¶ 1.1

MATH 101 B: Lecture 1

Variables, Expressions, and the Language of Algebra

At a Glance:

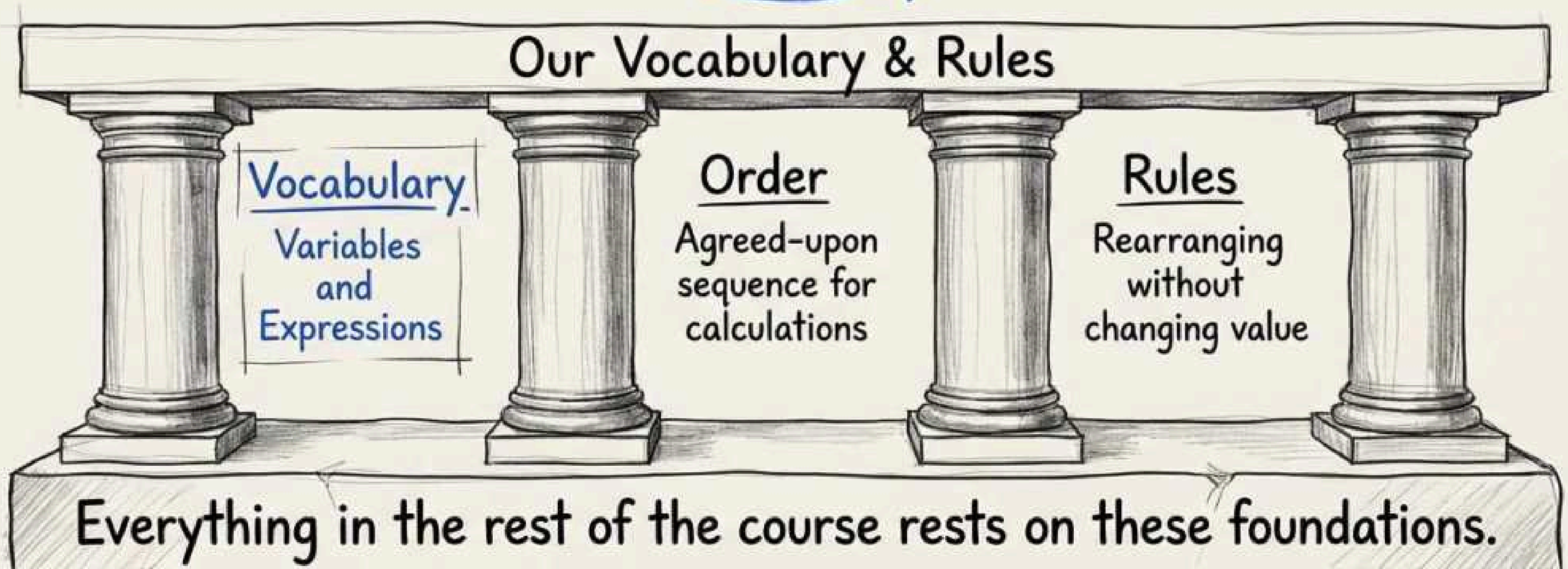
- How letters stand for numbers
- How to read, write, and simplify algebraic expressions



¶ 1.2

The Simple Idea: Use a letter to stand for a number.

$$A = 5$$





5-Mile Ride:	$3 + ($2 × 5) =	$13
10-Mile Ride:	$3 + ($2 × 10) =	$23

The calculation remains exactly the same; only the number of miles changes.

¶ 1.4

Variable (m)
A letter that stands for different numbers.
Let m = number of miles.

