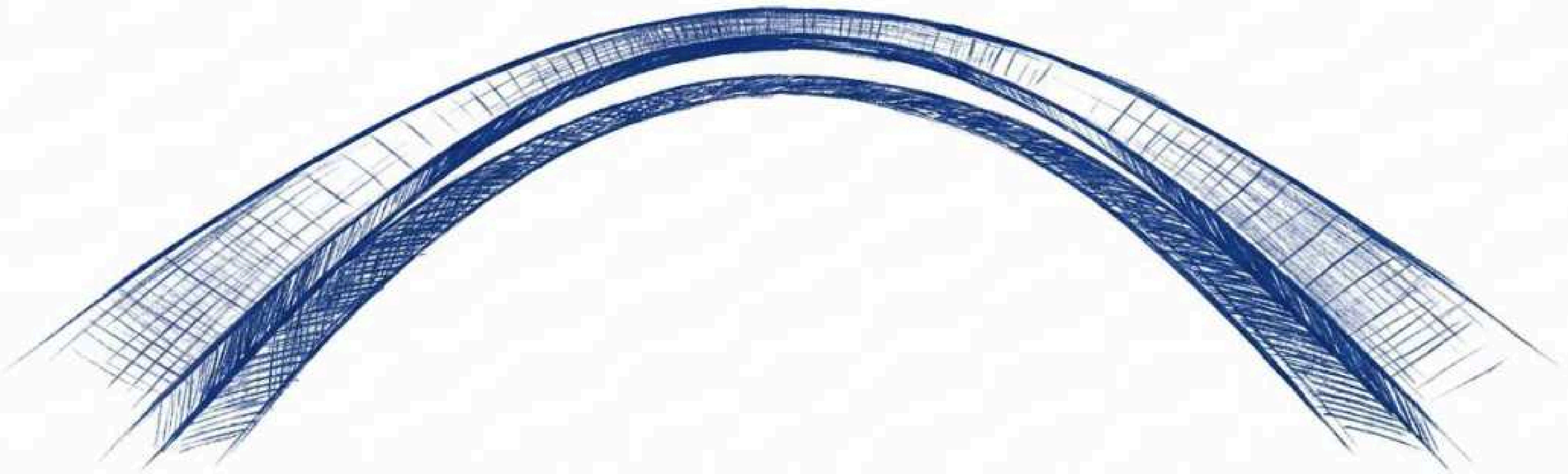
¶ 6.1

MATH 101 B: Lecture 6

Quadratic Equations.

