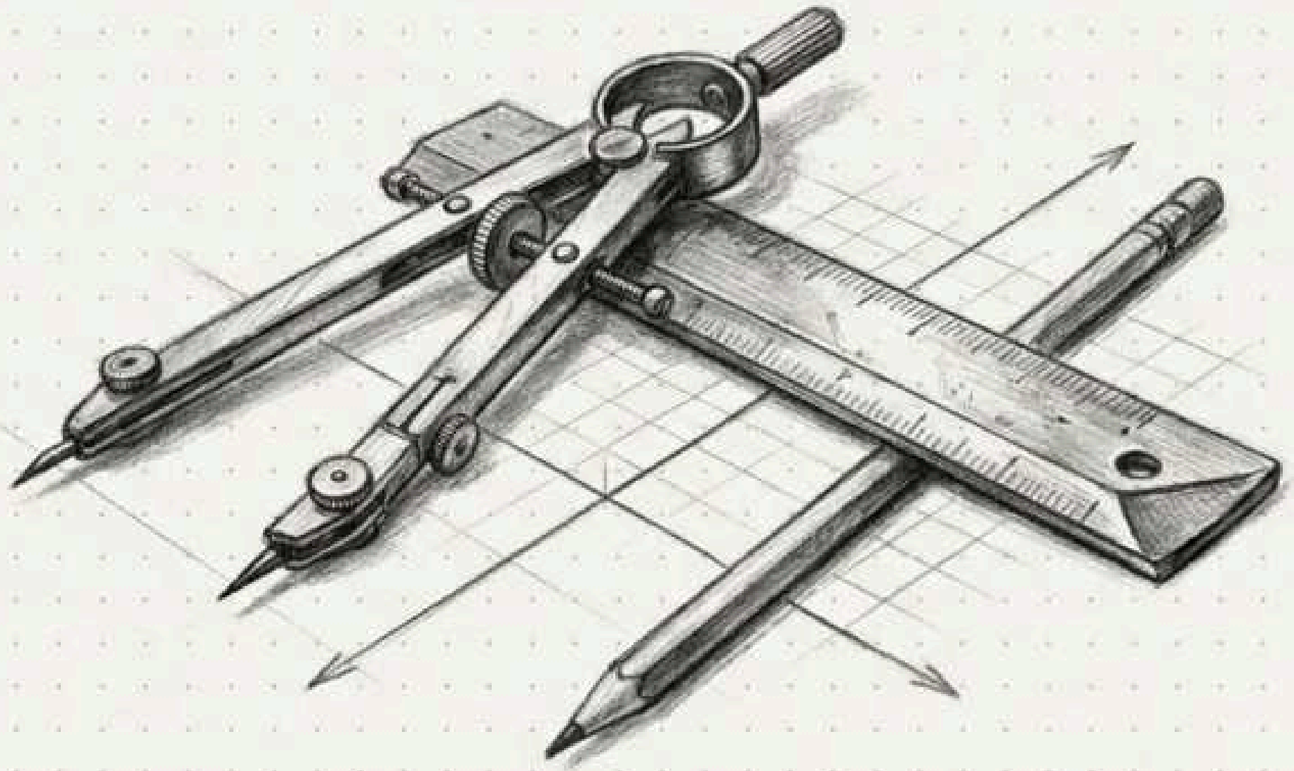
¶ 3.1

Lines: Slope, Intercepts, and Graphs

MATH 101 B — Lecture 3

At a Glance

- Describing a line (slope & intercepts)
- Graphing techniques
- Writing linear equations



¶ 3.2

Expanding from a single number to an infinite plane

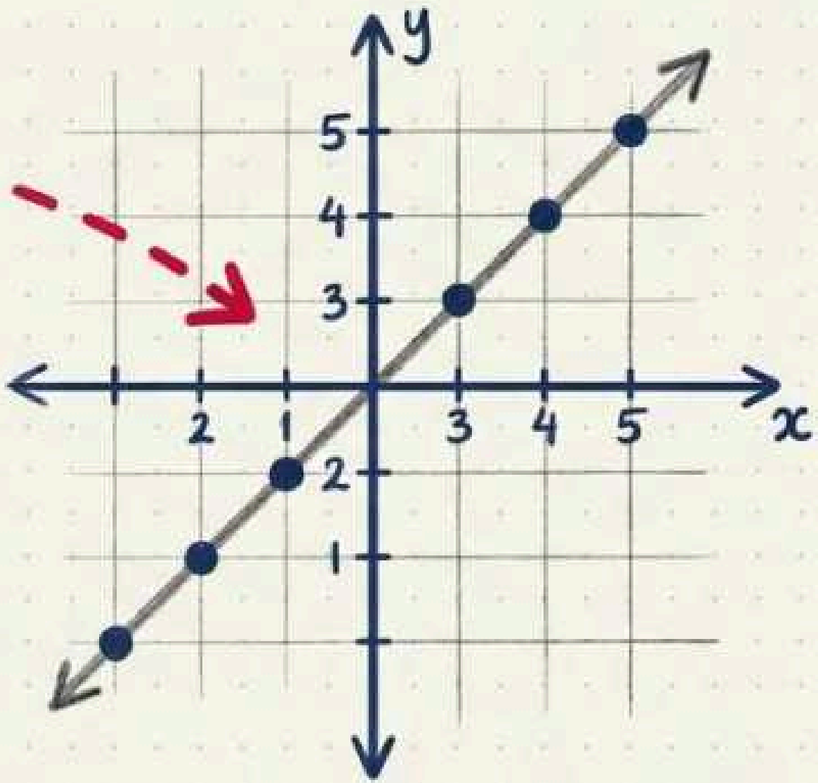
One Variable = Single Number



x = 4

(A point on a number line)