$$\text{Cost} = 3 + 2m$$

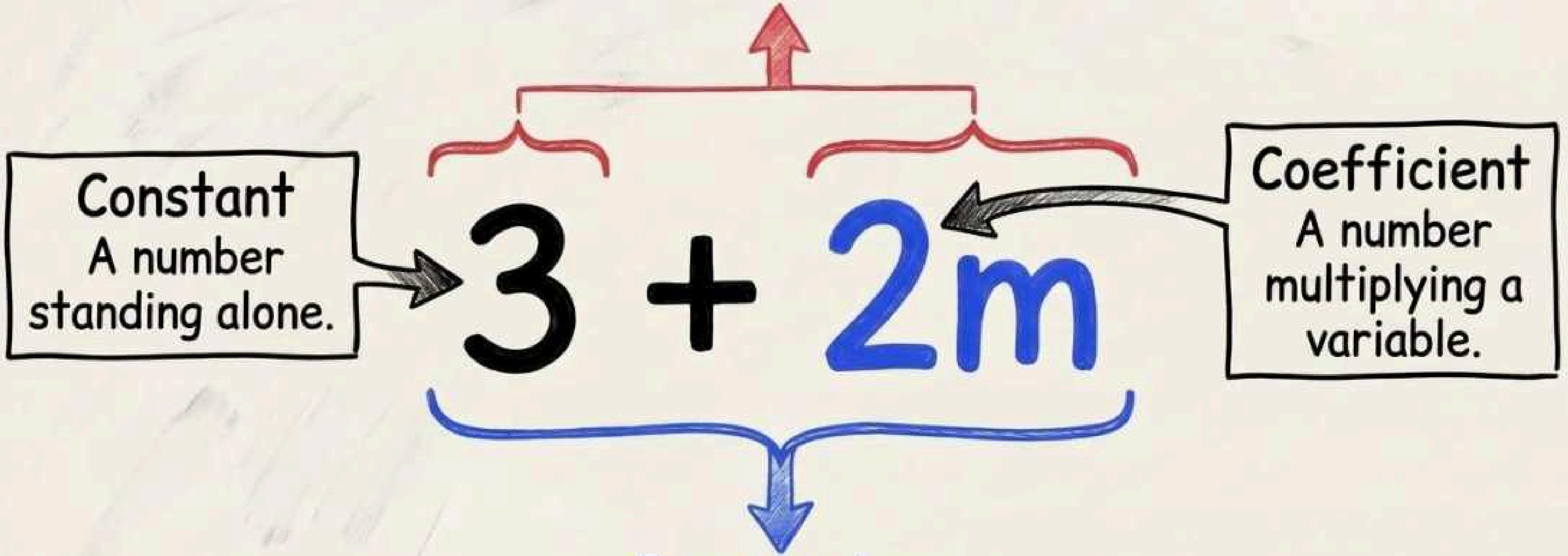
Two and m side-by-side means 2 × m.

Algebra captures sameness. This one short short phrase describes every possible ride.

