¶ 5.1

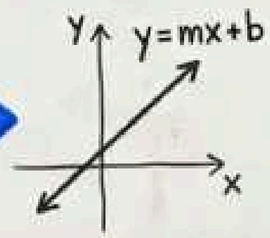
Exponents, Polynomials, and Factoring

MATH 101 B — Lecture 5

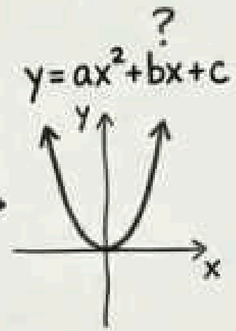
The Path from Lines to Curves

¶ 5.2

$y = mx + b$ ➡



x^2



1. Rules of Exponents (Understanding the shorthand)

2. Polynomial Operations (Adding, subtracting, multiplying)

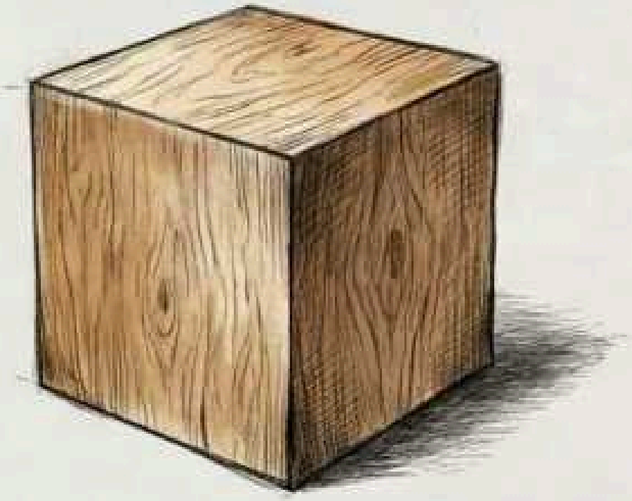
3. Factoring (Breaking expressions into products)

Factoring is the key to unlocking quadratic equations in Lecture 6.

Unpacking the Shorthand

¶ 5.3

$$x^3 = x \cdot x \cdot x$$



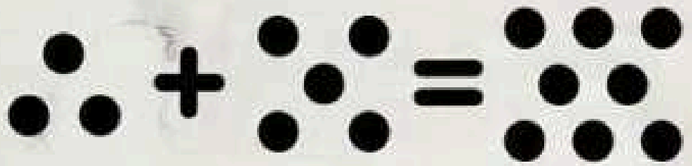
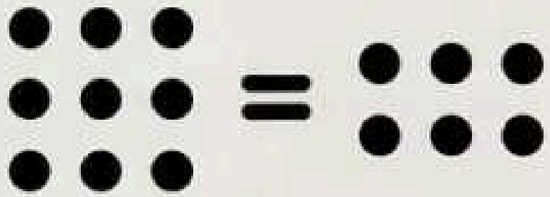
Exponent (how many times it is multiplied)

$$2^3 = 2 \cdot 2 \cdot 2 = 8$$

Base (the number being multiplied)

The Three Core Rules of Exponents

¶ 5.4

Same Base? Add Exponents.	$x^3 \cdot x^4 = x^{3+4} = x^7$	
Multiply the Exponents.	$(x^3)^2 = x^{3 \cdot 2} = x^6$	
Subtract the Exponents.	$\frac{x^5}{x^2} = x^{5-2} = x^3$	

Zero and Negative Exponents

¶ 5.5

$$\frac{x^3}{x^3} = x^0$$

↓

Anything divided by itself is 1.

↓

$$x^0 = 1 \text{ (where } x \neq 0)$$

$$x^{-2} = \frac{1}{x^2} \qquad 2^{-3} = \frac{1}{8}$$

WARNING:
A negative exponent means a reciprocal. It does NOT make the base number negative!

Anatomy of a Polynomial

¶ 5.6

Degree (highest exponent)

$$3x^2 + 2x - 5$$

3 Terms (Trinomial)

1 term = Monomial,
2 terms = Binomial

Rule: Must be a sum of terms with whole-number powers.

