and galaxies.

Rule of Growth: Small structures form first and merge into larger ones over billions of years.

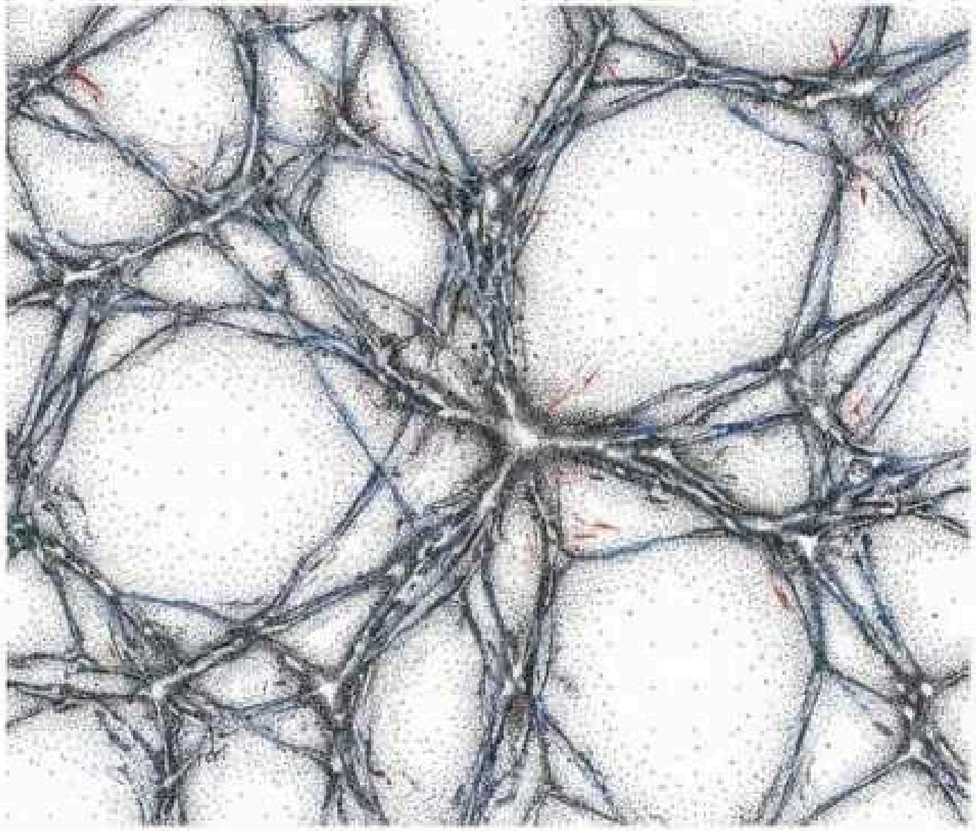
Digital Universes

$$\nabla^2\Phi = 4\pi G\rho$$

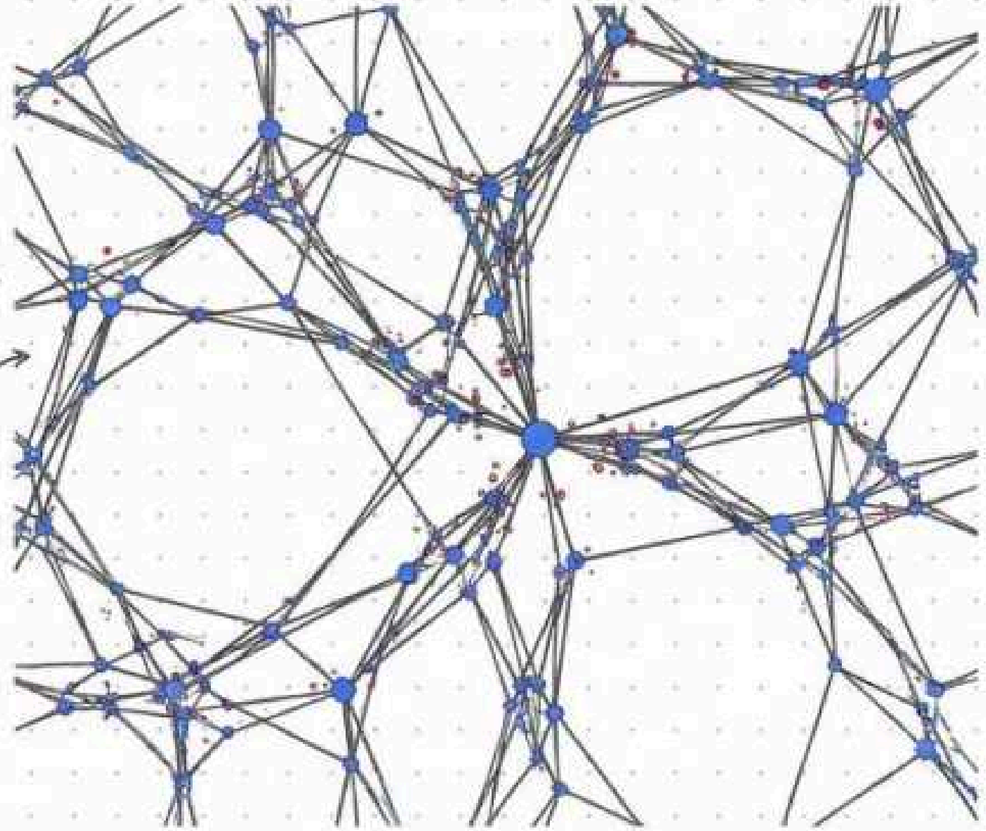
Reality (Observation)

