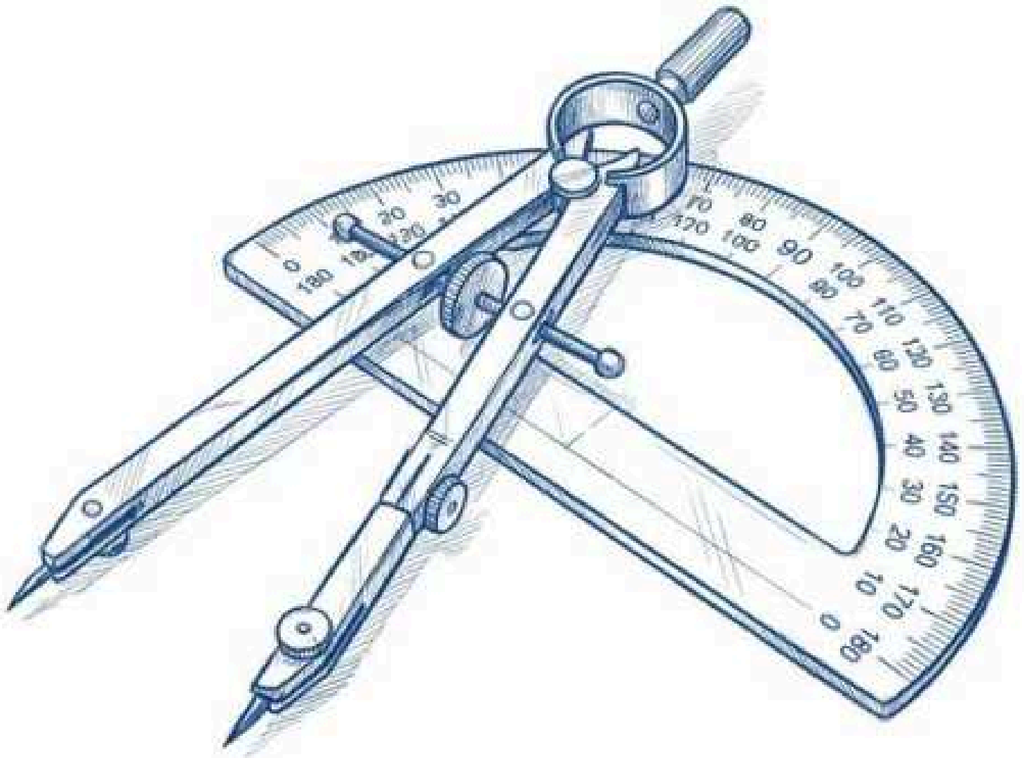
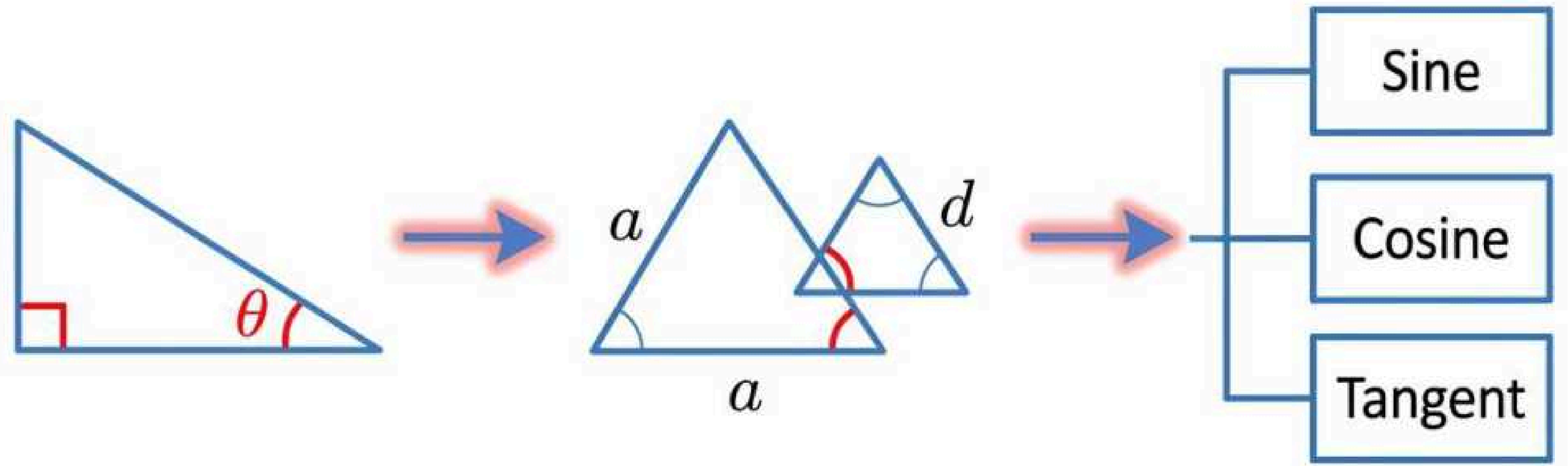
¶ 1.1