~~$\frac{1}{x}$~~ (variable in denominator)

~~$\sqrt{x}$~~ (under a root)

Adding Polynomials & Combining Like Terms

¶ 5.7

$$(3x^2 + 2x - 5)$$
$$+ (x^2 - 4x + 77)$$
$$\overline{\qquad\qquad\qquad\qquad}$$
$$4x^2 - 2x + 2$$

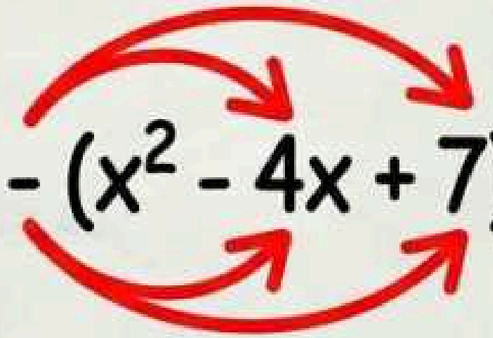
Standard Form: Terms are written from the highest power down.

Subtracting Polynomials (Watch the Signs)

¶ 5.8

Step 1: $(3x^2 + 2x - 5) - (x^2 - 4x + 7)$

Step 2: $(3x^2 + 2x - 5) - (x^2 - 4x + 7)$



Step 3: $3x^2 + 2x - 5 - x^2 + 4x - 7$

Result: $2x^2 + 6x - 12$

Quick Test (x=1)

Poly 1: $3(1)^2 + 2(1) - 5 = 0$

Poly 2: $1^2 - 4(1) + 7 = 4$

Difference $(0 - 4) = -4$

Our Answer:

$2(1)^2 + 6(1) - 12 = -4$

It matches! ✓

Multiplying Polynomials: The Area Model

¶ 5.9

Principle: Everything multiplies everything.

$(x + 3)(x + 5) \rightarrow x^2 + 5x + 3x + 15 \rightarrow x^2 + 8x + 15$

	x	5
x	x^2	$5x$
3	$3x$	15

Multiplication and Managing Signs

¶ 5.10

$$(2x-1)(x+4)$$

$2x^2$, $8x$, $-x$, and -4

Combine: $8x - x = 7x$

Result: $2x^2 + 7x - 4$

Test with x=1

Original:

$(2-1)(1+4) = 1 \cdot 5 = 5$

Result:

$2(1) + 7(1) - 4 = 5$

Match! ✓

Two Special Products to Memorize

¶ 5.11

The Square of a Sum

$$(x+4)^2 = x^2+8x+16$$

The middle term (8x) is twice the product of the two numbers.

COMMON MISTAKE:
Do not just write x^2+16!

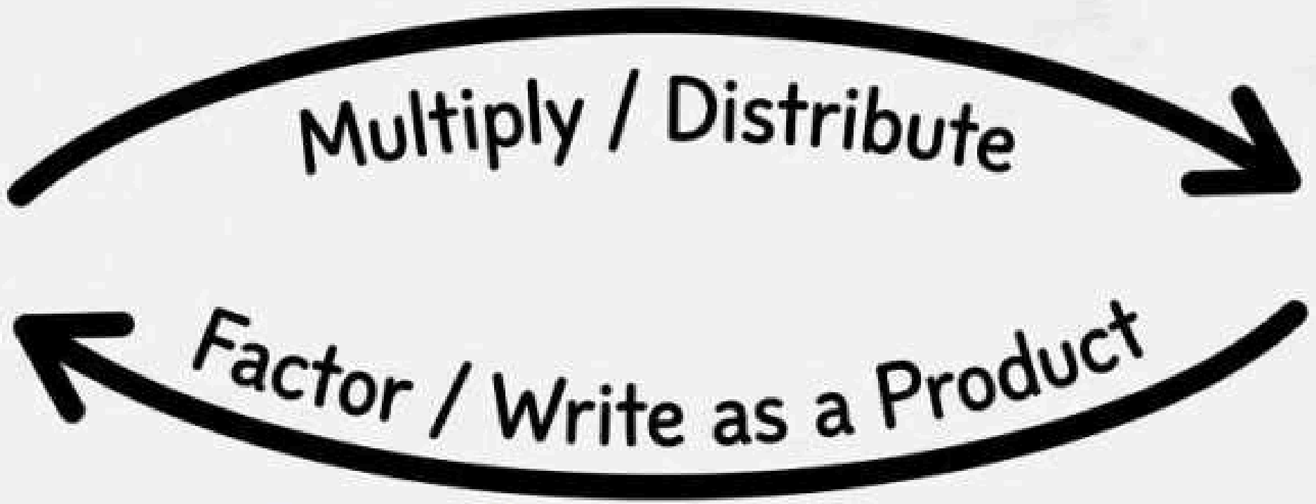
The Difference of Squares

$$(x-5)(x+5) = x^2-25$$

The middle terms (5x and $-5x$) cancel out perfectly.

