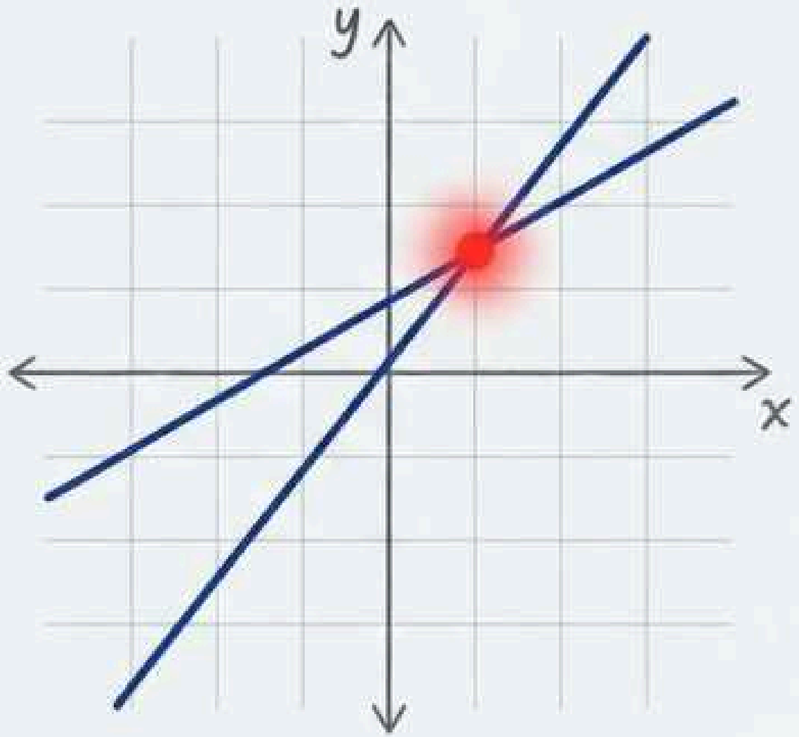
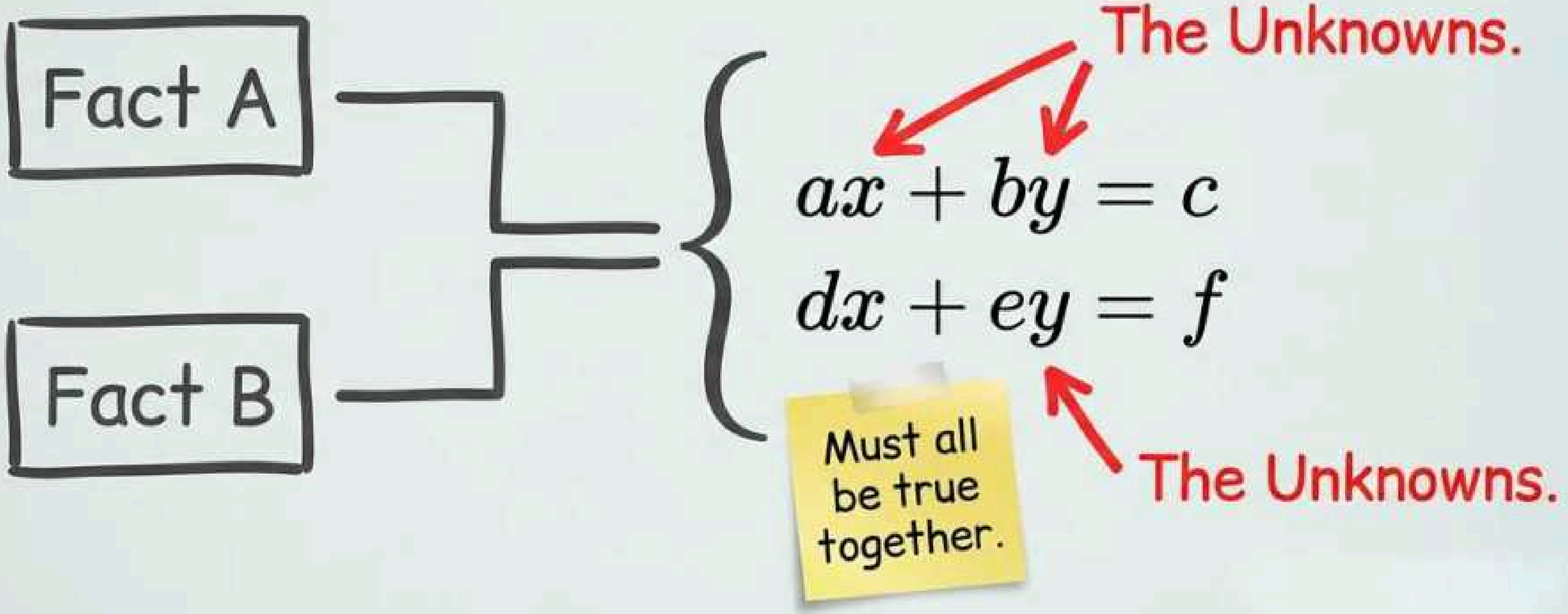
¶ 4.1