$$\frac{\delta\rho}{\rho} \propto a$$

$$\frac{\delta\rho}{\rho} \propto a$$



The Millennium Simulation (2005)

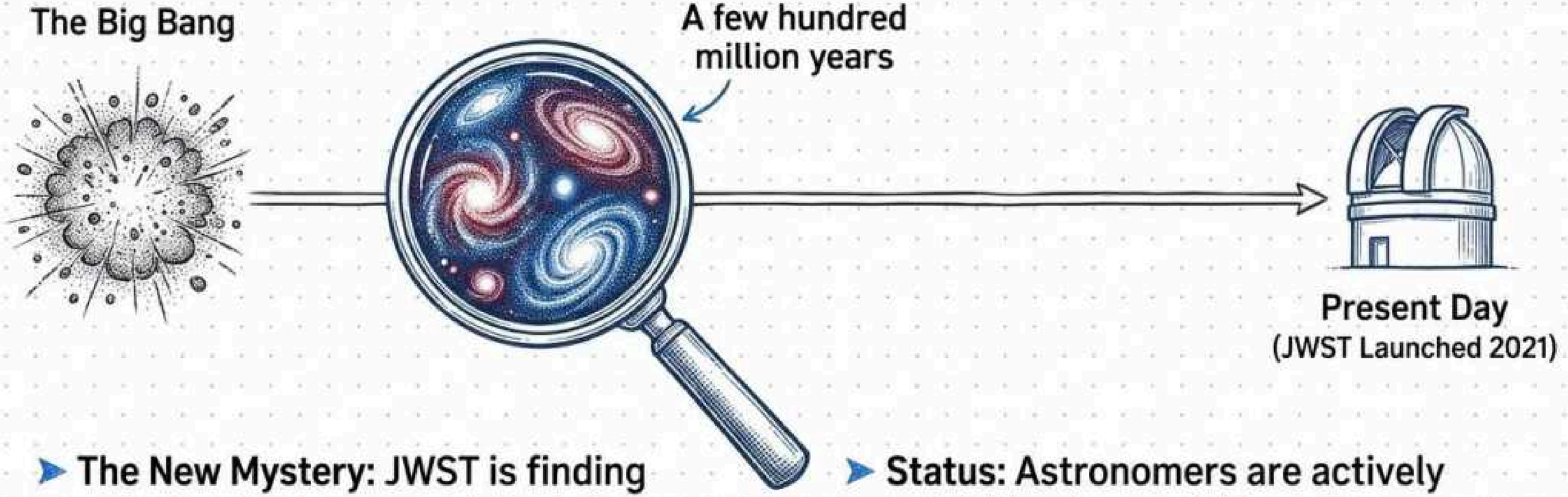


- **Process:** Supercomputer simulations start with measured CMB ripples and track billions of particles governed only by gravity.
- **Result:** The simulation naturally produces a cosmic web strikingly identical to our real universe.
- **Significance:** This is one of the strongest pieces of evidence that the dark matter model is fundamentally correct.