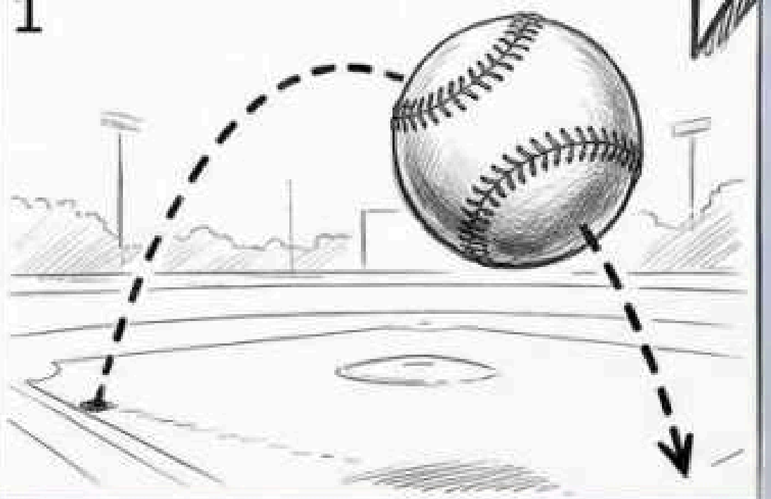


The Shape of Quadratics

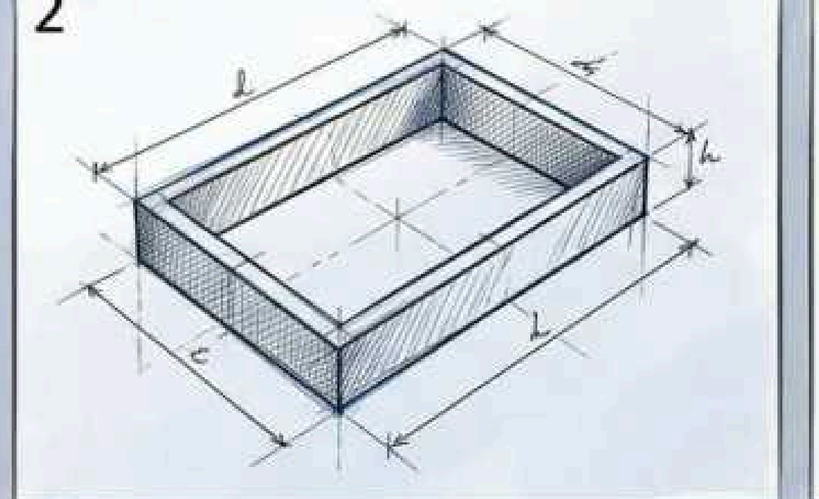
¶ 6.2

x^2 Highest power is 2

1



2



3



Factoring → Square Root → Completing the Square → Quadratic Formula

Anatomy of the Standard Form

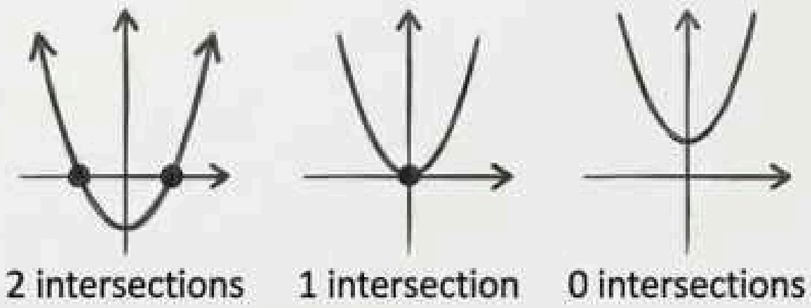
¶ 6.3

Coefficient
(Cannot be 0)

Highest power is 2

$$ax^2+bx+c=0$$

$$x^2+3x-4=0$$



Method 1: Factoring & The Zero Product Property

¶ 6.4

If $A \times B = 0$, then A=0 or B=0.

$$x^2 + 3x - 4 = 0$$

Factor to $(x+4)(x-1) = 0$

$$x + 4 = 0 \rightarrow x = -4$$

$$x - 1 = 0 \rightarrow x = 1$$

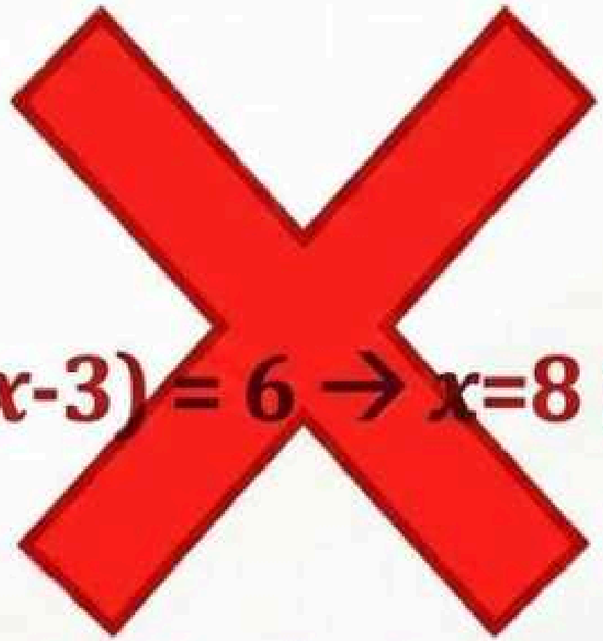
✓ $1^2 + 3(1) - 4 = 0$

The Factoring Trap

¶ 6.5

Only **ZERO** has the special property.

$(x\text{-}2)(x\text{-}3) = 6 \rightarrow x=8$ or $x=9$



WRONG: Each factor does **not** equal 6.

Expand → Subtract 6 to get zero → **THEN** factor.

