Solving Linear Equations and Inequalities

Lecture 2

$$3x + 7 = 22$$



$$x = 5$$

Undoing operations step by step.

Expression (The Phrase)

$$3x + 7$$

Equation (The Sentence)

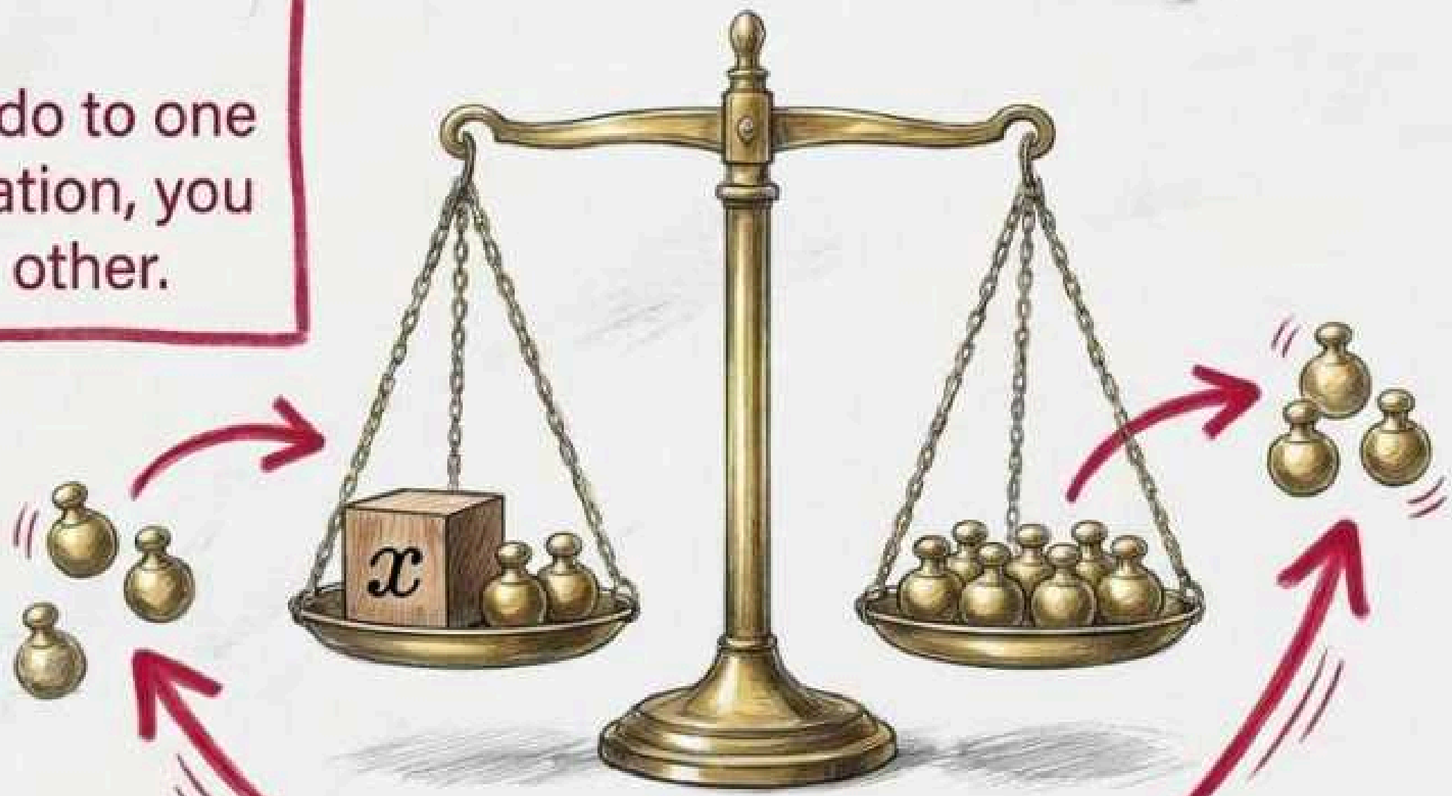
$$3x + 7 = 22$$

Solve: Find the value of the variable that makes the sentence true.

Note: Extends to inequalities, where the sentence says one side is bigger than the other.