¶ 1.5

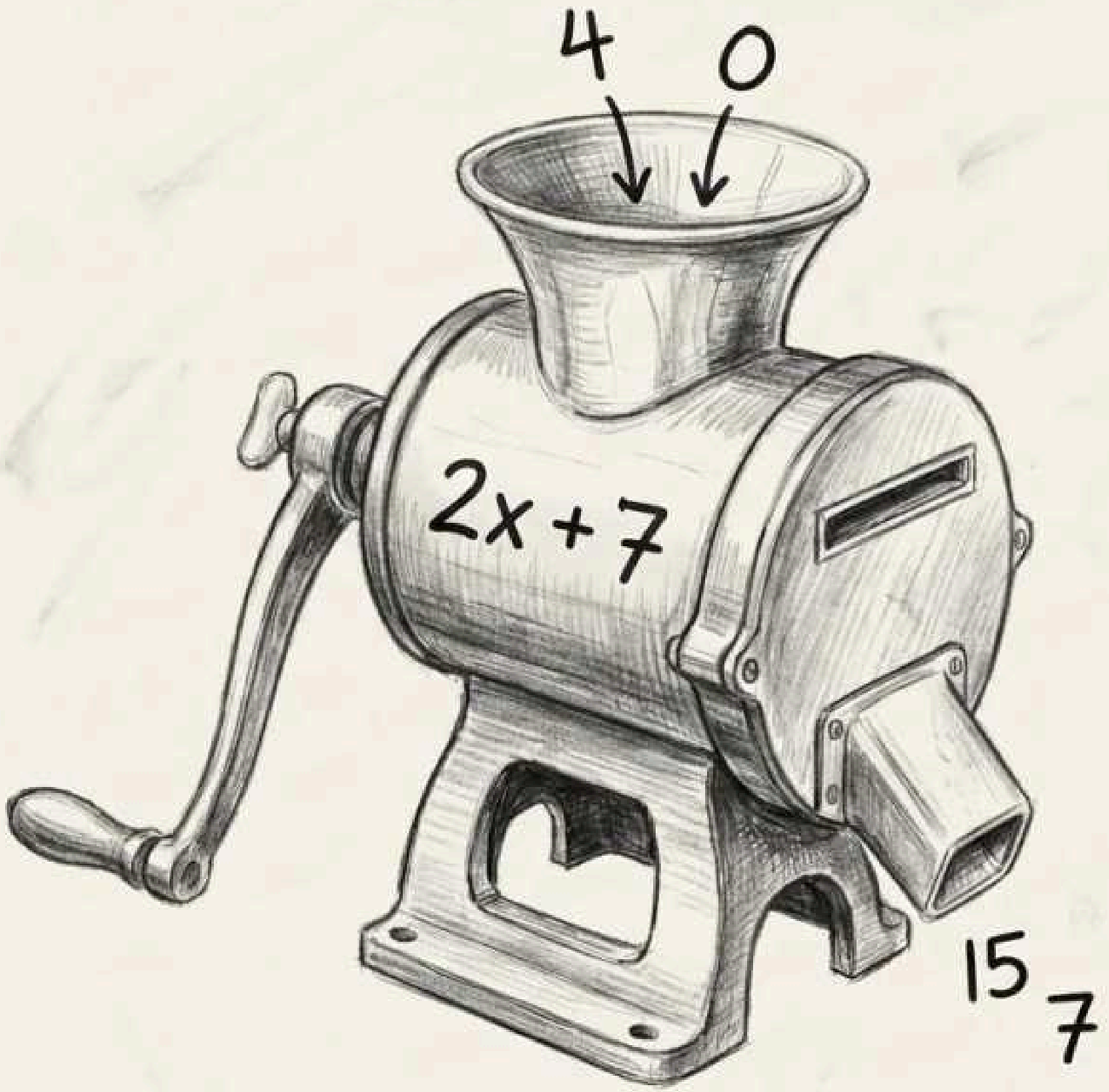
Terms

Parts joined by + or − signs.



Expression

Makes no claim; neither true nor false.
It just combines numbers, variables, and operations.



Evaluating Expressions: The Recipe

Input $x = 4$ → $2(4) + 7 = 8 + 7$ → Output 15

Input $x = 0$ → $2(0) + 7 = 0 + 7$ → Output 7

To evaluate means to replace the variable with a number and calculate.

Algebra lets you write the recipe once, and use it for any number.

¶1.7

When an expression has several operations, we need an agreement.

The agreed convention is 11, because multiplication is done before addition.

Without it, different people get different answers.