Method 2: The Square Root Method

$$x^2 - 9 = 0$$

$$x^2 = 9$$

$$x = \pm 3$$

-3^2 is also 9

$$(x - 2)^2 = 25$$

$$x - 2 = \pm 5$$

$$x = 7 \text{ or } x = -3$$

Always remember the ± (Plus and Minus)

Method 3: Completing the Square (Algebra)

$$x^2 + 6x + 2 = 0$$

$$x^2 + 6x = -2$$

Half of 6 is 3

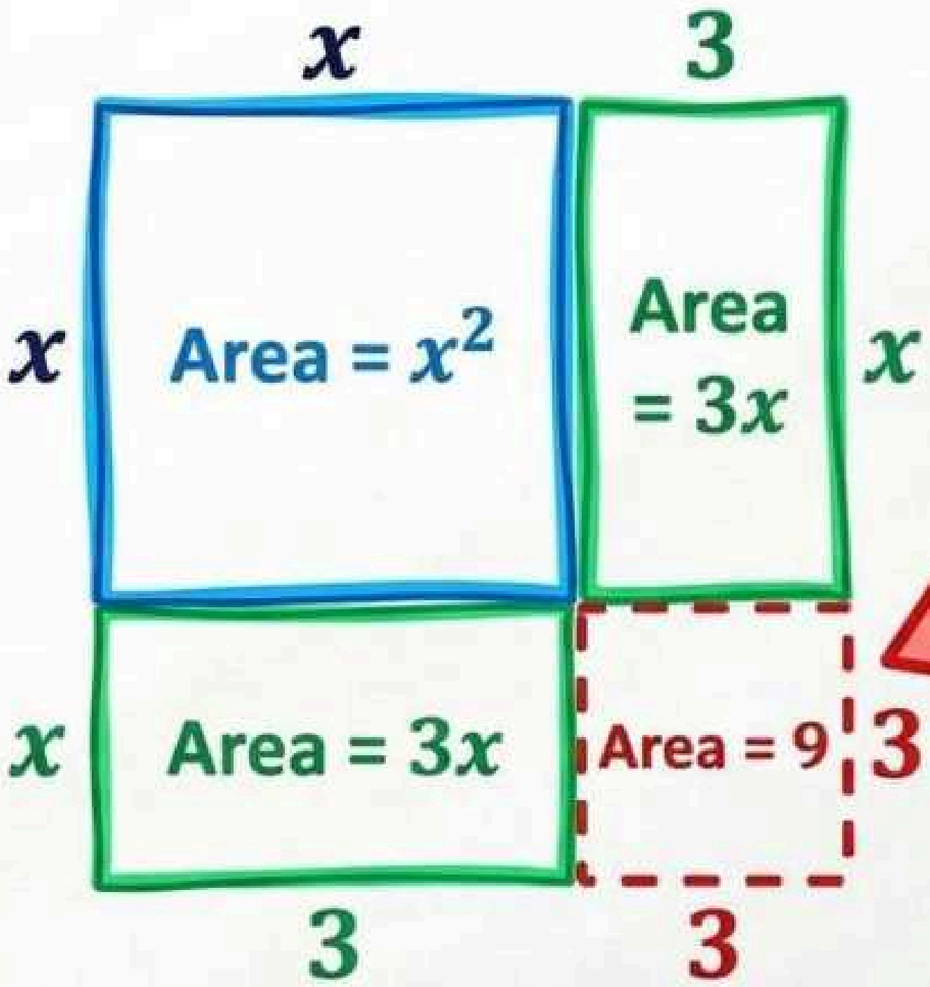
Three squared is 9

$$x^2 + 6x + 9 = -2 + 9$$

$$(x+3)^2 = 7 \rightarrow x + 3 = \pm\sqrt{7}$$

Completing the Square (Geometry)

¶ 6.8



Adding this missing corner completes the square.

$$x = -3 \pm \sqrt{7} \quad (x \approx -0.35 \text{ or } x \approx -5.65)$$

Method 4: The Quadratic Formula

$ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The most famous formula in school algebra.
Derived from completing the square.