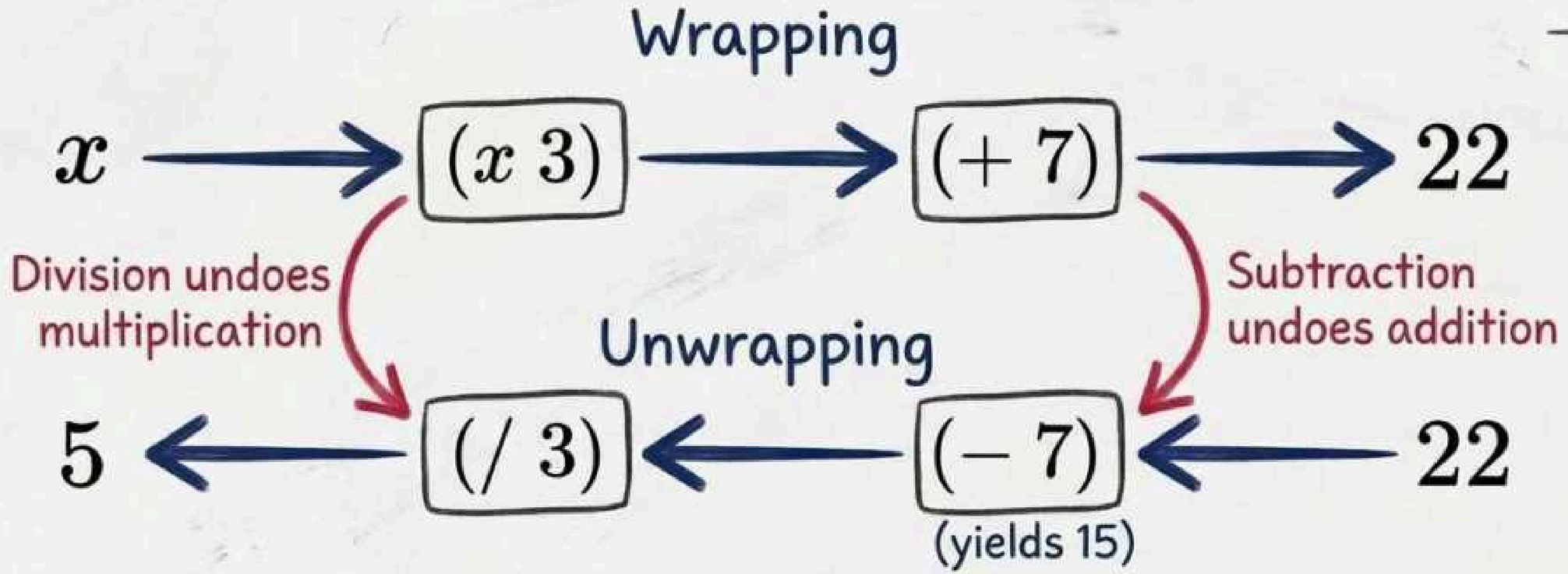
Golden Rule:

Whatever you do to one side of an equation, you must do to the other.