Add First

$(3+4) \times 2$

$\rightarrow 7 \times 2$

$= 14$

Multiply First

$3 + (4 \times 2)$

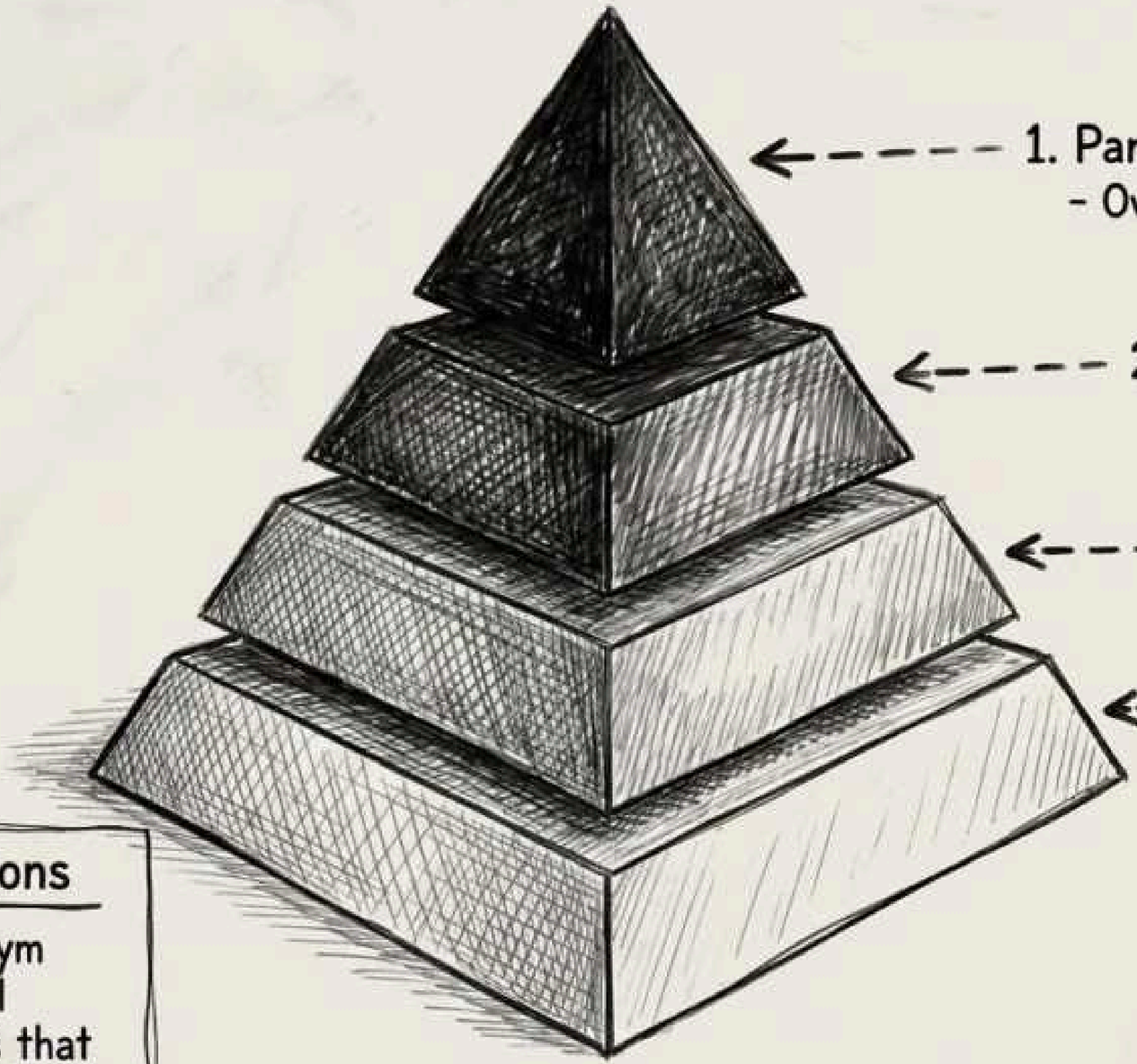
$\rightarrow 3 + 8$

$= 11$

The Problem:

$3 + 4 \times 2$

¶ 1.8



1. Parentheses (P)
   - Overrides everything.

2. Exponents (E)
   - Handled second.

3. Multiply & Divide (M/D)
   - Working left to right.

4. Add & Subtract (A/S)
   - Working left to right.

The Order of Operations

While many use the acronym PEMDAS, the fundamental concept is that operations that bind more tightly happen first.

Evaluate: $6+2(7-3)^2$

¶ 1.9

$6+2(7-3)^2$ Step 1 (Parentheses): $7-3=4$

$6+2(4)^2$ Step 2 (Squaring): $4^2=16$

$6+2(16)$ Step 3 (Multiplying): $2\times16=32$

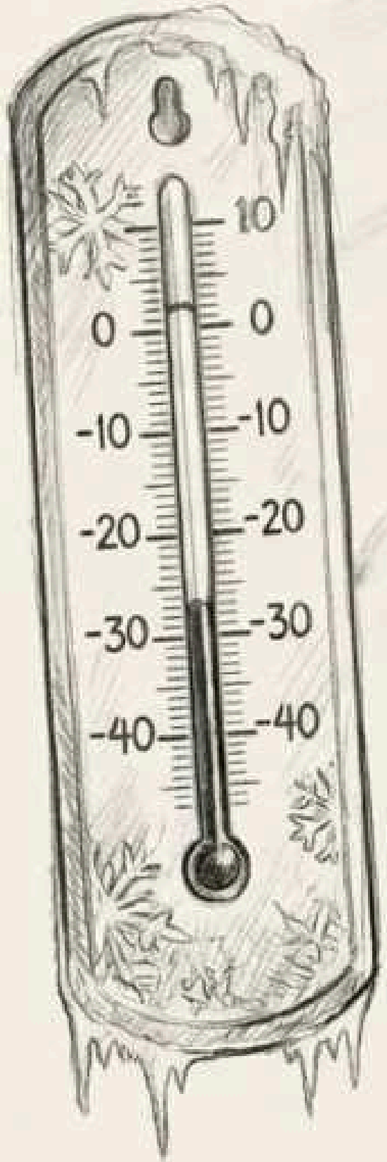
$6+32$ Step 4 (Adding)

↓

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Like an experienced cook seeing the stages of a recipe, you are seeing the layers of an expression.