Applying the Quadratic Formula

$$2x^2 - 5x - 3 = 0$$

$a = 2$ $b = -5$ $c = -3$

$$(-5)^2 - 4(2)(-3) = 25 - (-24) = 49 \rightarrow \sqrt{49} = 7$$

$$x = \frac{5 \pm 7}{4}$$

$$x = \frac{12}{4} = 3 \qquad x = \frac{-2}{4} = -0.5$$

✓ $18 - 15 - 3 = 0$

Traps in the Quadratic Formula

¶ 6.11

Signs

If b is negative, $-b$ is positive.

$-(-5) \rightarrow +5$

The Radical

Square b first.

Watch the sign on c when subtracting $4ac$.

The Fraction

The entire numerator is divided by $2a$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ ✓

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ ✗

Put parentheses ( ) around each number as you substitute, and the errors disappear.

The Discriminant: $b^2 - 4ac$

¶ 6.12

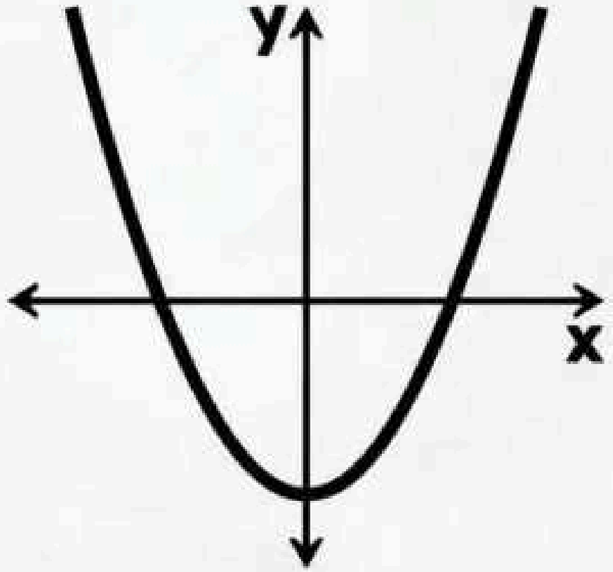
Positive (>0)	Zero (=0)	Negative (<0)
2 real solutions.	Exactly 1 repeated root. $x^2 - 6x + 9 = 0$ $\rightarrow x=3$	0 real solutions (cannot take square root of negative in real numbers). $x^2 + 2x + 5 = 0$ $\rightarrow$ negative 16