$$x + 3 = 8$$

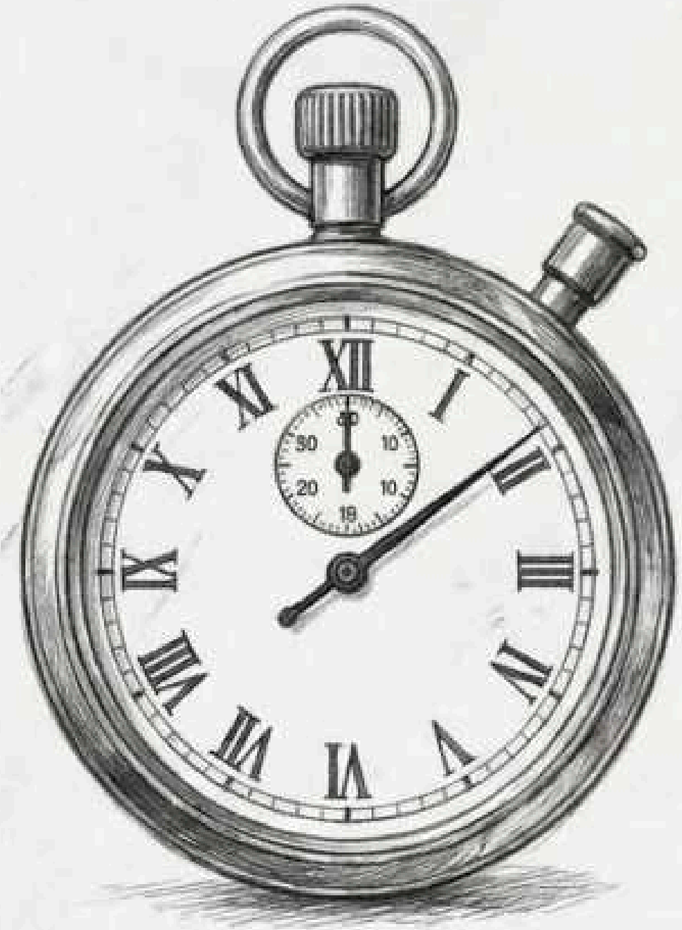
$$x = 5$$



$$3x + 7 = 22$$

$$3x = 15$$

$$x = 5$$



The 10-Second Verification.

Catches almost every mistake. The great strength of algebra is you can always verify your own answer.

Original: $3x + 7 = 22$

Substitute: $3(5) + 7 = 22$

Calculate: $15 + 7 = 22$

Result: $22 = 22$



Subtract 2x

$$6x - 5 = 2x + 11$$

$$4x - 5 = 11$$

Add 5

$$4x = 16$$

Divide by 4

$$x = 4$$

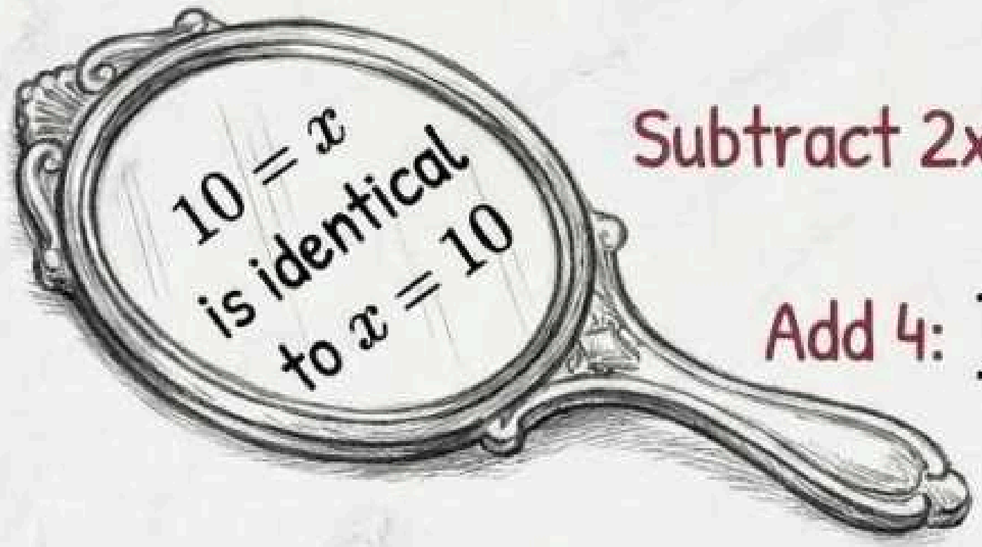
Check: Left (24 – 5 = 19) balances
Right (8 + 11 = 19).

$$2(x+3) = 3x - 4$$

$$2x + 6 = 3x - 4$$

Subtract 2x: $6 = x - 4$

Add 4: $10 = x$



Check: 2(13) = 26 and 30 - 4 = 26

$\frac{x}{3} + 2 = 7 \rightarrow \frac{x}{3} = 5 \rightarrow x = 15$

$$\left[\frac{x}{2} + \frac{x}{3} = 10\right] \quad [\times 6]$$

$$3x + 2x = 60$$

$$\downarrow$$

$$5x = 60$$

$$\downarrow$$

$$x = 12$$

Multiply every term by a common denominator first to eliminate fractions.