N-body simulation: $\hat{F}_i = \sum_j \frac{Gm_i m_j}{r_{ij}^2}\hat{r}_{ij}$

JWST & The Early Universe

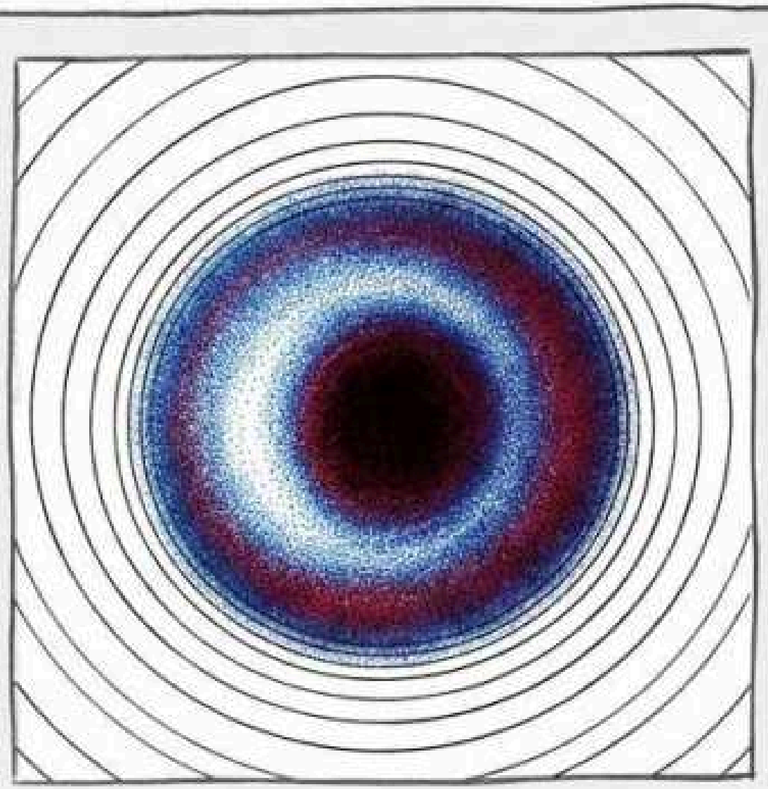
¶ 4.14



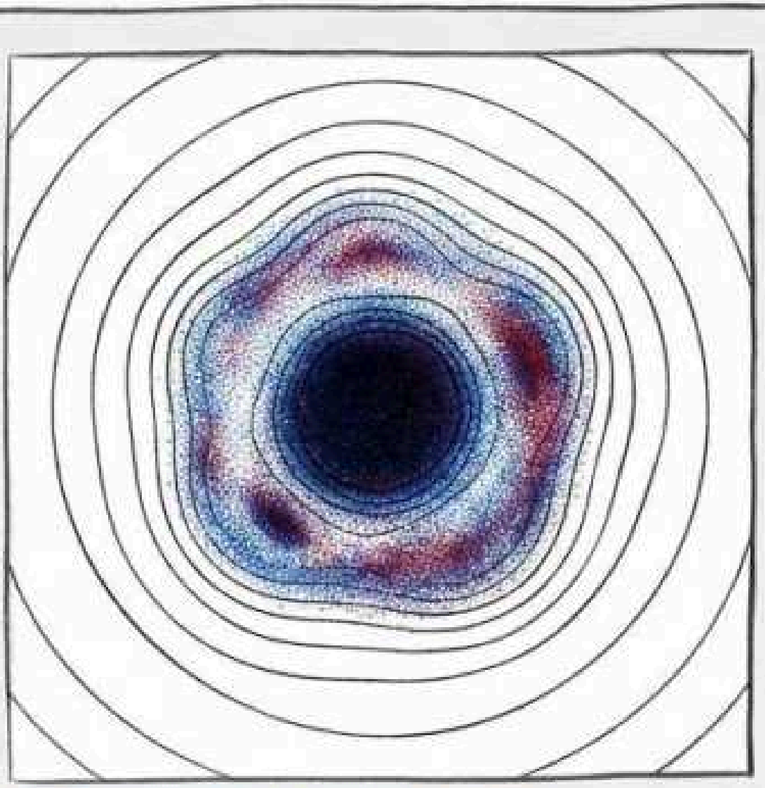
- **The New Mystery:** JWST is finding galaxies that are surprisingly large and bright just a few hundred million years after the beginning.
- **Status:** Astronomers are actively rethinking how quickly ordinary matter can form such immense structures in the early universe.