Graphing the Parabola

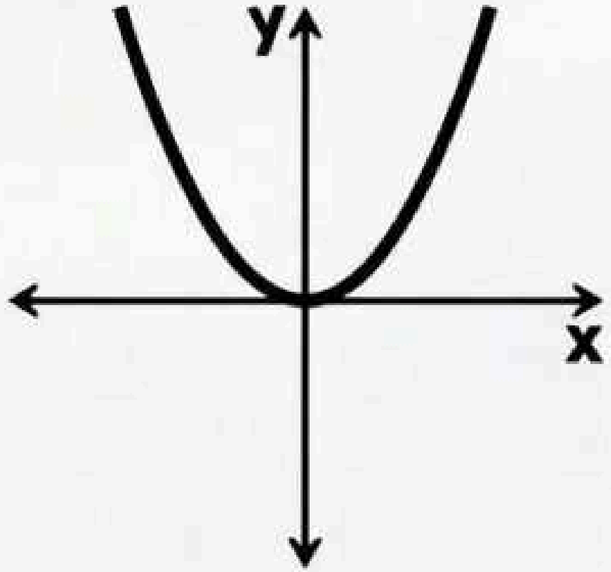
¶ 6.13



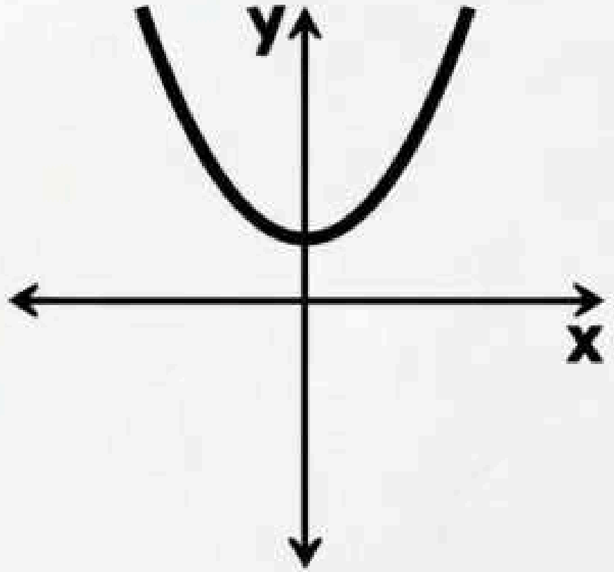




2 x-intercepts



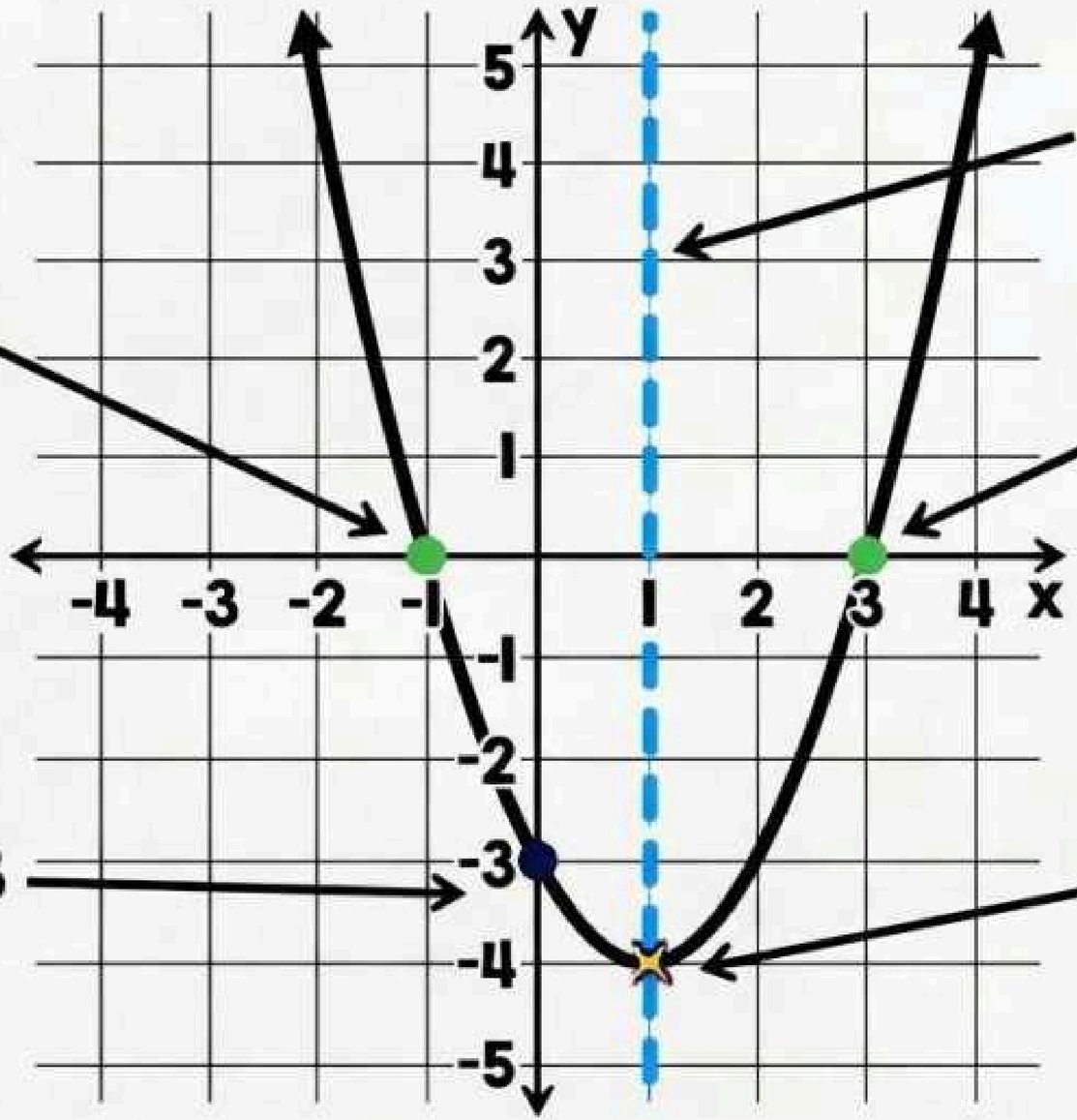
1 x-intercept (vertex)