Introduction to Factoring (Multiplication Run Backwards)

¶ 5.12



Step 1: Always look first for a Greatest Common Factor (GCF).

$$6x^2 + 9x$$

$$\downarrow$$

$$\underbrace{3x}(2x + 3)$$

GCF pulled out

Check: Distribute the 3x to prove you get the original back.

Factoring Simple Trinomials ($x^2 + bx + c$)

¶ 5.13

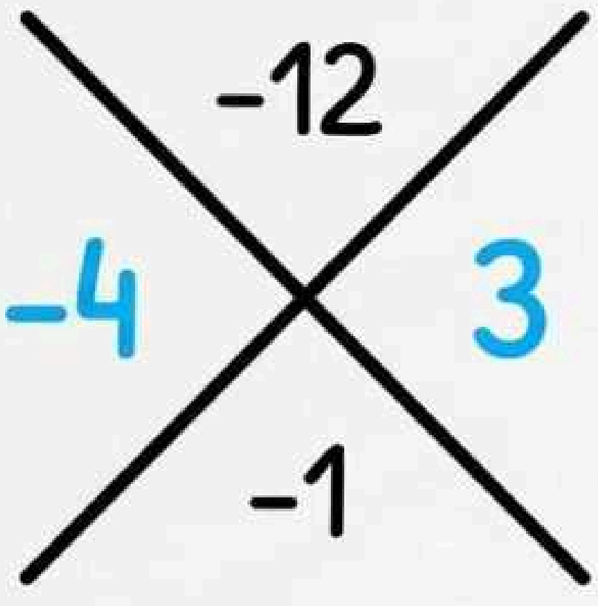
Rule: Find two numbers that multiply to c and add to b.

$x^2 + 7x + 12$



$(x + 3)(x + 4)$

$x^2 - x - 12$



$(x - 4)(x + 3)$

¶ 5.14

Factoring Special Patterns

Difference of Squares: $x^2 - 49 \rightarrow (x - 7)(x + 7)$ Factors at once.

Sum of Squares: $x^2 + 49$ Cannot be factored using real numbers.



Perfect Square Trinomial: $x^2 + 10x + 25 \rightarrow (x + 5)^2$ Shortcut: $5 \cdot 5 = 25$ and $2 \cdot 5 = 10$.

Memorizing these patterns saves massive amounts of time.

Complex Trinomials: The Splitting Method

¶ 5.15

Used when the leading coefficient is NOT 1.