Two Variables = Pairs of Numbers



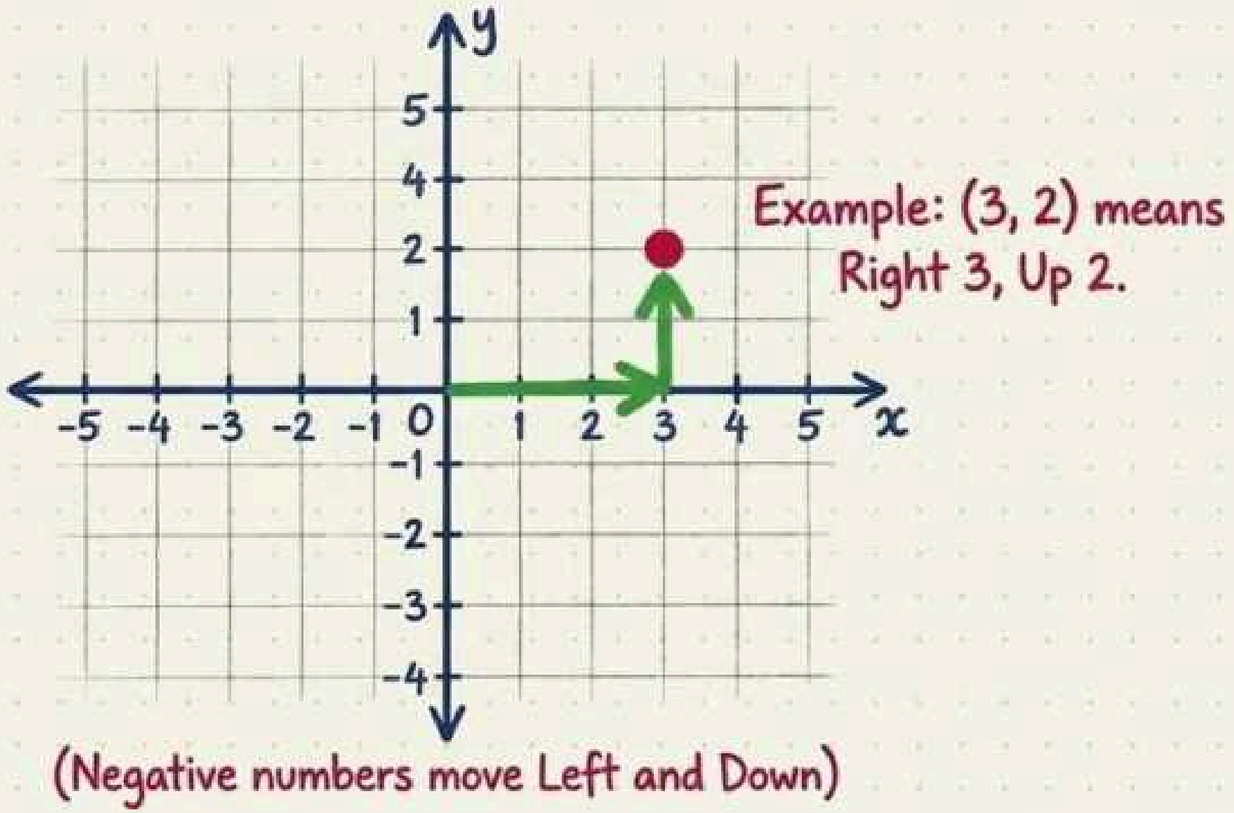
The Essence of a Line: Described perfectly by just two numbers—**Slope** & **Intercept.**

¶ 3.3

Navigating the Cartesian Coordinate Plane

x-axis (Horizontal) & y-axis (Vertical) meet at the Origin (0,0)

Ordered Pair: (x, y) — Right/Left first, then Up/Down.





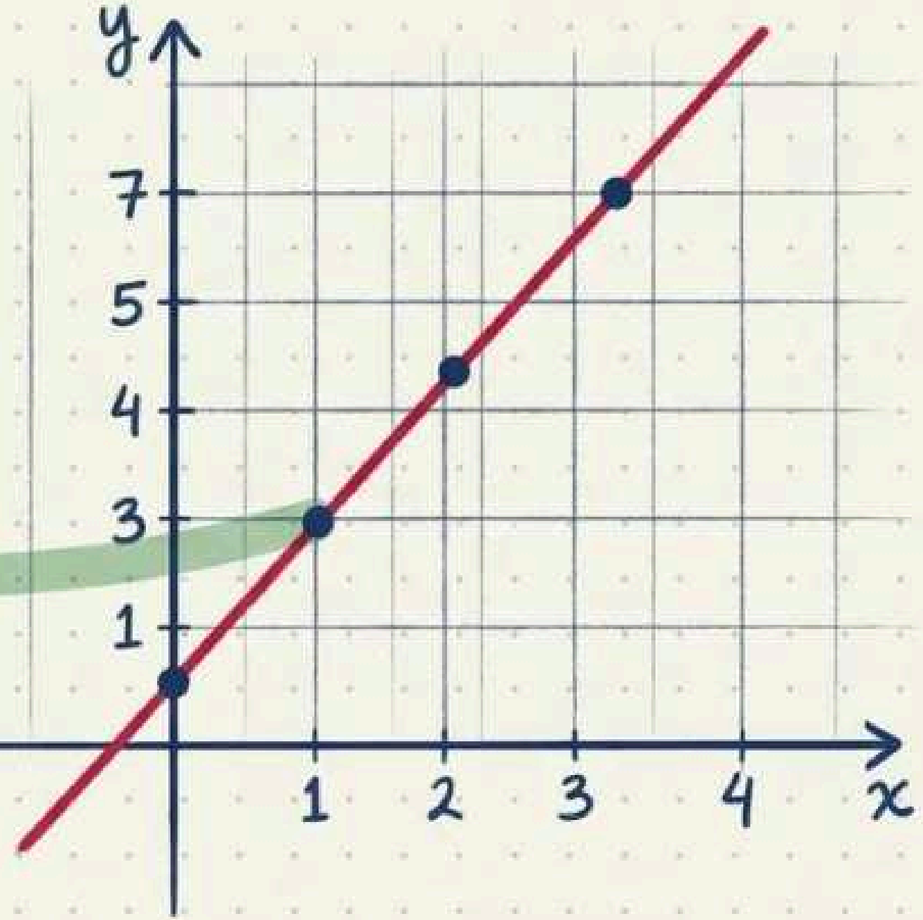
René Descartes: Joined algebra to geometry, 17th Century.