# Lecture 4: Systems of Equations

At a glance — graphing, substitution, and elimination



# Defining a System of Equations




One equation cannot settle two unknowns, but two equations can.

¶ 4.3

# The Theatre Problem







Adult Ticket

+




Child Ticket

= 20 total tickets



Cash from Adults

+



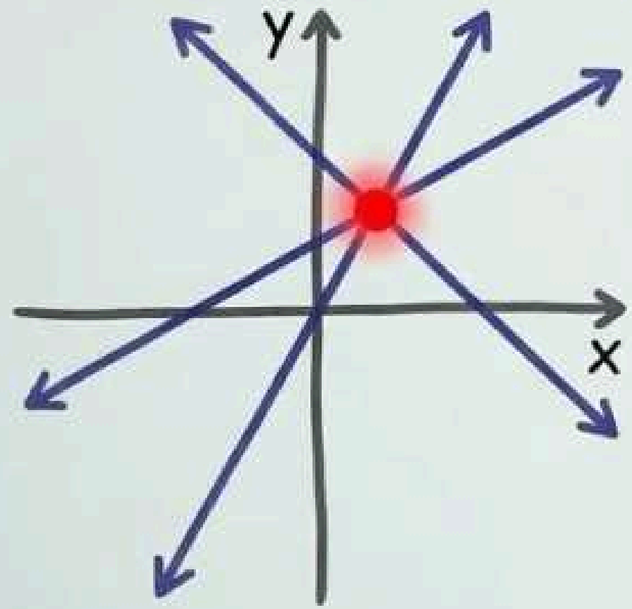
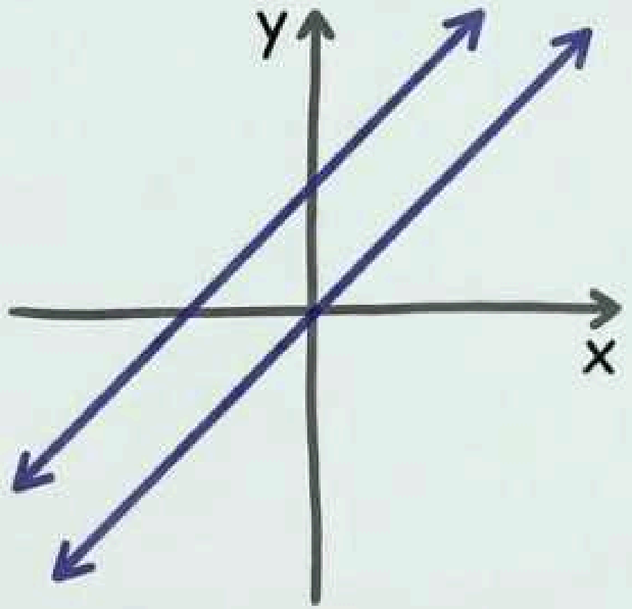
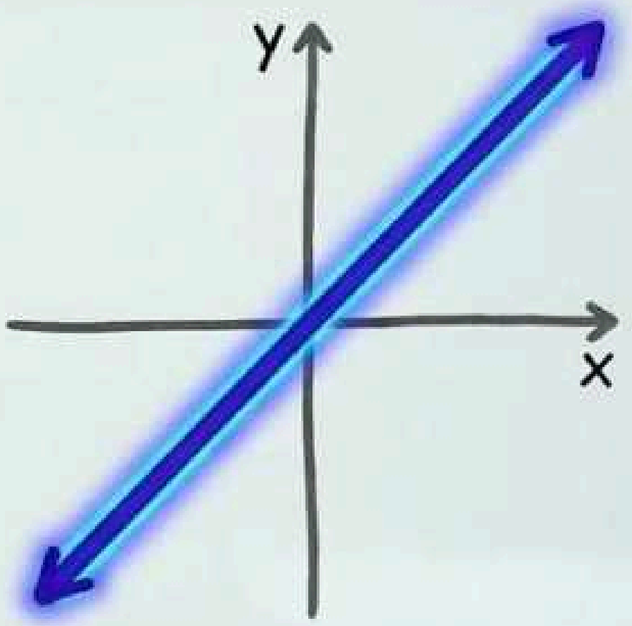
Cash from Children