Factor $2x^2 + 7x + 3$

↓

Step 1 (Multiply ends): $2 \cdot 3 = 6$

↓

Step 2 (Find factors): Multiply to 6, add to 7 → The numbers are 6 and 1

↓

Step 3 (Split): $2x^2 + 7x + 3$

$6x$ x

$2x^2 + 6x + x + 3$

↓

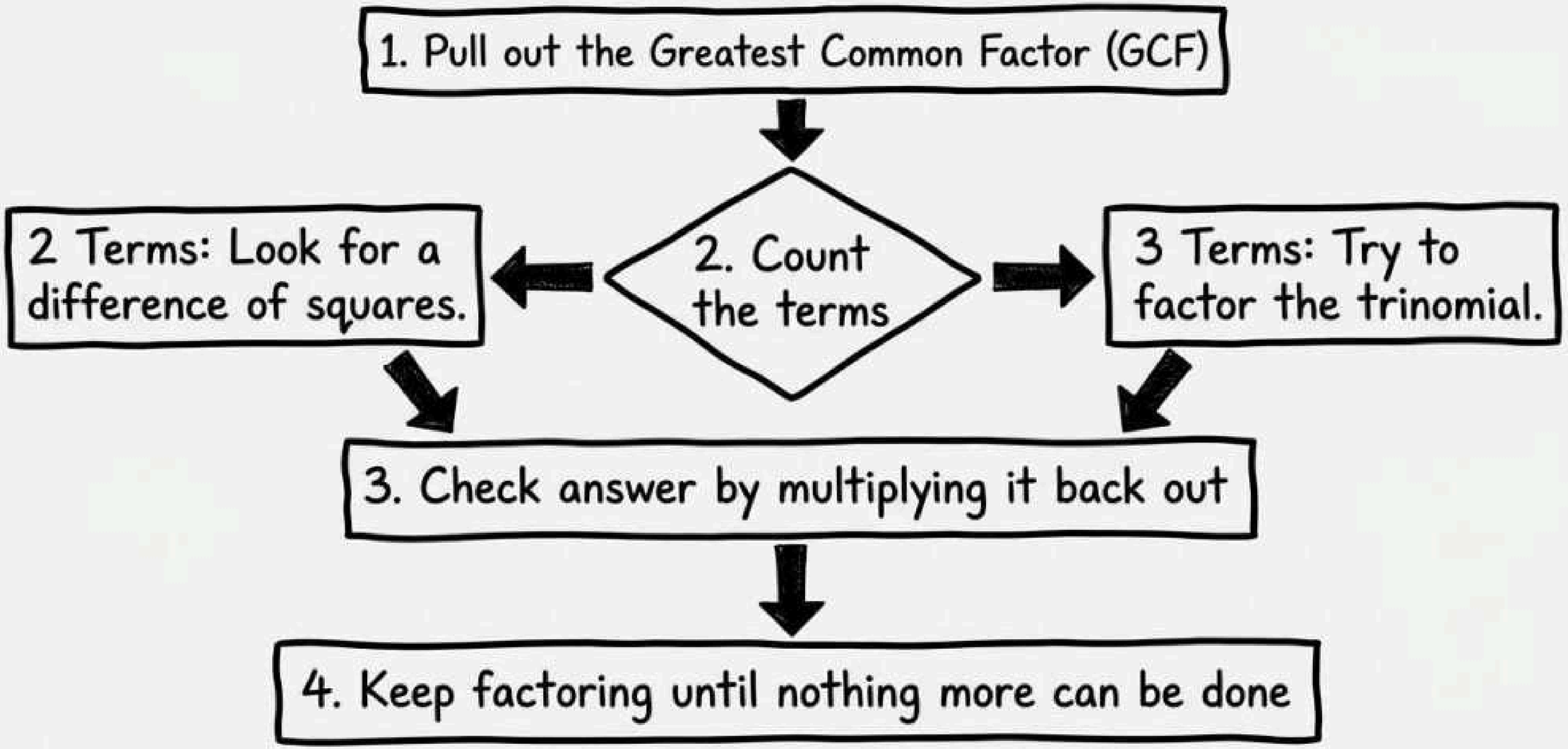
Step 4 (Group): $2x(x + 3) + 1(x + 3)$

↓

Final Result: $(2x + 1)(x + 3)$

The Universal Factoring Checklist

¶ 5.16



Note: Not all polynomials factor neatly. Lecture 6 provides a method that always works.

The Big Question: Why Factor?

¶ 5.17

Multiplying and factoring undo each other. Why is a quantity easier to understand as a product of simple pieces rather than a long sum?

The Long Sum

$ax^5 + bx^4 - cx^3 + dx^2 - ex + f$ → $3x^2 - 8x + 12$

$7x^3y - 5x^2y^2 + 9xy^3 - 11y^4$ → $7x^2 + 6x^2 - 4y + 6y^4$

$3x^2 - 8x + 12$ → $-x^3 + 4x^2 - 6x + 3$

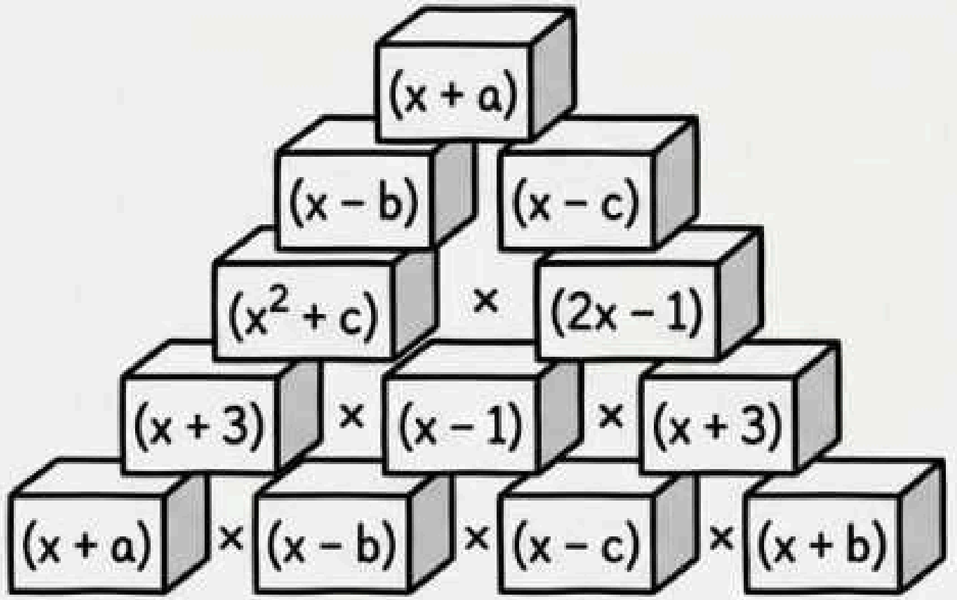
$x^7 + 2x^5 - x^3 + 4x^2 - 6x + 3$ → $6x + 3$