Angles, Triangles, and the Three Ratios

MATH 110 B Lecture 1

The Trigonometric Roadmap



Start: The Right Triangle

Bridge: One observation on similarity

Goal: Name the sides & calculate the three ratios.

Everything in the later lectures rests on this one.

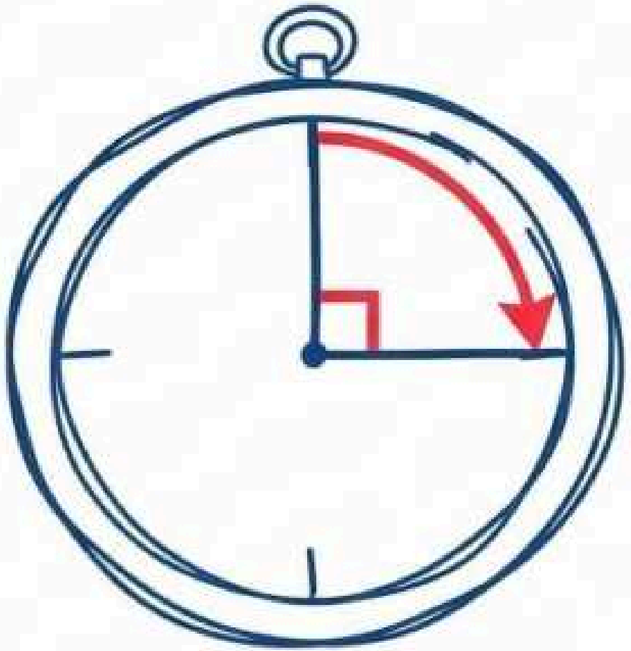
Angles Measure the Amount of Turning