= $127 total

How many of each were sold?

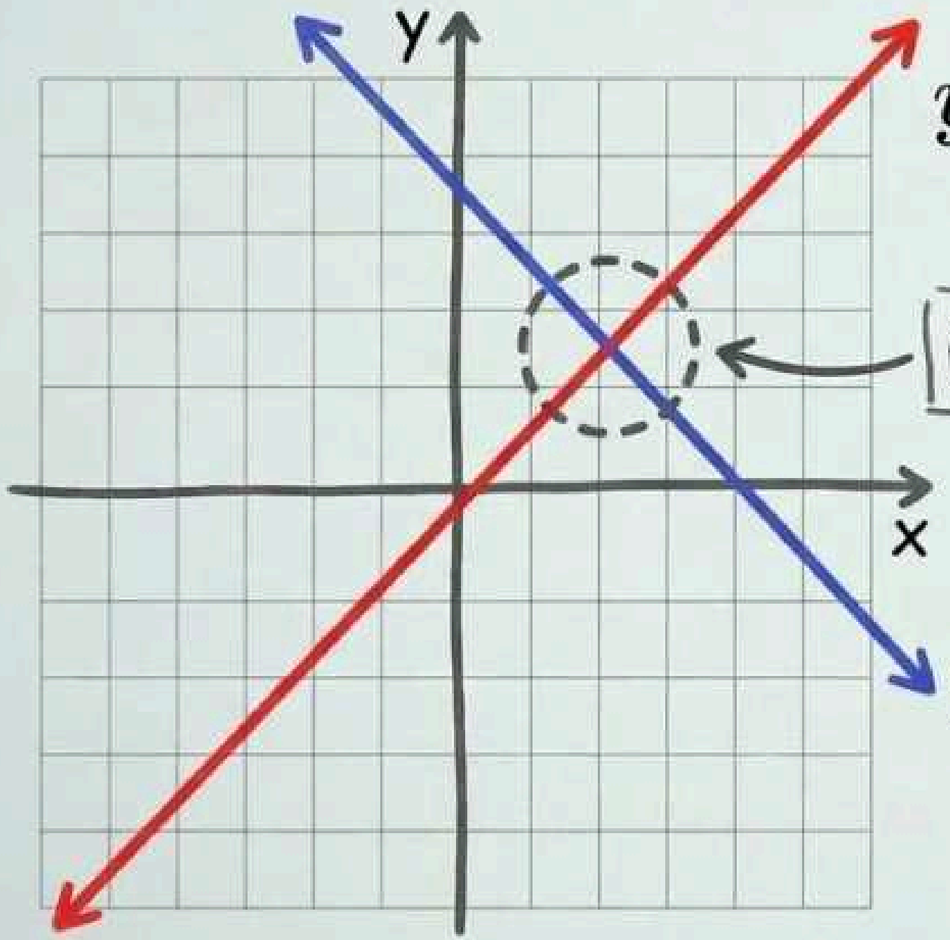
¶ 4.4

# The Geometry of Systems

| Exactly One Solution | No Solution | Infinitely Many Solutions |
| --- | --- | --- |
|   |   |   |
| (The point where they cross) | (They never meet) | (The very same line) |

¶ 4.5

# Method 1: Graphing




$y = x + 1$

(2, 3)

$y = -x + 5$

| Check Your Work |
| --- |
| 3 = 2 + 1 ✓ |
| 3 = -2 + 5 ✓ |

**Caution**

Wonderfully visual, but imprecise when the crossing does not fall neatly on grid lines.