Hearts of Darkness: Supermassive Black Holes

Most large galaxies, including the Milky Way, host extraordinarily **dense centers**.



M87 (Imaged 2019) – The first image of a black hole ever captured.



Sagittarius A* (Imaged 2022) – The Milky Way's own central black hole, weighing four million times the mass of the Sun.

Why ‘Cold’ Dark Matter?

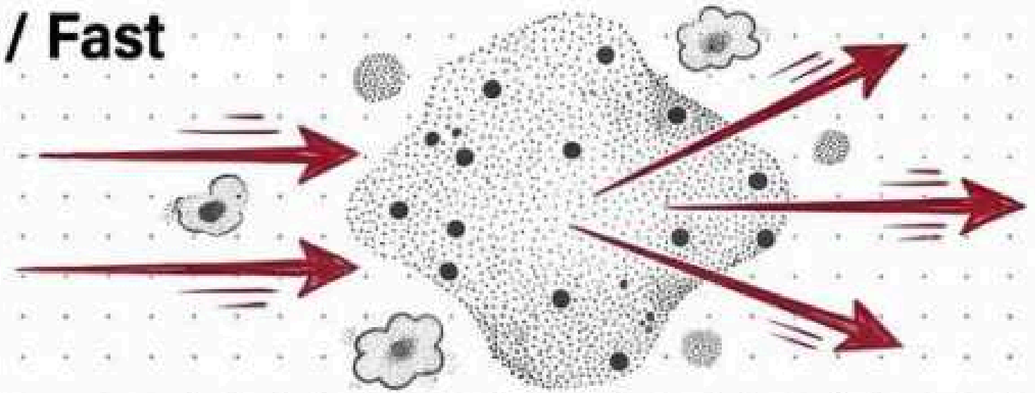
$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$

¶ 4.16

Definition: In physics, ‘Cold’ simply means particles move slowly compared to the speed of light.

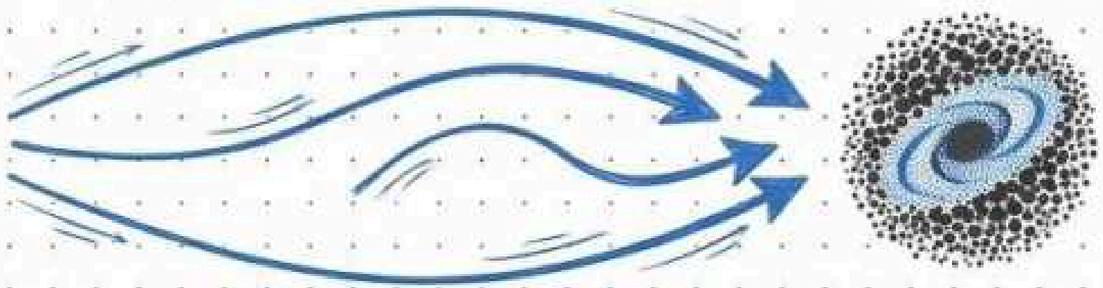
$E = mc^2$

Hot / Fast



The Hot Problem: Fast particles smear out small density clumps, preventing early galaxy formation.

Cold / Slow



The Cold Solution: Slow particles allow small structures to form first and merge later.

$E = mc^2$

Conclusion: This perfectly matches astronomical observations, making Cold Dark Matter the standard cosmological model.

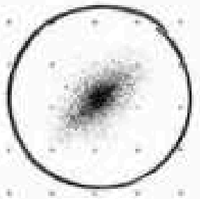
$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$

$$\Omega_m + \Omega_\Lambda + \Omega_k = 1$$

¶ 4.17

A Closing Exercise: Look Up

Action: On a dark autumn night in the northern hemisphere, find the Andromeda galaxy. As you look at that faint smudge, remember its stars are moving as if held by an immense, invisible halo of mass far larger than what your eyes can see.



Andromeda Galaxy

Next: In Lecture 5, we turn to Dark Energy and the ultimate fate of the universe.