$x^7 + 2x^5 - x^3 + 4x^2 - 6x^2 + 2x^3$

$x^7 + 2x^5 - x^3 + 4x^2 - 6x + 3$ → $2x^3y^1 + 2x^5 - x^3 + 4x^2 - 6x + 3$

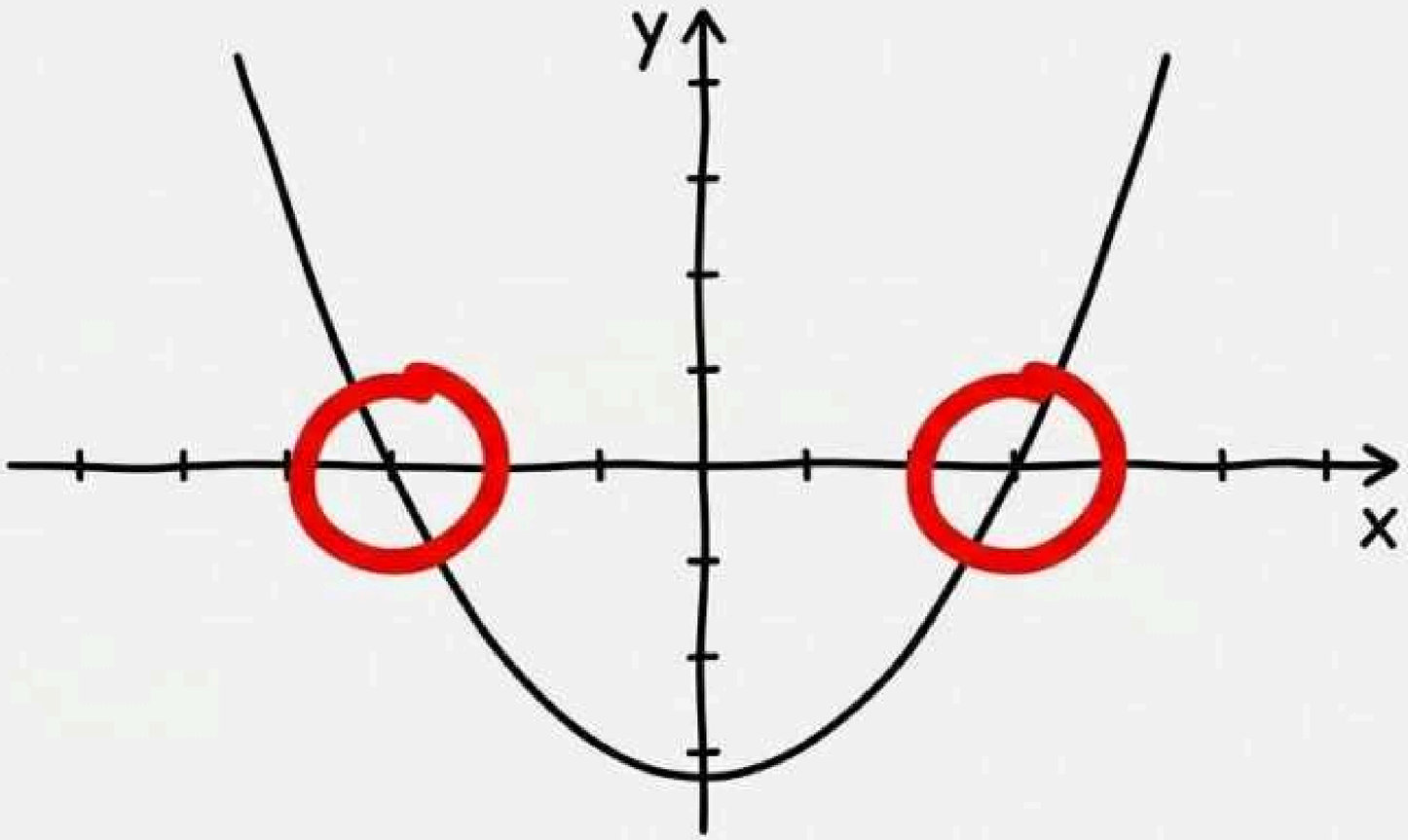
$7x^3 + 5x^2 - 9x^3 - 11y^4$

The Product of Simple Pieces



Looking Ahead to Lecture 6

¶ 5.18



We use factoring to solve quadratic equations. Meet the Parabola.