¶ 4.6

# Method 2: Substitution In Action

$$y = 2x - 1 \qquad 3x + y = 14$$

$$3x + (2x - 1) = 14$$

$$5x - 1 = 14$$

$$5x = 15$$

$$x = 3$$

$$y = 2(3) - 1$$

$$\boxed{y = 5}$$

¶ 4.7

# The Steps of Substitution

1. Isolate one variable.

2. Substitute into the other equation.

3. Solve for that variable.

4. Substitute back to find the first unknown.

5. Check the answer in both original equations.

| Proof |
| --- |
| $3(3) + 5 = 14$ ✓ |
| $5 = 2(3) - 1$ ✓ |

¶ 4.8

# Method 3: Simple Elimination

$$x + y = 10$$
$$+\ x - y = 2$$

$$2x = 12 \rightarrow x = 6 \rightarrow y = 4$$

| Verification |
| --- |
| 6 + 4 = 10 ✓ |
| 6 - 4 = 2 ✓ |

¶ 4.9

# Elimination with Multiplication

$$2x + 3y = 12 \xrightarrow{\text{Multiply by 2}} 4x + 6y = 24$$

$$5x - 2y = 11 \xrightarrow{\text{Multiply by 3}} 15x - 6y = 33$$

$$19x = 57 \rightarrow x = 3$$

$$3y = 6, \text{ so } y = 2$$

Check: 15 - 4 = 11 ✓

¶ 4.10

"Adding equal amounts to equal amounts keeps the balance."

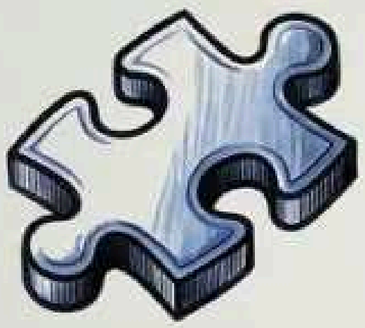
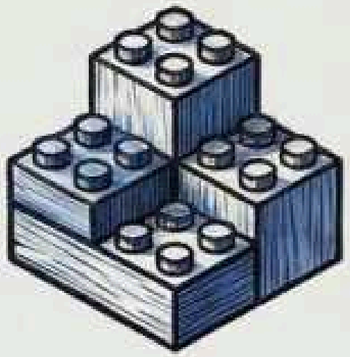
# The Balance Rule




Multiplying a whole equation is also allowed because you do the exact same thing to both sides.

# Synthesis Matrix: Choosing a Method

¶ 4.11

| Substitution | Elimination | Graphing |
| --- | --- | --- |
|  |  |  |
| Use when a variable is already isolated, or has a coefficient of one. | Use when both equations are in standard form, with variables aligned on the left. | Use when you want a visual picture, or a rough estimate. |