Rules of Care



Subtracting a number = adding its opposite:

$5 - 8 = 5 + (-8) = -3$

Negative × Negative = Positive:

$(-3) \times (-4) = 12$

Evaluation Tracker

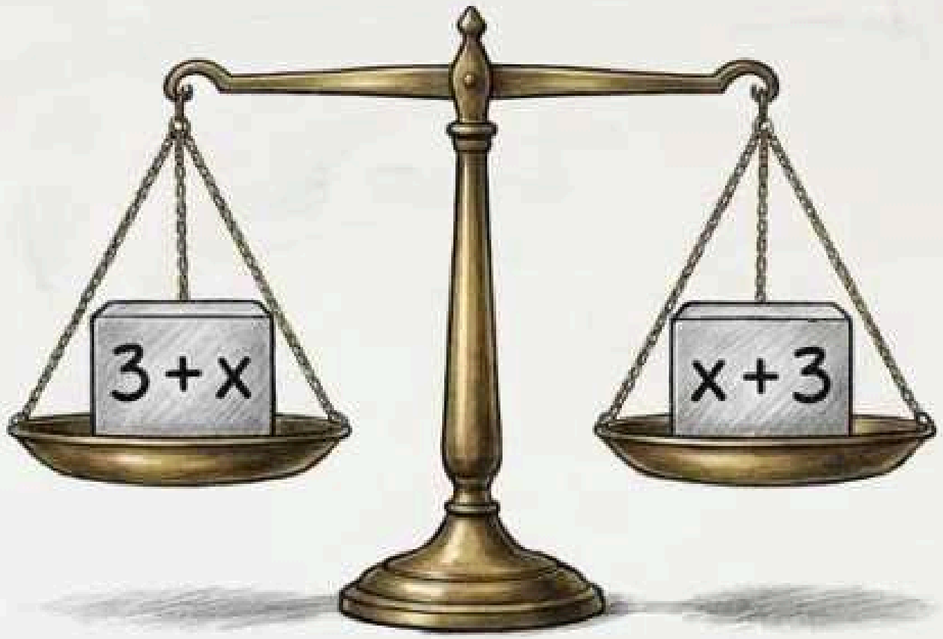
¶ 1.10

Evaluate $3x^2 - x + 1$ when $x = -2$

- Square first: $(-2)^2 = 4$ (not -4)
- Multiply: $3 \times 4 = 12$
- Subtract negative: $12 - (-2)$
  $= 12 + 2 = 14$
- Add one: $14 + 1 = 15$

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¶ 1.11



Commutative Rule: Order doesn't matter for addition or multiplication.

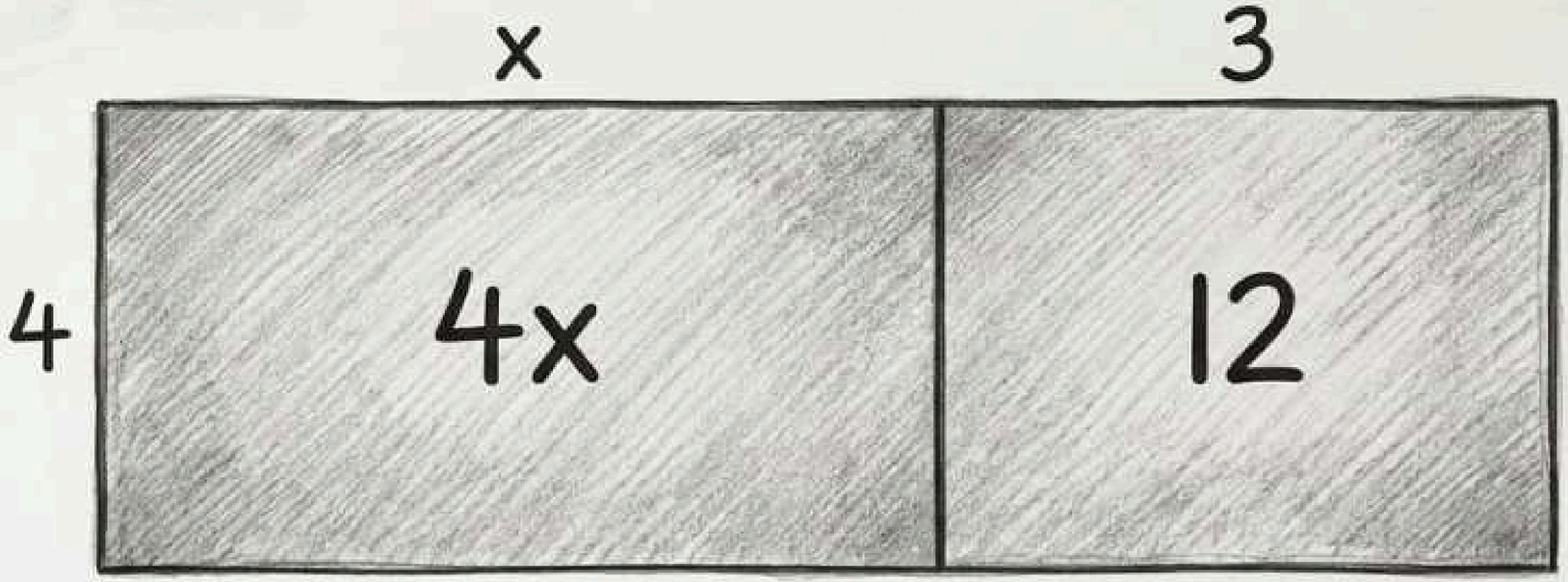
Associative Rule: Regrouping a sum or product freely doesn't change the value.