¶3.4

Every point on the line is a solution, and every solution is on the line

Equation: $y = 2x + 1$

x	y
0	1
1	3
2	5
3	7



A four-step blueprint for graphing linear equations

¶ 3.5

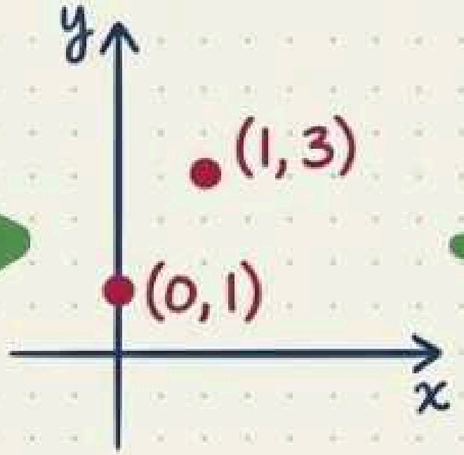
Step 1:
Pick values for x

x	y

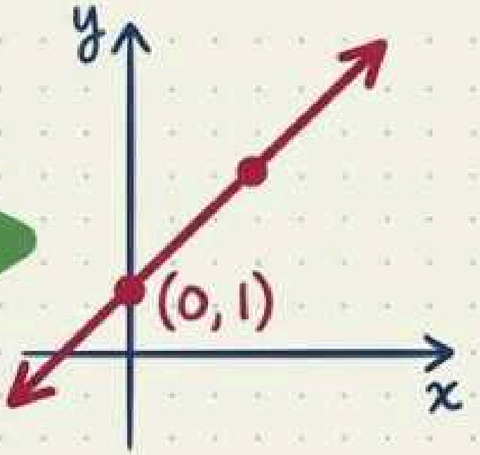
Step 2:
Compute y

x	y
0	1
1	3
2	5

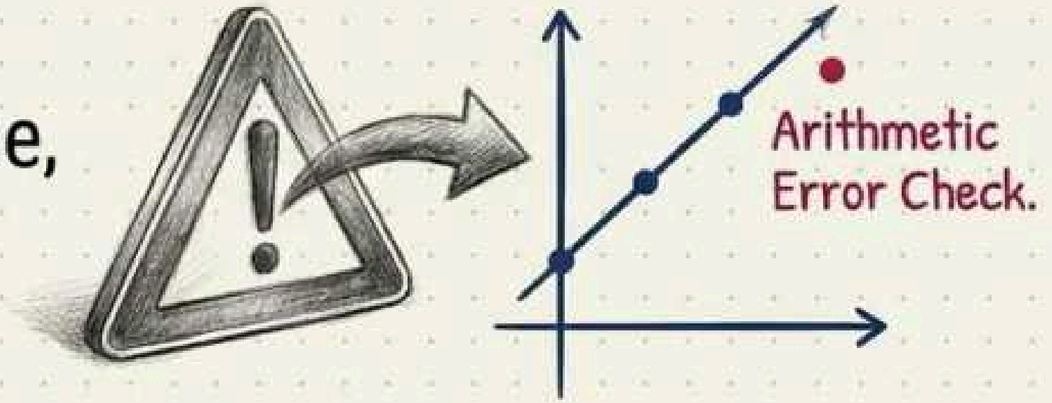
Step 3:
Plot the points



Step 4:
Draw the line



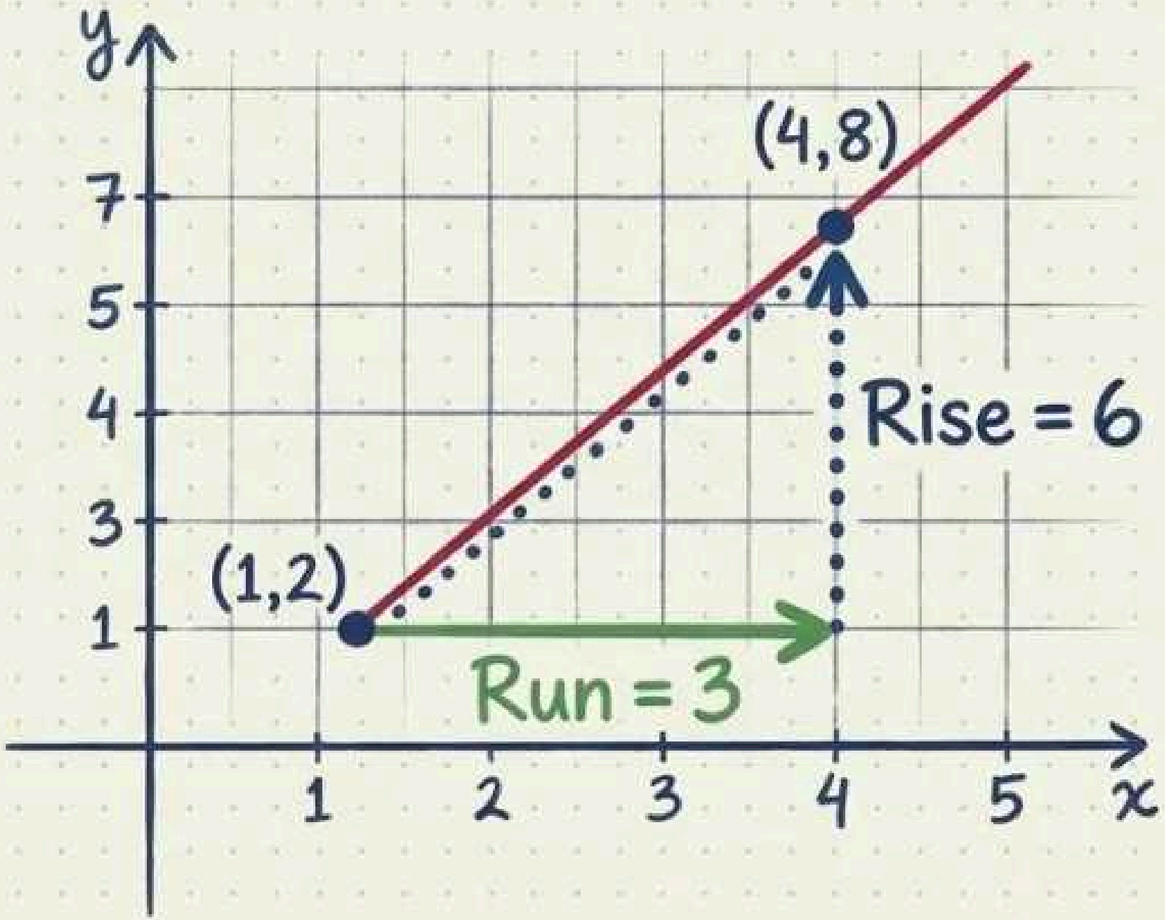
The **3-Point Check**: Two points fix a line, the third catches arithmetic errors.



¶ 3.6

Measuring steepness with Rise over Run

$$\text{SLOPE } (m) = \frac{\text{Rise}}{\text{Run}} = \frac{\Delta y}{\Delta x}$$



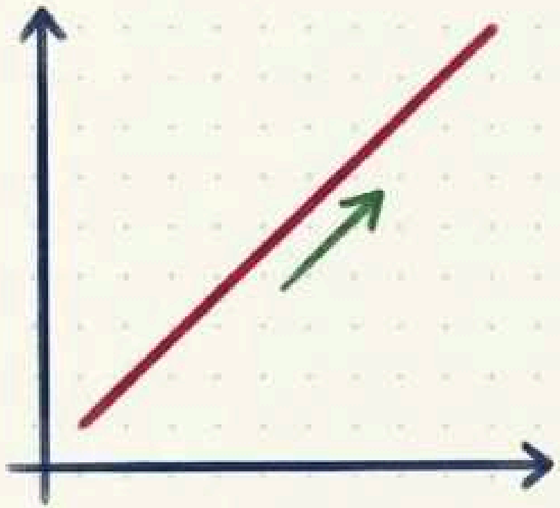
$$\text{Slope} = \frac{6}{3} = 2$$

For every step **right**,
the line climbs two.