**All three methods give the same exact answer. Pick whichever is quickest.**

# Resolving the Theatre Problem

¶ 4.12

$$a + c = 20 \longrightarrow a = 20 - c$$

$$8a + 5c = 127$$

$$8(20 - c) + 5c = 127$$

$$160 - 3c = 127$$

$$33 = 3c$$

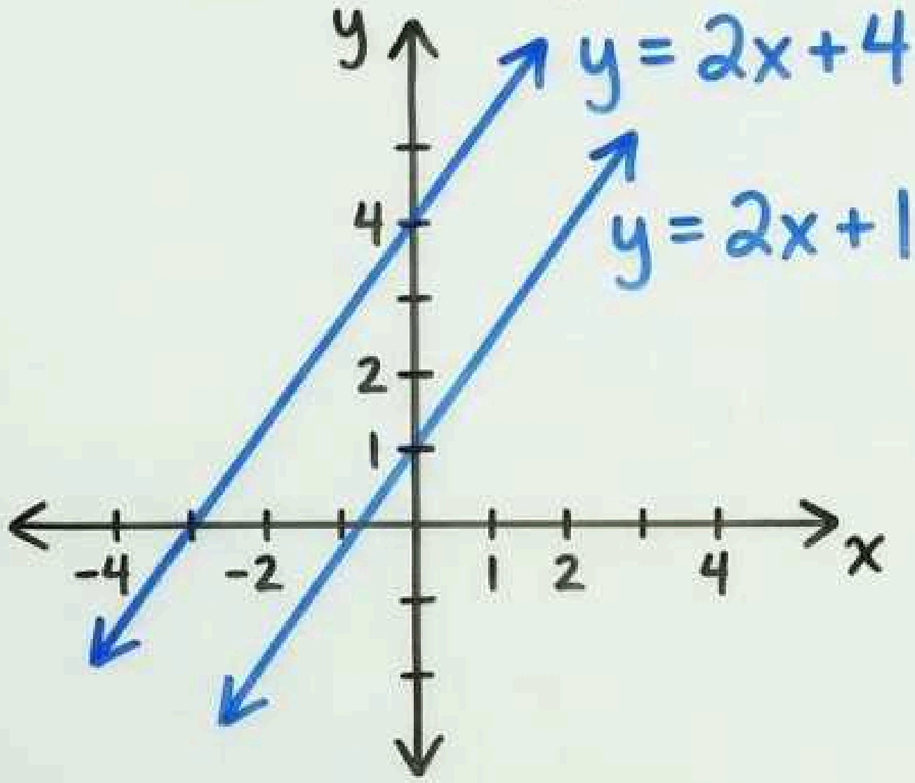
**$c = 11$ (Children), $a = 9$ (Adults)**

☑ $72 + 55 = 127$

# Odd Case 1: No Solution

¶ 4.13

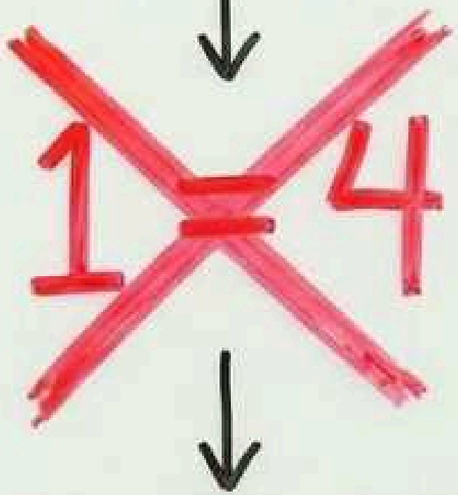
## Geometry




Same slope, different intercepts.