Warning: Neither rule works for subtraction or division ($10 - 4 \neq 4 - 10$).

The Distributive Property (The Workhorse of Algebra)

¶1.12



Rule: A number multiplying a sum multiplies each part of the sum.

Addition: $4(x+3) = 4x+12$

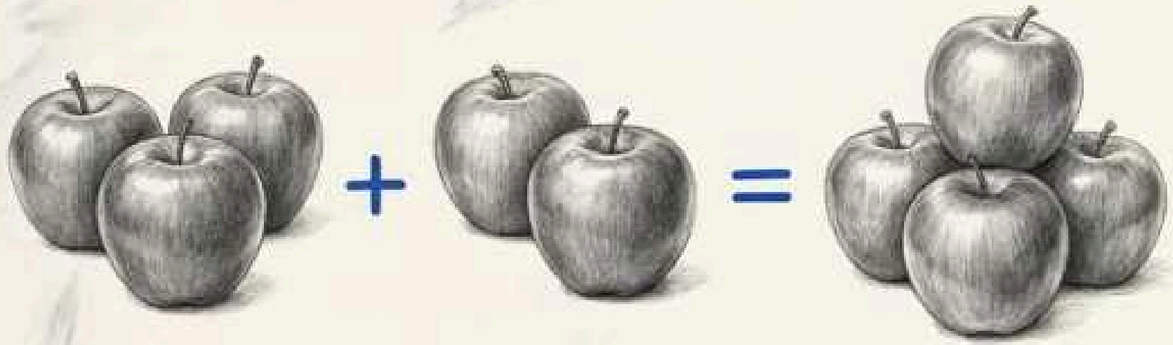
Subtraction: $3(2x-5) = 6x-15$

¶ 1.13

Combining Like Terms

Like Terms

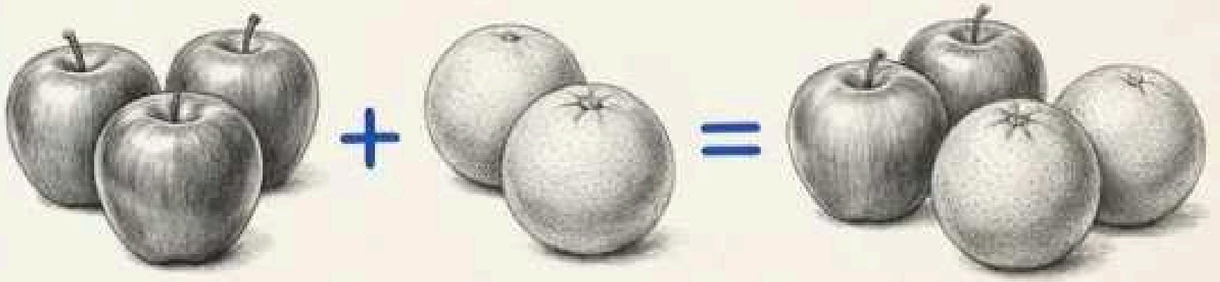
Exactly the same variable part.
Combine by adding coefficients.