¶ 3.7

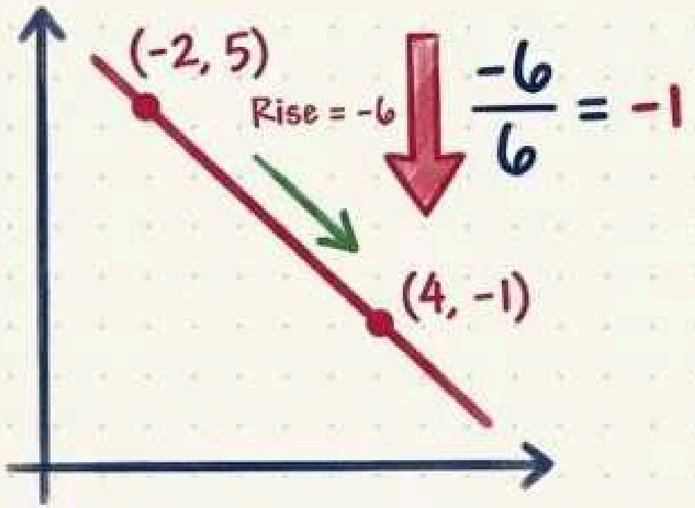
The sign of the slope dictates the direction of the line

Positive (+)



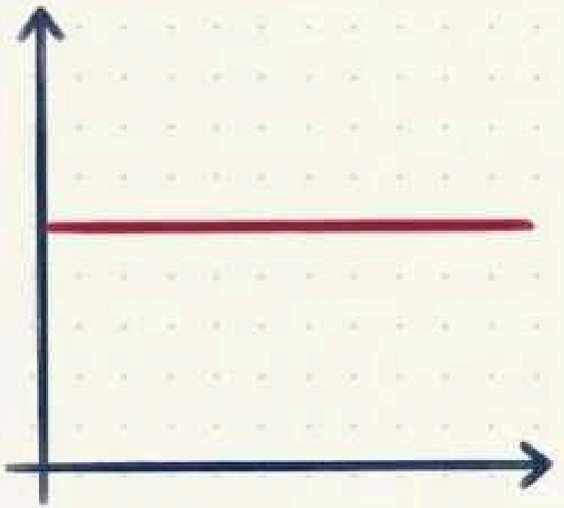
Positive:
Rises Left to Right

Negative (–)



Negative:
Falls Left to Right

Zero (0)



Zero:
Flat, Horizontal

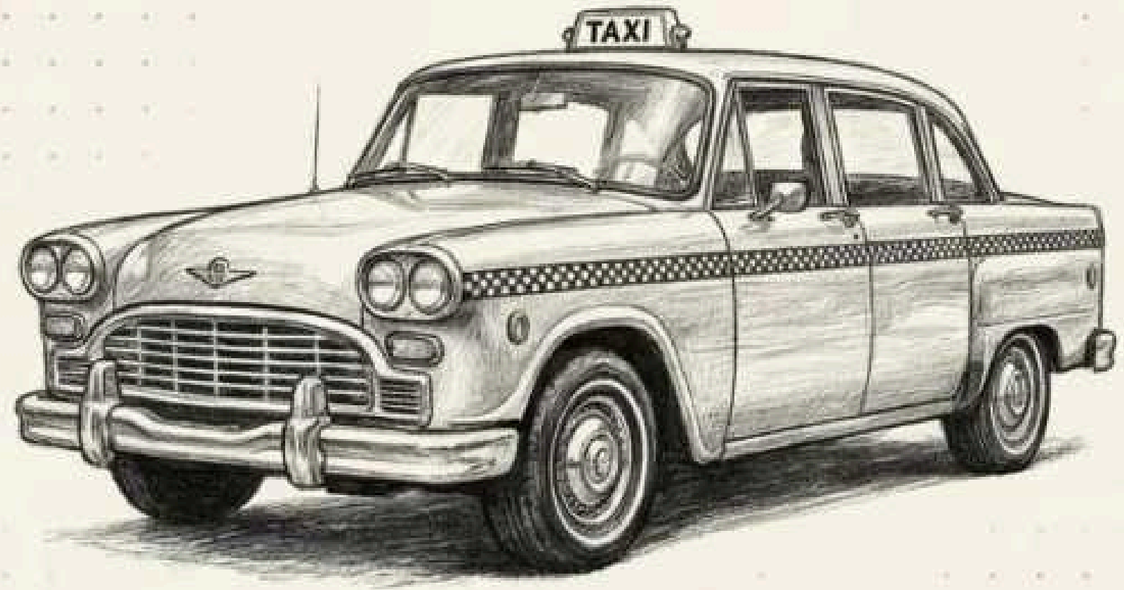
⚠ Caution: Always subtract coordinates in the same order, top and bottom.

Slope-intercept form translates directly to real-world costs

¶ 3.8

$$y = \overbrace{mx}^{\text{Rate/Slope}} + \overbrace{b}^{\text{Starting value/y-intercept where } x=0}$$

Abstract Math: $y = 2x + 1$ (Slope is 2, crosses at 1)



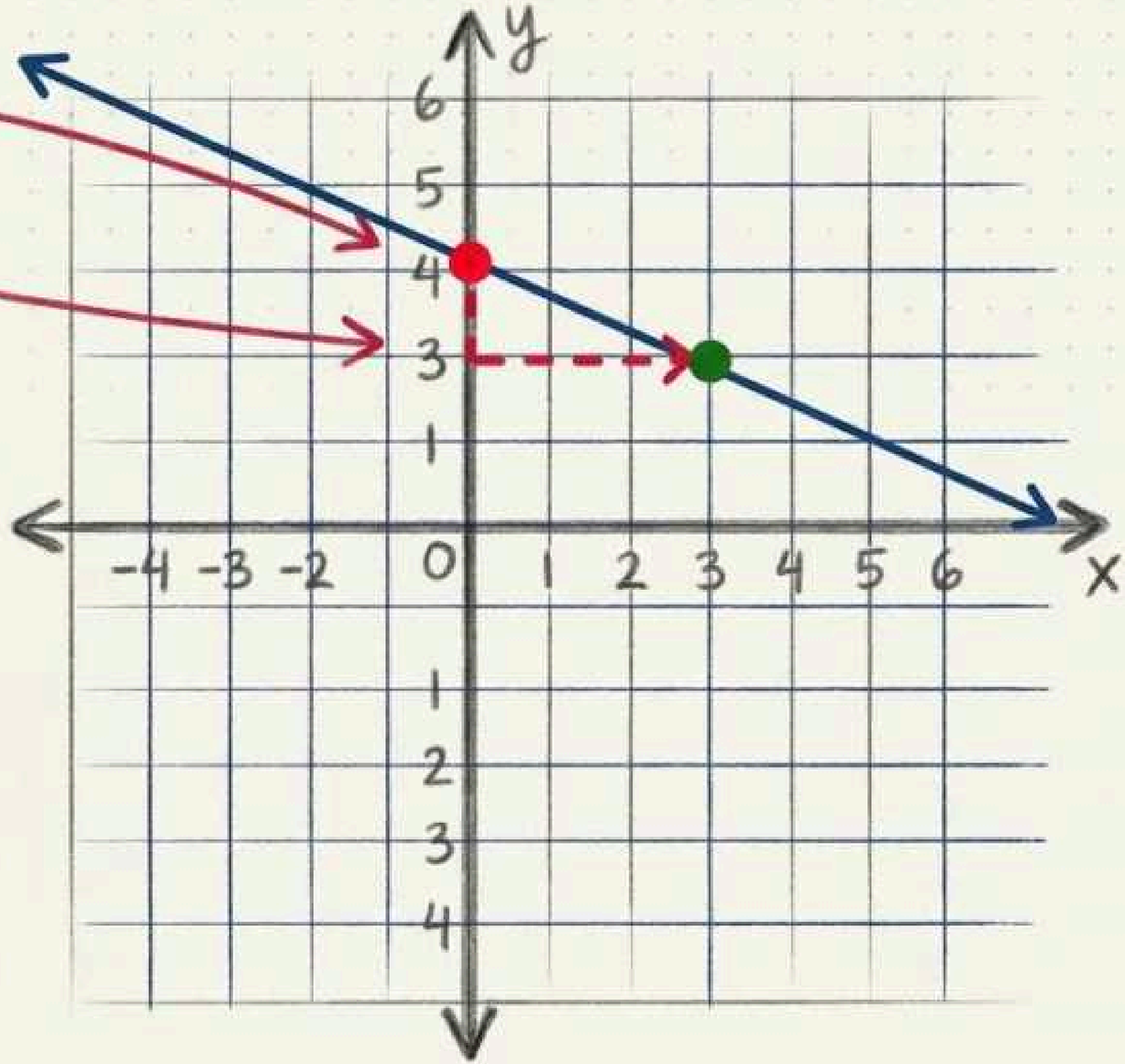
$$\text{Cost} = \underbrace{2m}_{\text{Price per mile (slope)}} + \underbrace{3}_{\text{Flat fee (intercept)}}$$