0 x-intercepts

Anatomy of a Parabola

¶ 6.14



Axis of Symmetry at x=1

factors (x-3)(x+1)

factors (x-3)(x+1)

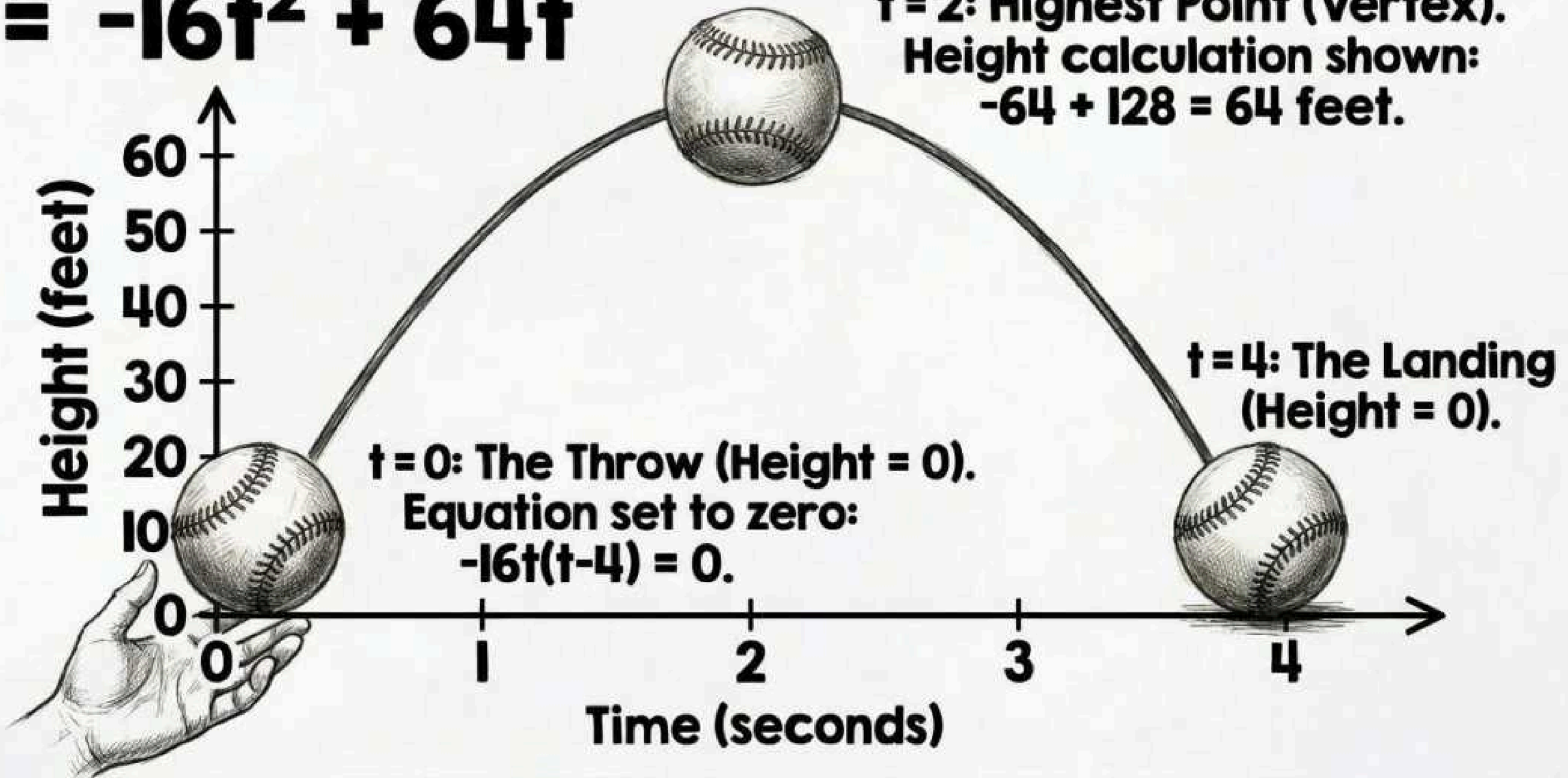
y-intercept = -3

Vertex always lies at x = -b/2a.