The algebra tells you which case you are in.

$$2x + 3 = 2x + 5$$

$$3 = 5$$

False whatever x is

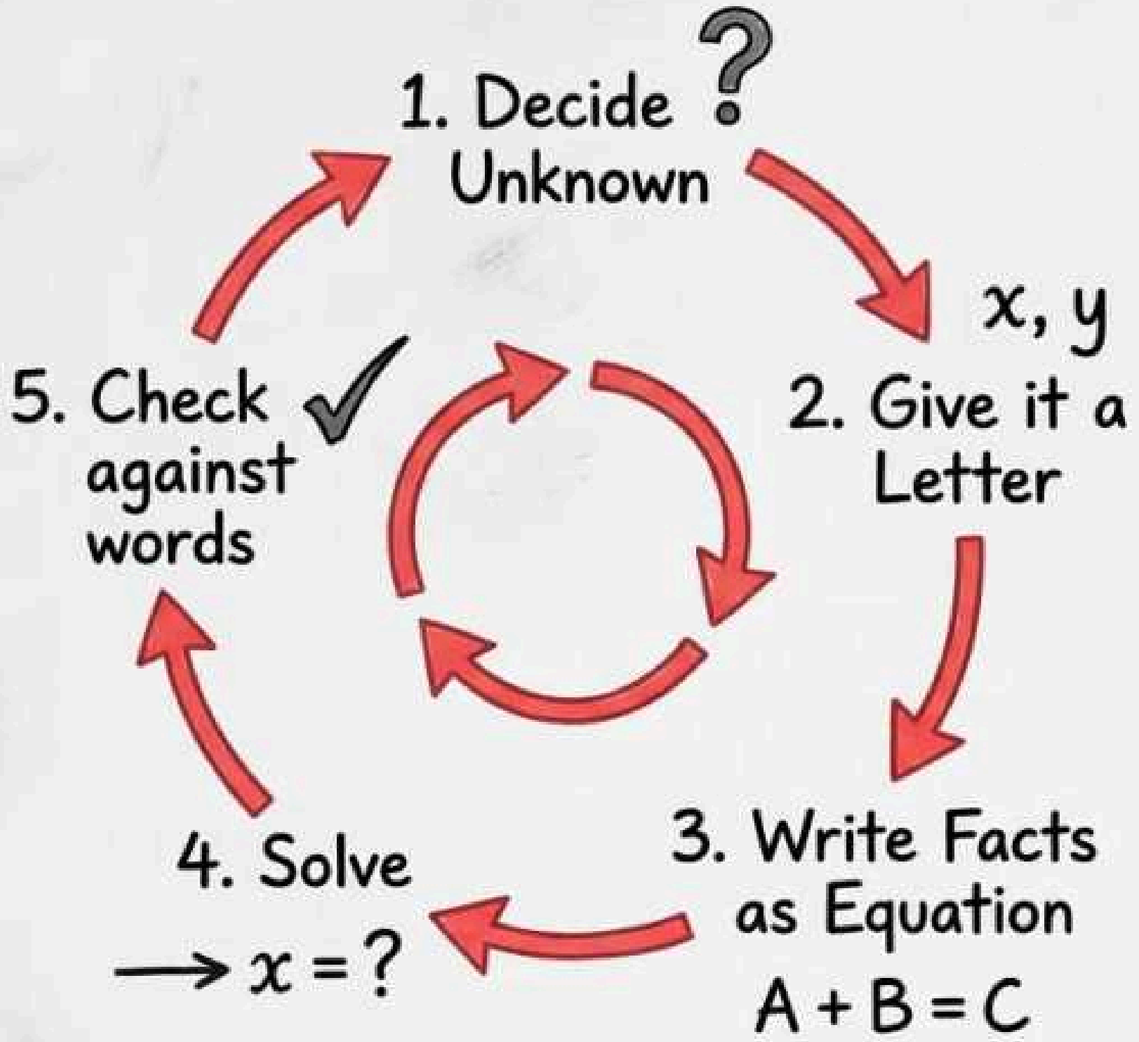
→ No Solution.

$$2(x + 1) = 2x + 2$$

$$2x + 2 = 2x + 2$$

True for every x

→ Infinite Solutions.





Two friends spend $46 total. One spends $6 more than the other.

Let $x =$ the smaller amount.

$$x + (x + 6) = 46$$

$$2x = 40$$

$$x = 20$$

Conclusion: The friends spent $20 and $26.