## Algebra

$$2x + 1 = 2x + 4$$

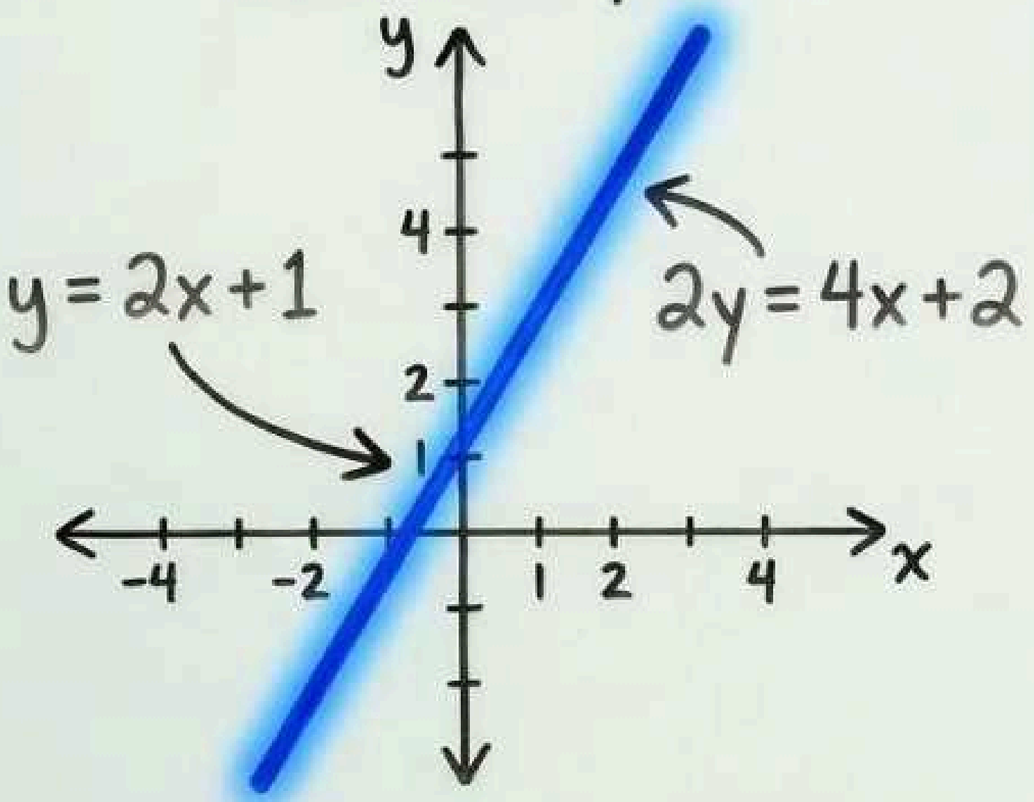



FALSE STATEMENT = NO SOLUTION

# Odd Case 2: Infinitely Many Solutions

¶ 4.14

## Geometry




## Algebra

$$2y = 4x + 2$$
$$\downarrow$$
$$y = 2x + 1$$
$$\downarrow$$
$$2x + 1 = 2x + 1$$
$$\downarrow$$
$$1 = 1$$
$$\boxed{0 = 0}$$

Always true = Every point on the line is a solution (Infinitely many).