$3x + 2x = 5x$

Unlike Terms

Like apples and oranges;
cannot be combined.



$3x$ and $2x^2$ are unlike.

Constants combine only with other constants.

Basic Simplification

$$3x + 5 + 2x - 1$$

- Group variables: $3x + 2x = 5x$
- Group constants: $5 - 1 = 4$

$5x + 4$

Harder Case (Distribute & Combine)

$$2(x+6) - 3x$$

- Distribute: $2x + 12 - 3x$

$12 - x$

¶ 1.15



Trap 1: Distributing a Negative Sign

$$-(x+4)$$

~~$-x+4$~~ $-x-4$

The minus sign multiplies everything inside.

Trap 2: Forgetting to Multiply Both Terms

$$2(x+3)$$

~~$2x+3$~~ $2x+6$

The two must multiply both terms.

The Habit: Test with a number to check your own work without an answer key. If expressions are truly equivalent, they agree for every value.

Does $2(x+3) = 2x + 3$? Let $x=1$.

Left Side

$2(1+3)$

$= 2(4)$

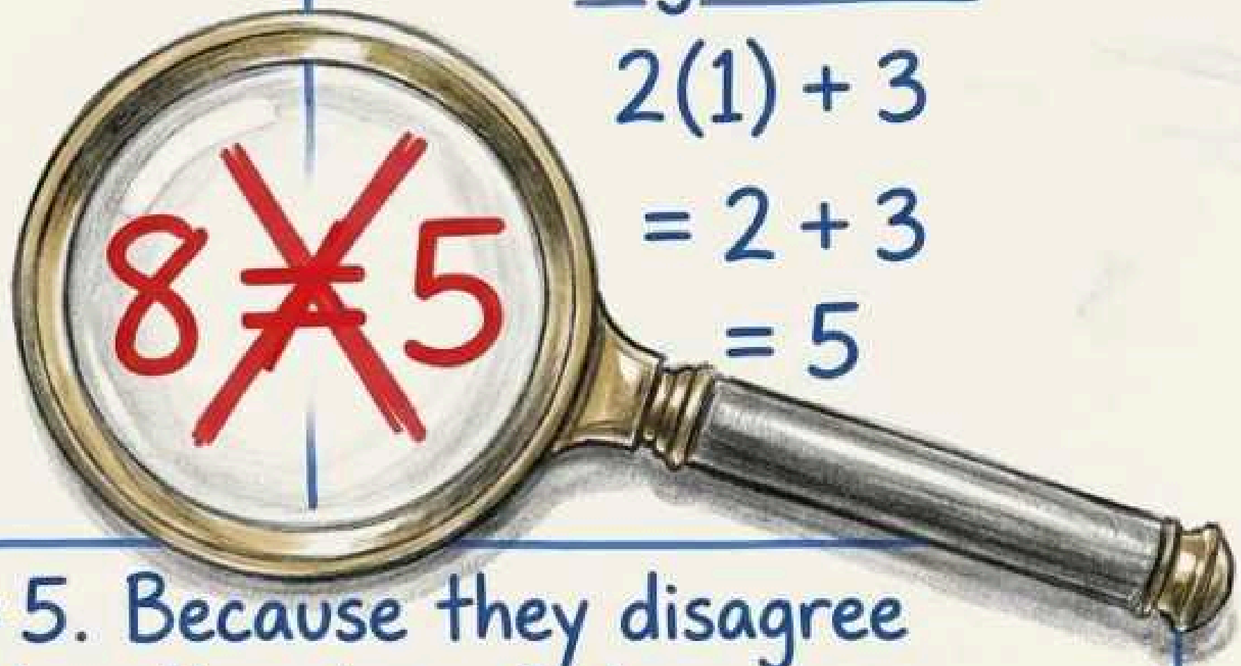
$= 8$

Right Side

$2(1) + 3$

$= 2 + 3$

$= 5$



Conclusion: $8 \neq 5$. Because they disagree for even <u>one</u> value, the claim fails.

The Takeaway Question:

Think of a calculation you do repeatedly.

Reflect:

- Can you write it as an expression with one variable?
- What does the variable stand for?
- What do the constants and coefficients mean in real life?