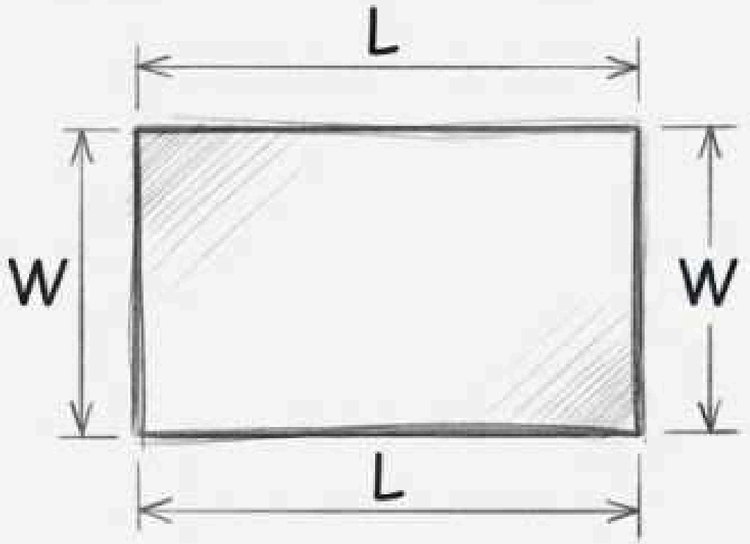


A plan costs $20/month plus $5/GB.
Budget is $65. How many GBs?

$$20 + 5g = 65$$

$$20 + 5g = 65$$
$$5g = 45$$
$$g = 9$$

Check: 9 gigabytes x $5 = $45.
Add $20 base = $65 total.



Solve for W

$$P = 2L + 2W$$

Subtract $2L$

$$P - 2L = 2W$$

Divide by 2

$$\frac{P - 2L}{2} = W$$

Test Case

Test: P=22, L=7

$$\rightarrow W = \frac{22 - 14}{2} \rightarrow W = 4$$

Symbol Matrix	Mini-matrix
$>$ (greater than)	$<$ (less than)
$\geq$ (at least)	$<=$ (at most)

$$3x - 5 > 10$$
$$3x > 15$$
$$x > 5$$

6 qualifies ✓
5.5 qualifies ✓
5 itself does not ✗

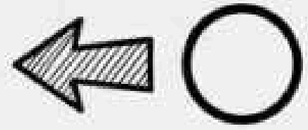
The solution is usually a whole range of numbers.

Remember:
$2 < 3$, but $-2 > -3$.

$$-2x \leq 8 \quad /-2$$

$$x > -4$$

Test $x = 0$ (inside set):	$0 < 8 \rightarrow$ True ✓
Test $x = -5$ (outside set):	$10 < 8 \rightarrow$ False, as it should be ✓


number itself is not included


number is included

$$-1 < 2x + 3 \leq 9$$

Subtract 3 throughout:

$$-4 < 2x \leq 6$$

Divide by 2 throughout:

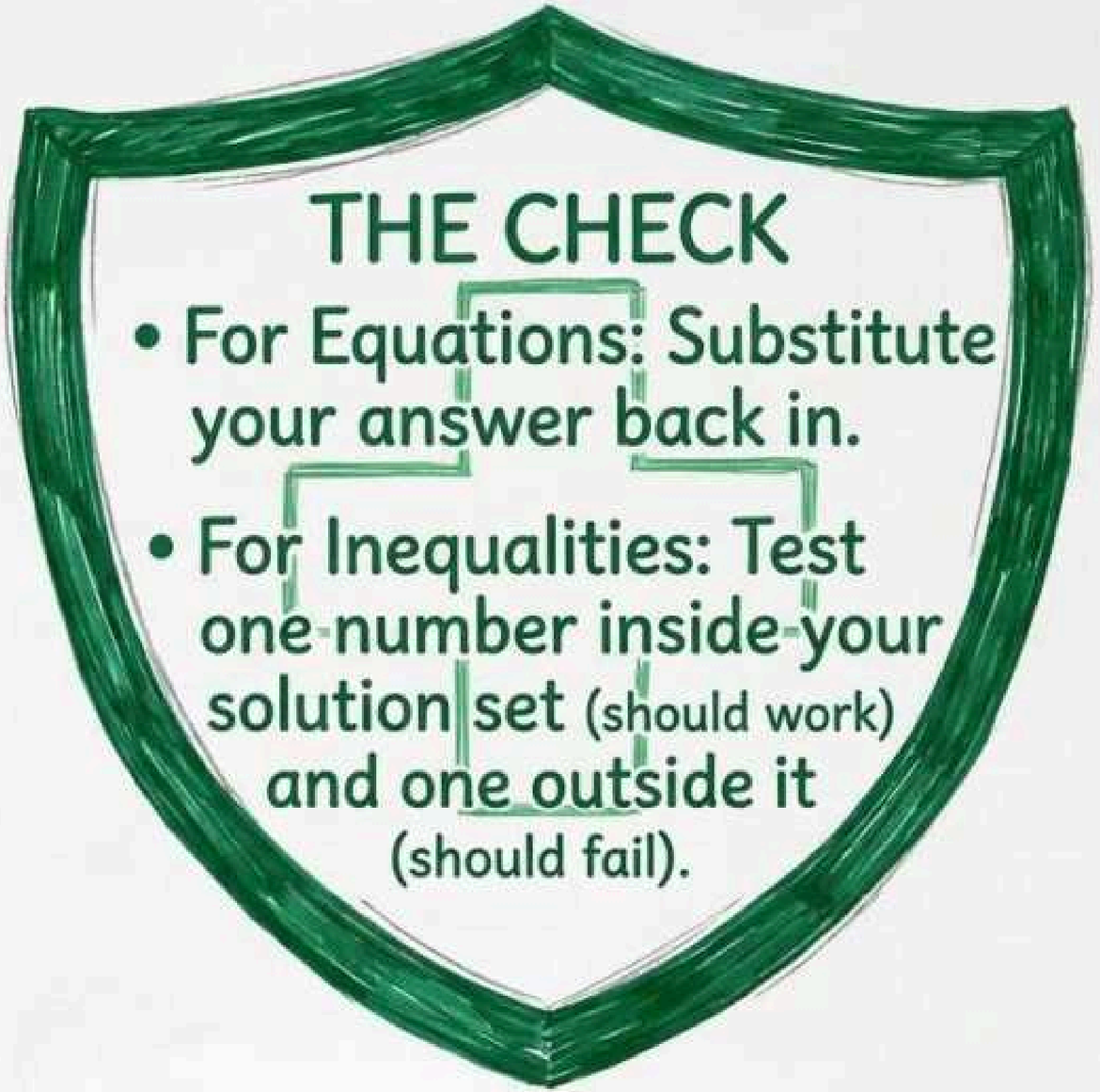
$$-2 < x \leq 3$$

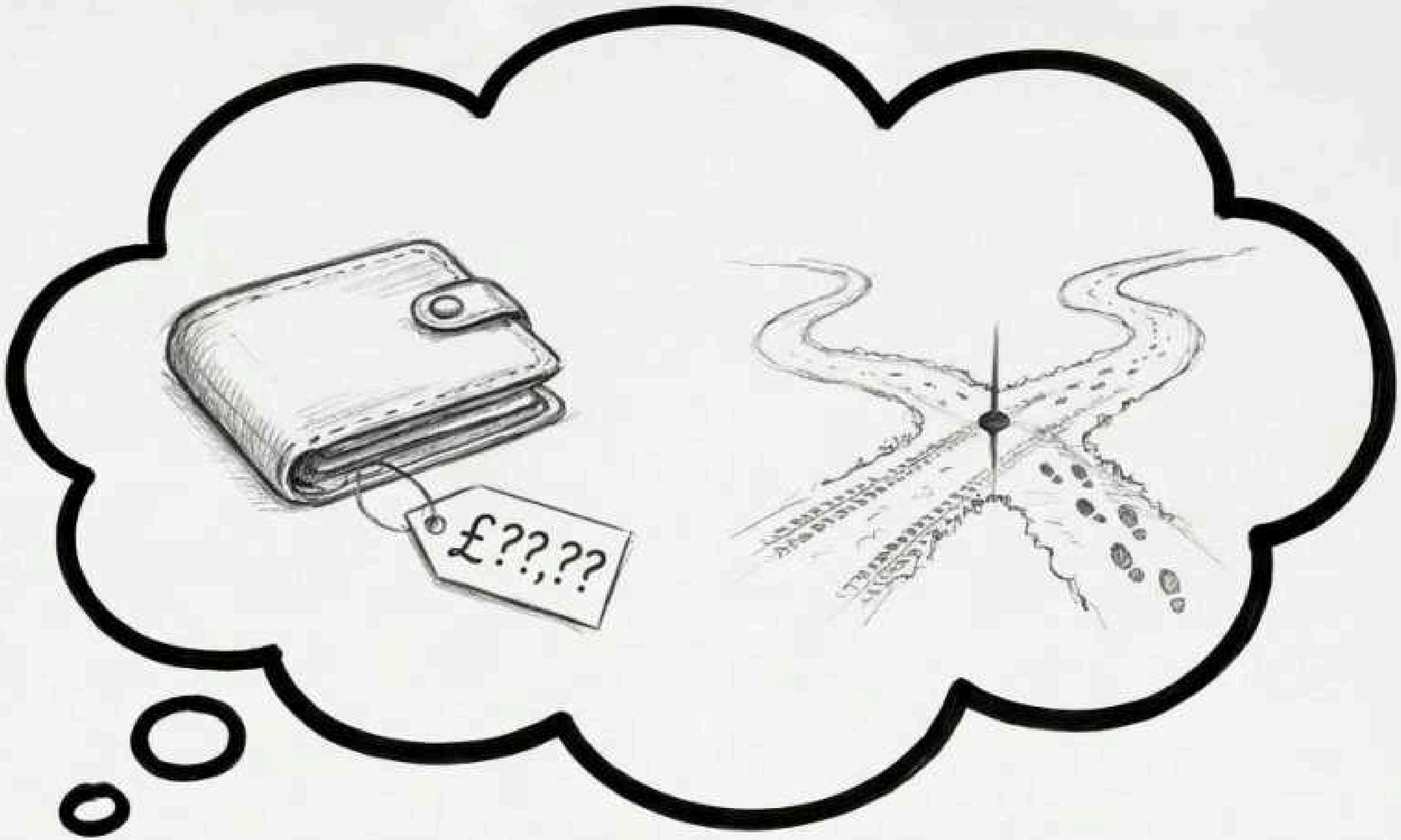


Predictable Mistakes

1. **Missed Terms**
   (forgetting to apply operations to every term)
2. **Forgot the Flip**
   (ignoring the negative division rule)