# Comparison Questions & The Break-Even Point

¶ 4.15

**Plan A**: \$10/mo + \$3/GB



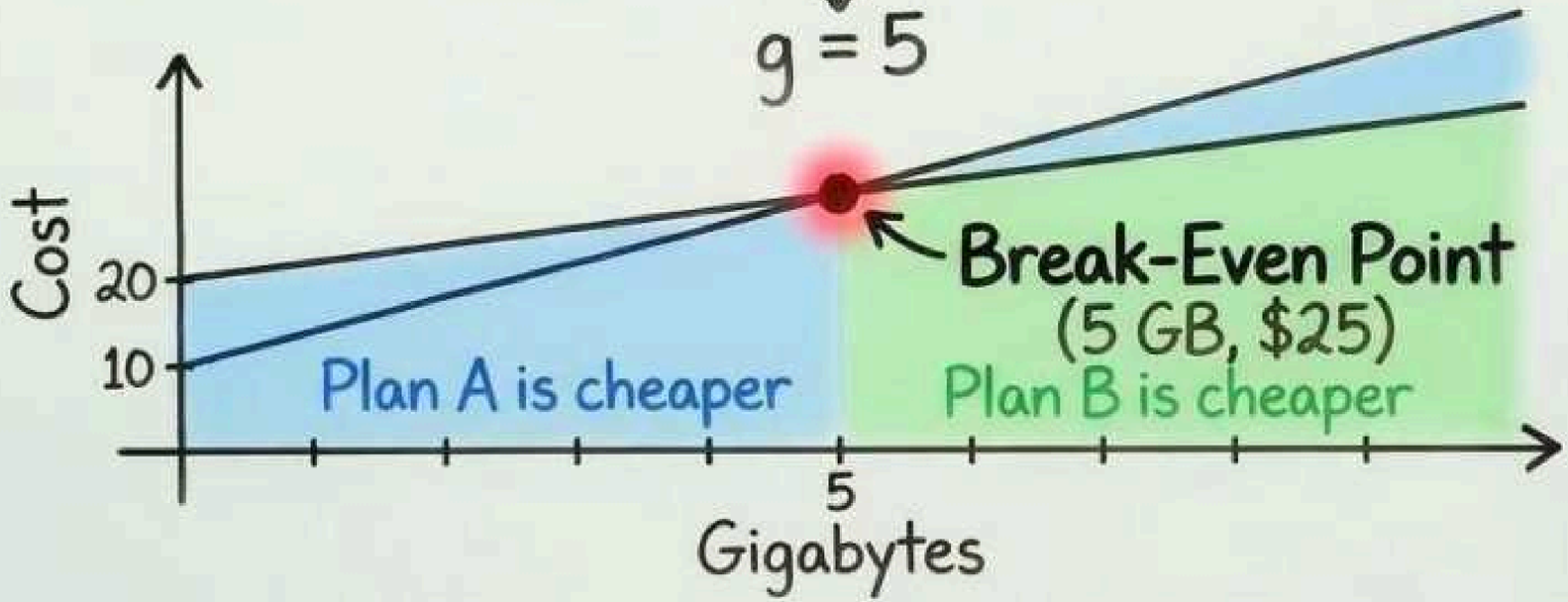




**Plan B**: \$20/mo + \$1/GB



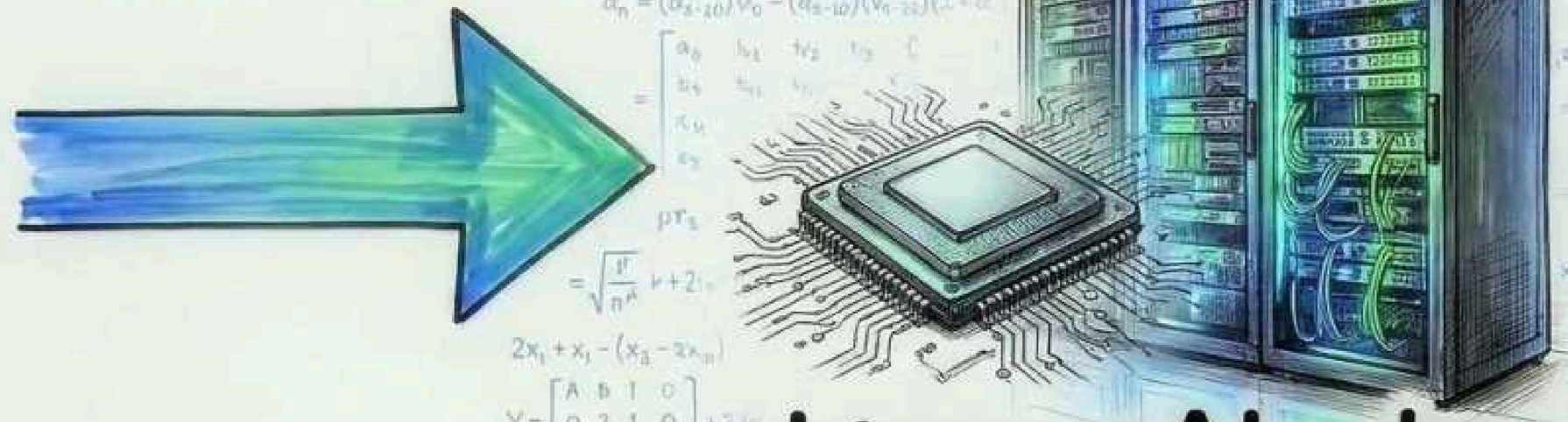
$$10 + 3g = 20 + g$$
$$\downarrow$$
$$2g = 10$$
$$\downarrow$$
$$g = 5$$



# Scaling Up: Linear Algebra

¶ 4.16

$$\begin{cases} ax + by = c \\ dx + ey = f \end{cases}$$



## Linear Algebra

The exact same elimination methods, scaled up by computers for engineers and economists.