Graphing on the fly without a data table

¶ 3.9

$$y = -\frac{1}{2}x + 4$$

1. Start at intercept: (0, 4)
2. Use slope: Down 1, Right 2
3. Arrive at new point: (2, 3)



Finding intercepts by zeroing out variables in Standard Form

¶ 3.10

Standard Form: $3x + 2y = 12$

Slope check: Solving for y gives $y = -\frac{3}{2}x + 6$ (Rise -6, Run 4).

Algebra (Zeroing Out):

y-intercept (set x=0):

$\cancel{3x} + 2y = 12$

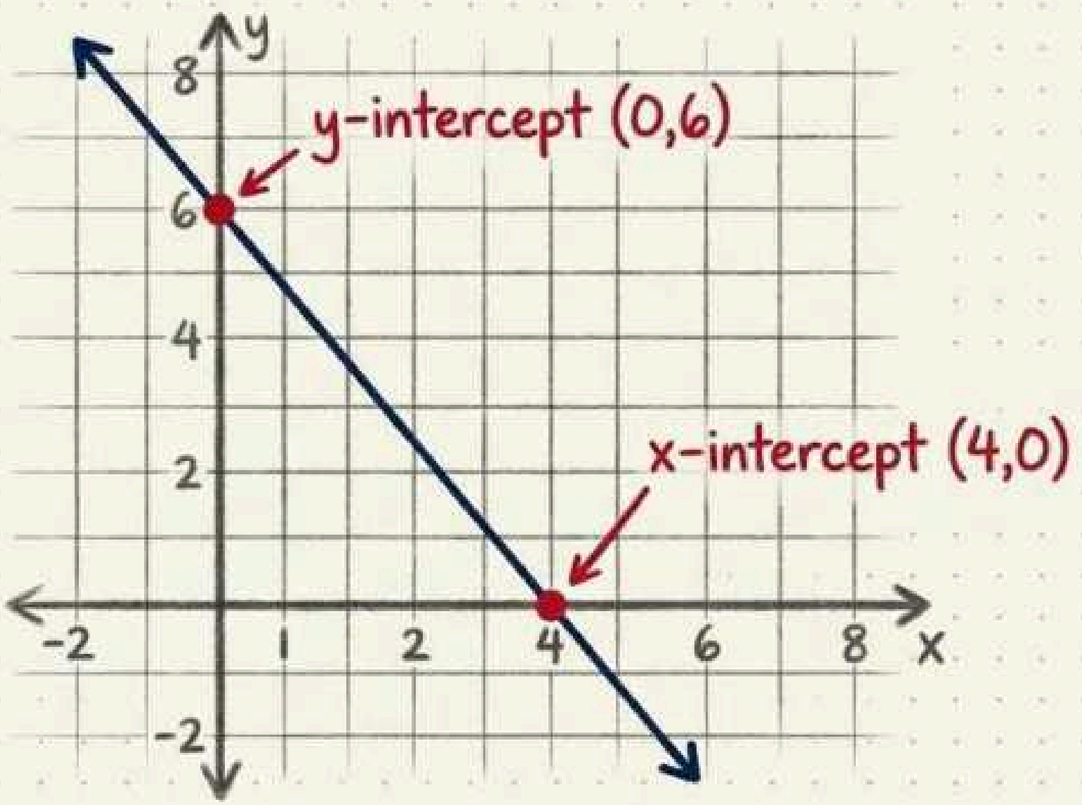
$2y = 12 \rightarrow y = 6 \rightarrow$ Point (0,6)

x-intercept (set y=0):

$3x + \cancel{2y} = 12$

$3x = 12 \rightarrow x = 4 \rightarrow$ Point (4,0)

Geometry (Axis Crossings):



Building an equation from a single point and a slope

¶ 3.11

Given: Slope = 3, Point = (2, 1)

Point-Slope Form: $y - y_1 = m(x - x_1)$

Substitute: $y - 1 = 3(x - 2)$

Distribute & Add: $y = 3x - 5$

Validation

If $x=2$,
$y = 3(2) - 5 = 1$. ✓

Deriving the full equation from just two points

¶ 3.12

Given: Points (1,2) and (4,8)

Step 1:
Find slope

$m = 2$

→

Step 2: Use point-slope form (with either point)