3. **Sign Errors**

The Antidote



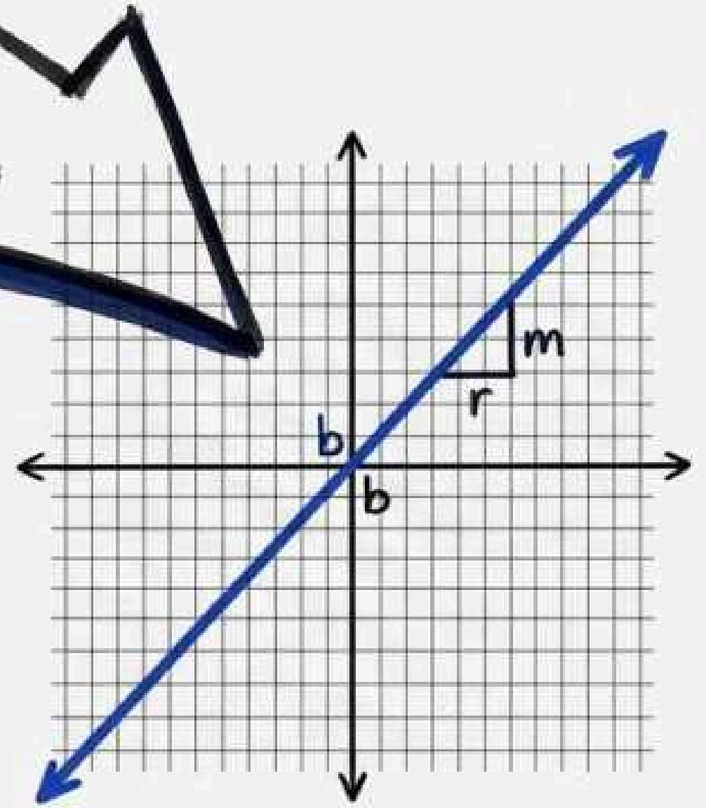


Think of a real decision. What is the unknown variable, and what does the solution tell you?

¶ 2.18

$y = mx + b$

y = m x + b



Looking Ahead to Lecture 3:
Turning equations into pictures.