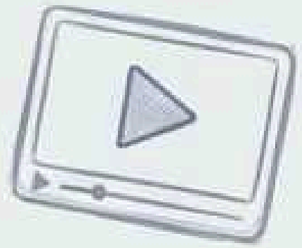
# Reflection

9/4.17



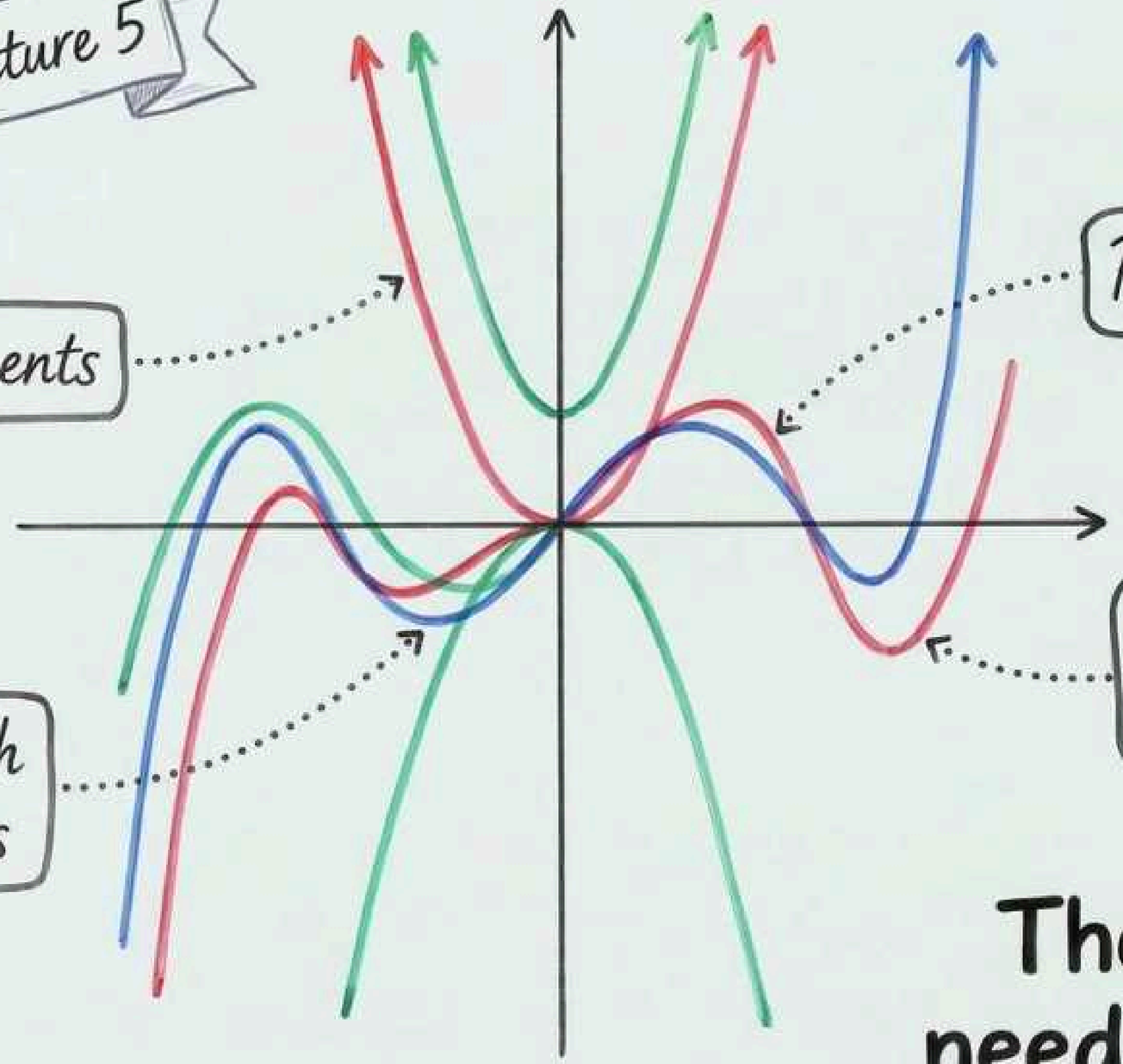
Think of a choice between two options whose costs change at different rates... Where would the lines cross, and what would the crossing point tell you?





## Coming Up in Lecture 5

¶ 4.18



Exponents

Polynomials

Geograph Edamals

The Art of Factoring

**The tools we need for curves.**