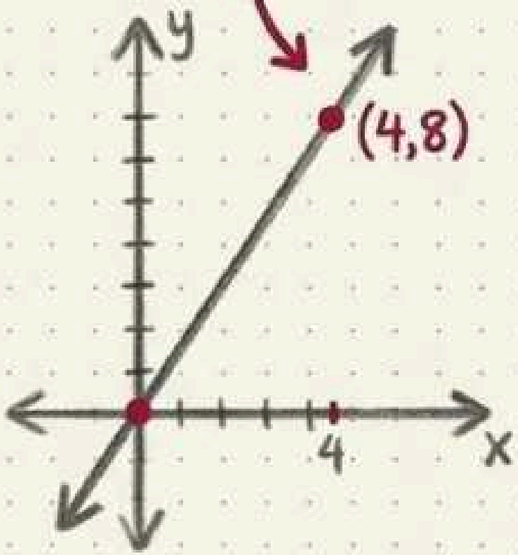
$y - 2 = 2(x - 1)$

→

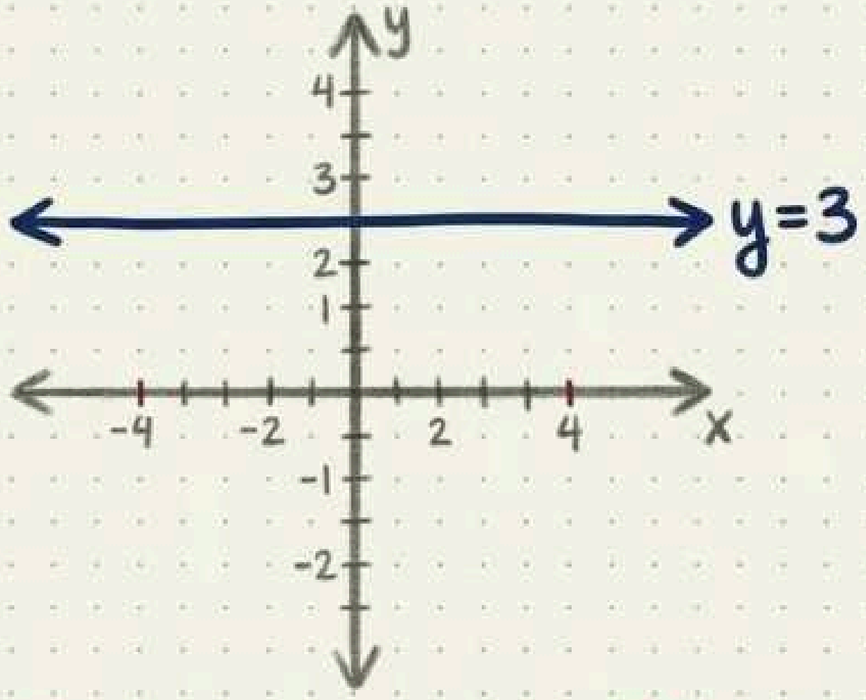
Step 3:
Simplify

$y = 2x$

Visual Proof

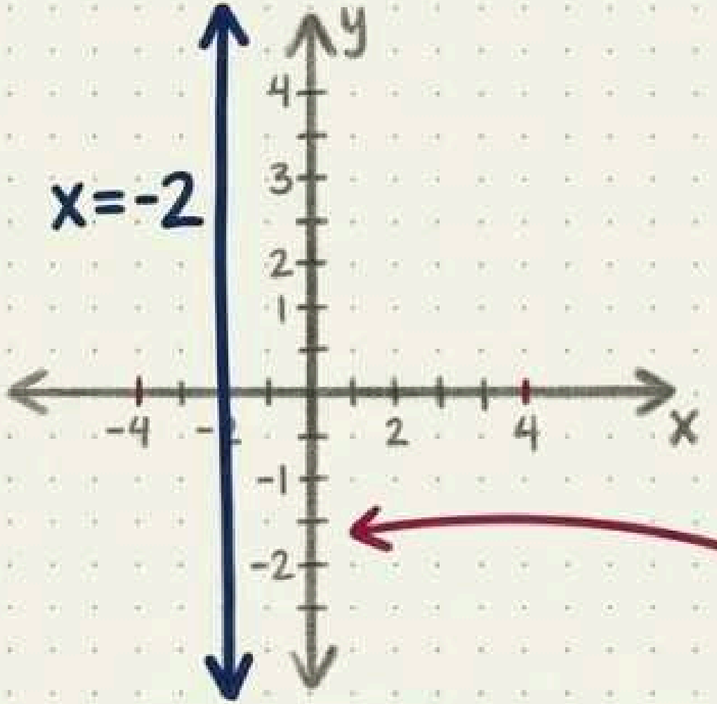


The critical difference between Zero slope and Undefined slope ¶ 3.13



SLOPE = 0

(It never rises)



SLOPE IS UNDEFINED

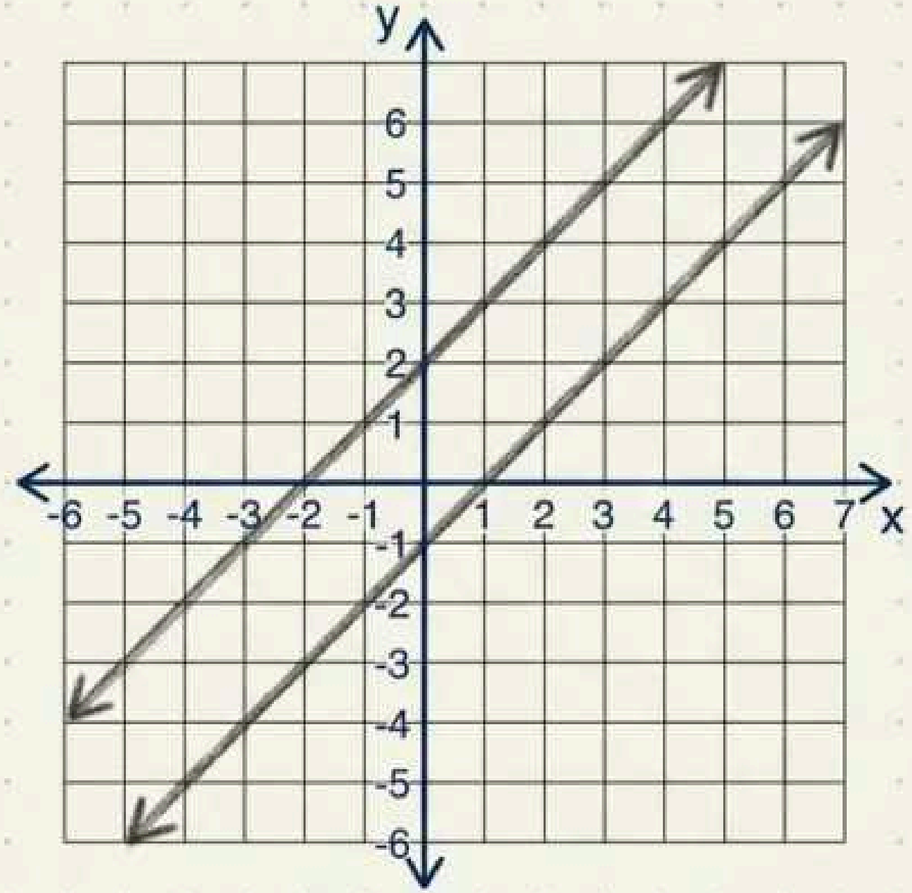
$\frac{x}{0}$

Common Mistake!

Vertical lines don't have zero slope; they have NO slope at all.