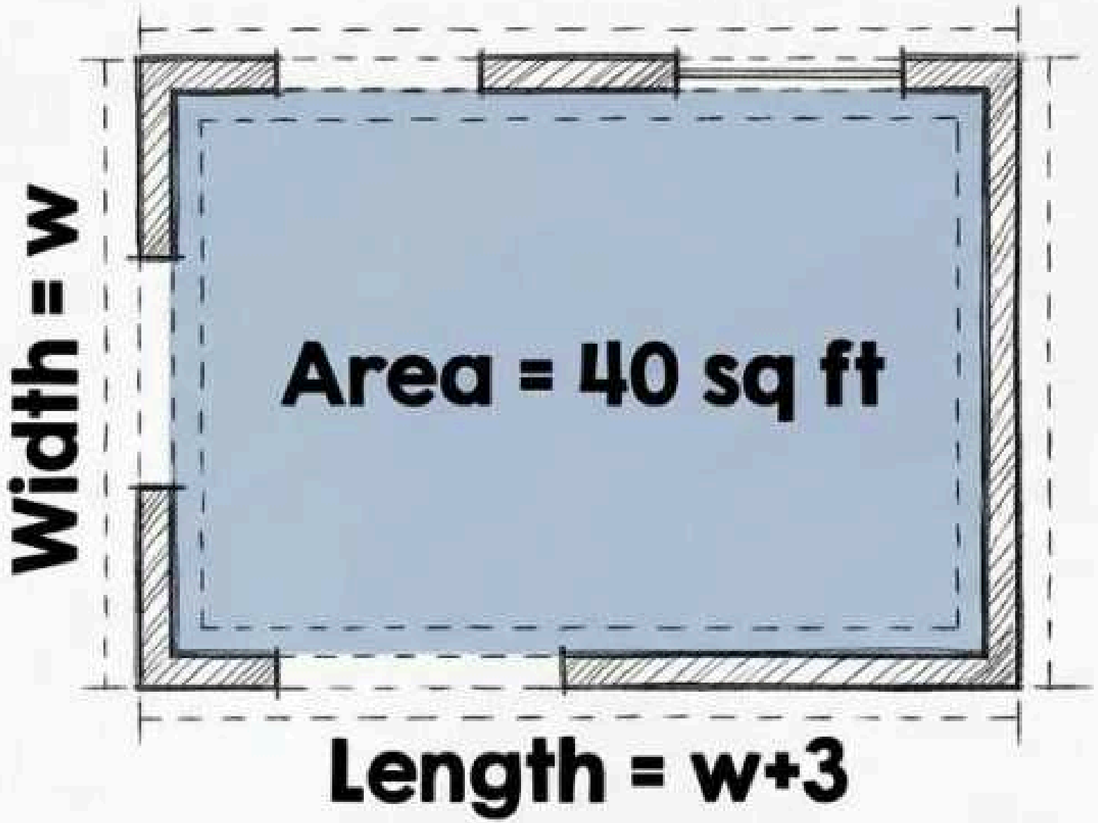
Calculation: 1 - 2 - 3 = -4.

Application I: Motion

$h = -16t^2 + 64t$



Application 2: Areas



$w(w+3) = 40 \rightarrow w^2 + 3w - 40 = 0$

Factors to $(w+8)(w-5) = 0$

~~$w = -8$~~ ← A width cannot be negative.

$w = 5$ ← Width = 5 ft, Length = 8 ft.

Always ask whether each solution makes sense in reality.

Choosing Your Method

¶ 6.17

How do the numbers look?

Friendly numbers?
→ Try Factoring.

Is the equation a square?
→ Use Square Root Method.

Unsure?
→ Use the Quadratic Formula (It always works).

Completing the square explains the formula, and returns in later mathematics.

The golden rule of the course: Do the same thing to both sides, and check your answer.

Pick something that changes. Would a line describe it, or does it need a curve? What would the solutions mean in that situation?

Thank you for taking Mathematics 101 B.

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Academy

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