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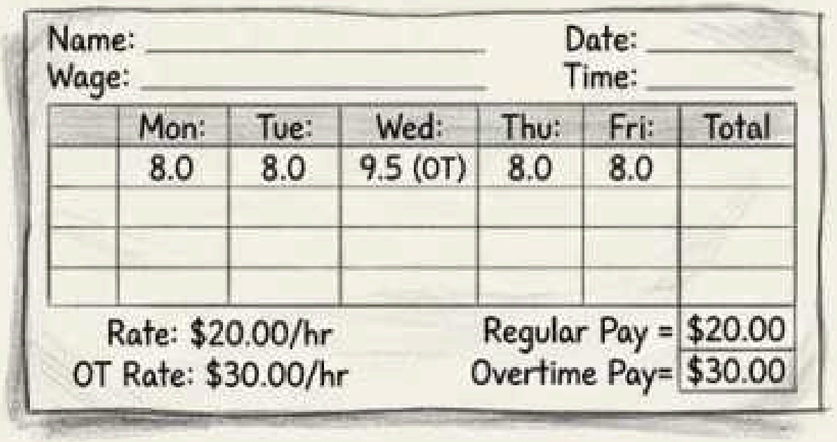
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1	Soda	$2.00
1	Soda	2.50
1	Powder Corns	5.00
1	Cocasn Tisp	4.50
1	Fres	$3.00
2	Over time	$2.00
1	Total	$2.00
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Tax: $2.00
Tip (20%): $5.40
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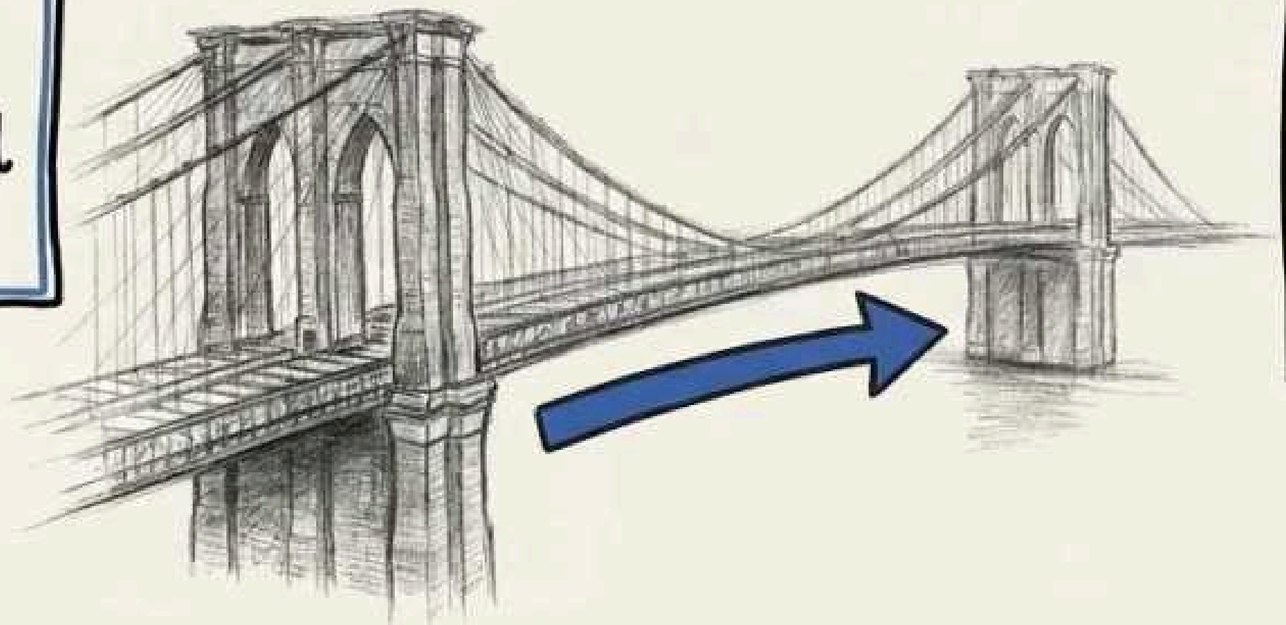
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Rate: $20.00/hr — Regular Pay = $20.00
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EXPRESSION

$(3x + 2)$

A mathematical recipe.



EQUATION

$(3x + 2 = 11)$

A complete mathematical statement.

Looking Ahead to Lecture 2:
We move from expressions to equations.
We will learn how to find the
specific number that makes a
mathematical statement true.