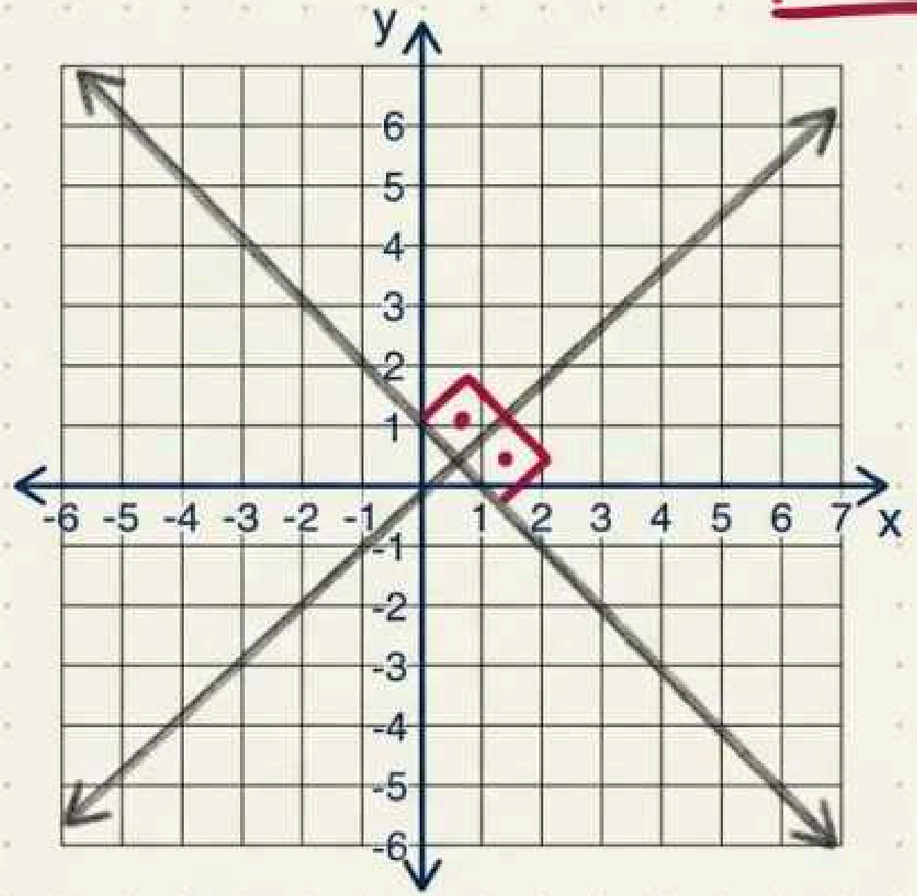
Geometric relationships: Parallel vs. Perpendicular lines

¶ 3.14



Same Slope

Never meet.

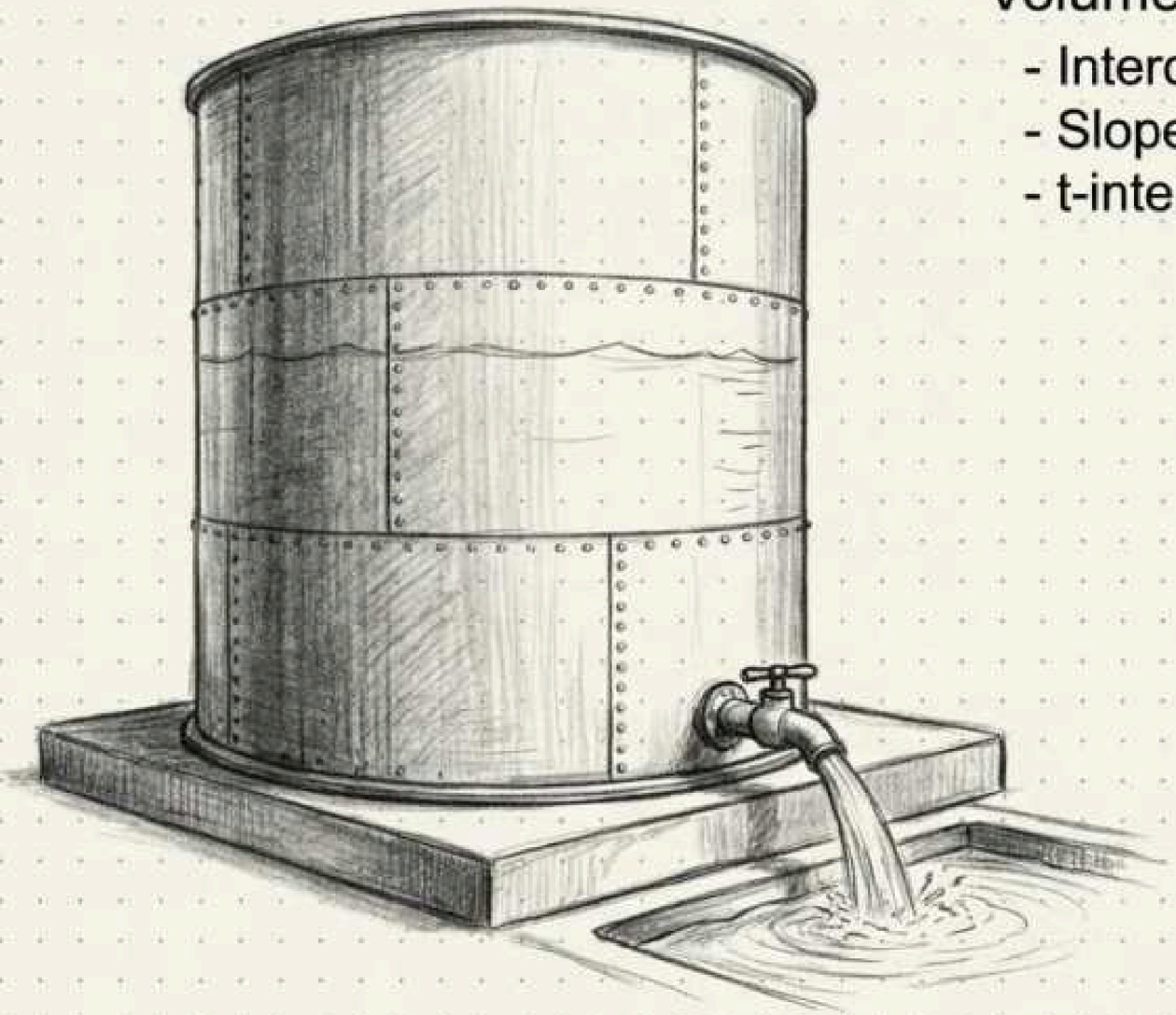


Negative Reciprocals

$m_1 = 2$, $m_2 = \frac{-1}{2}$ $2 \times \frac{-1}{2} = -1$ ✓

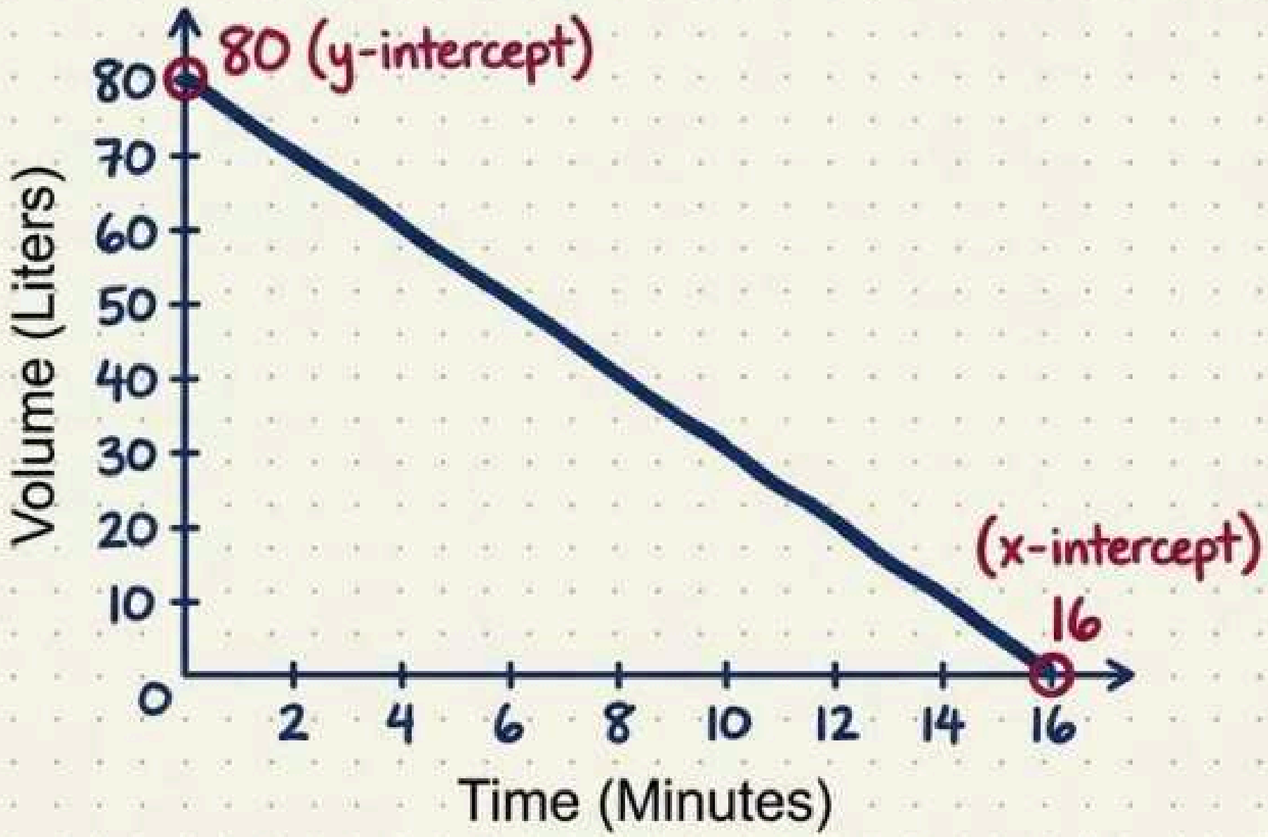
Modeling physical reality: The draining tank

¶ 3.15



Volume = 80 - 5t

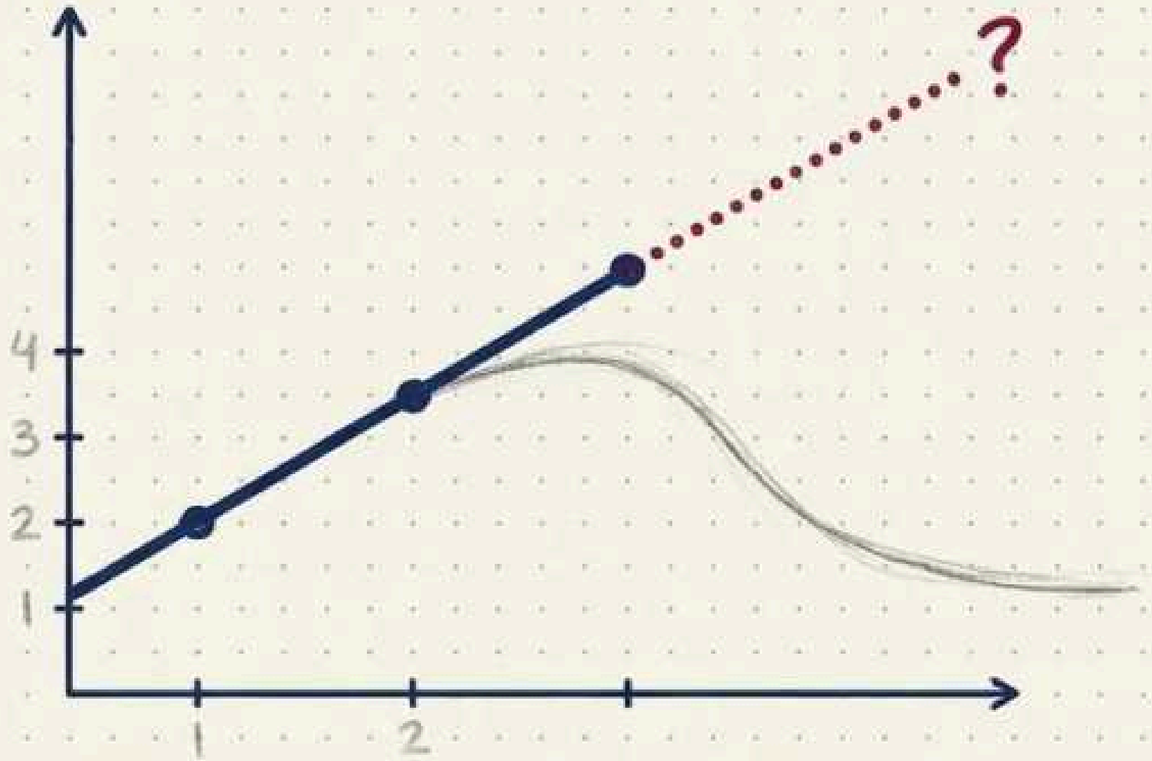
- Intercept (80) = Starting amount (Liters)
- Slope (-5) = Rate of draining (Liters/minute)
- t-intercept (16) = Time until empty



The deepest use of lines (and their limits)

¶ 3.16

Slope = Rate of Change
Intercept = Starting Value



WARNING: Reality rarely changes at a constant rate forever. Extrapolating too far can mislead.

¶ 3.17

Reflection: Where do you see constant rates in your life?

Think of something that changes at a steady rate. What is the slope? The intercept? How long is a straight line sensible?



Bank Balance



Growth



Fuel Tank

Slope = _____

Intercept = _____

Sensible until? _____

Coming in Lecture 4...

¶ 3.18

What happens when two lines cross?