1 Full Turn $= 360°$



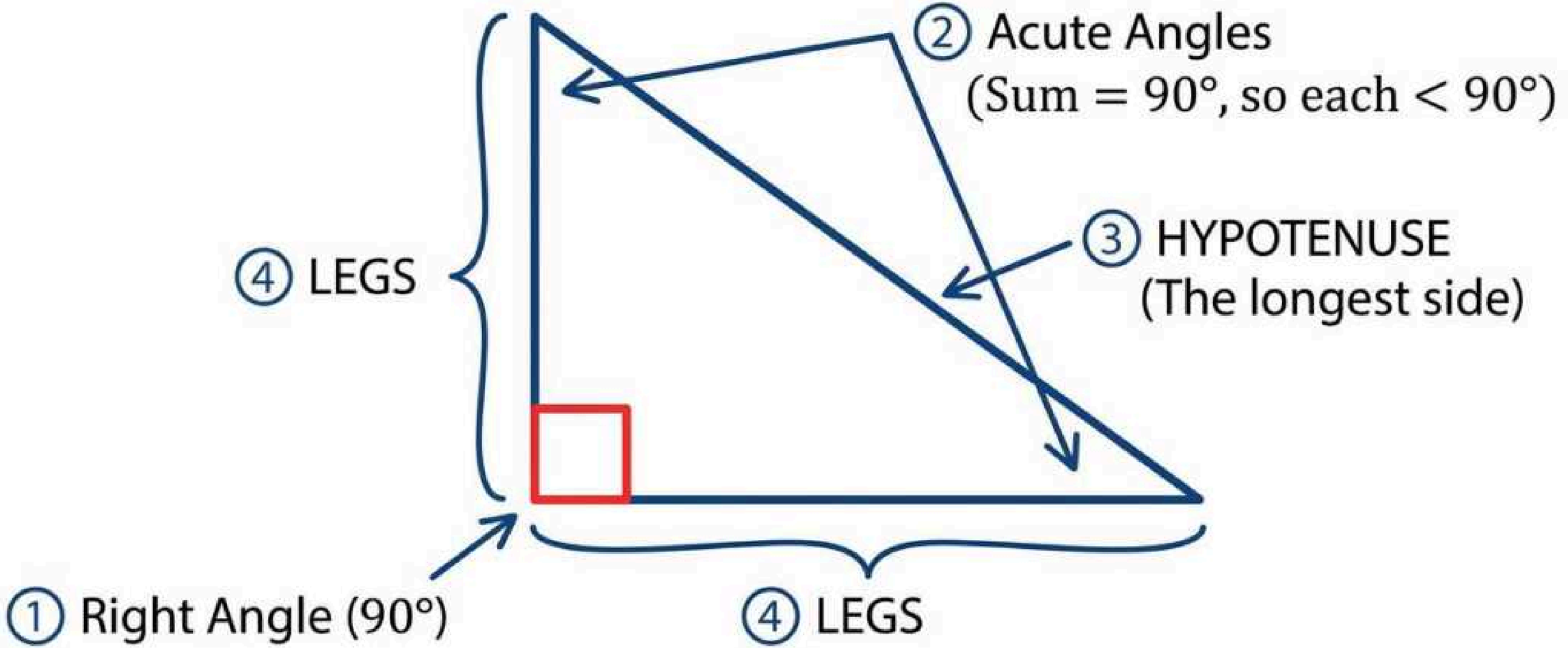
$\frac{1}{2}$ Turn $= 180°$



$\frac{1}{4}$ Turn $= 90°$ (Right Angle)

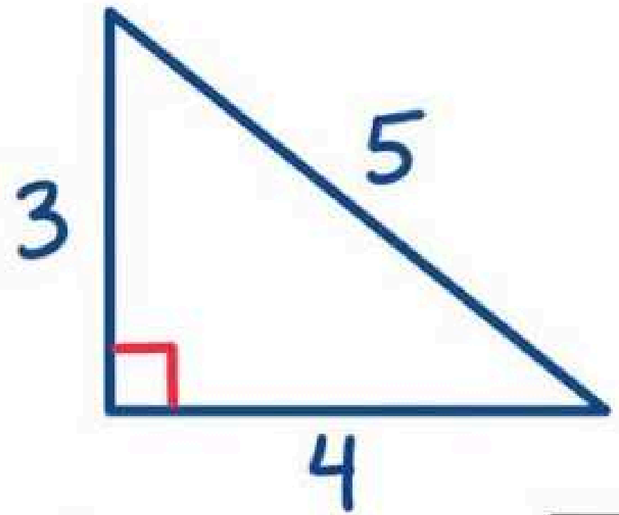
$\sum$ Angles in Any Triangle $= 180°$

Anatomy of the Right Triangle

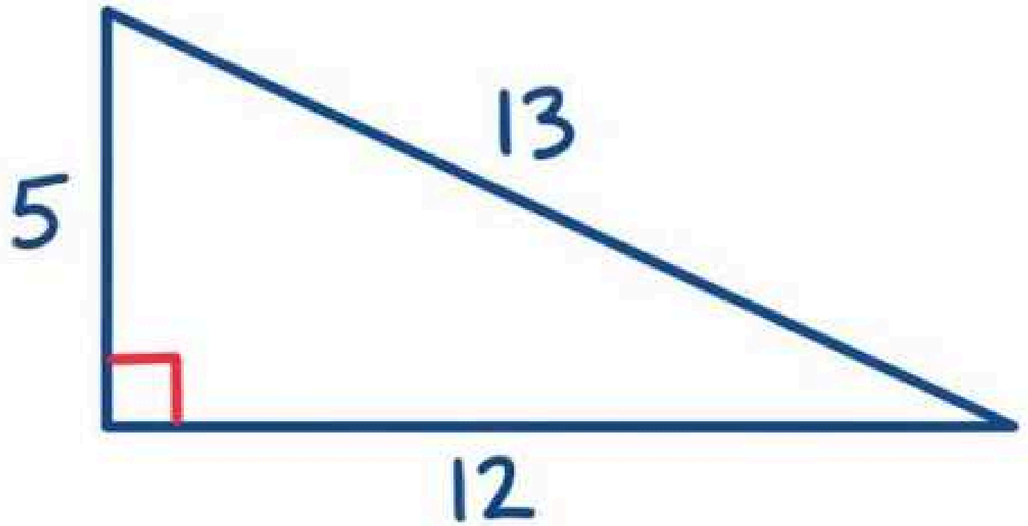


The Pythagorean Theorem

$$a^2 + b^2 = c^2$$

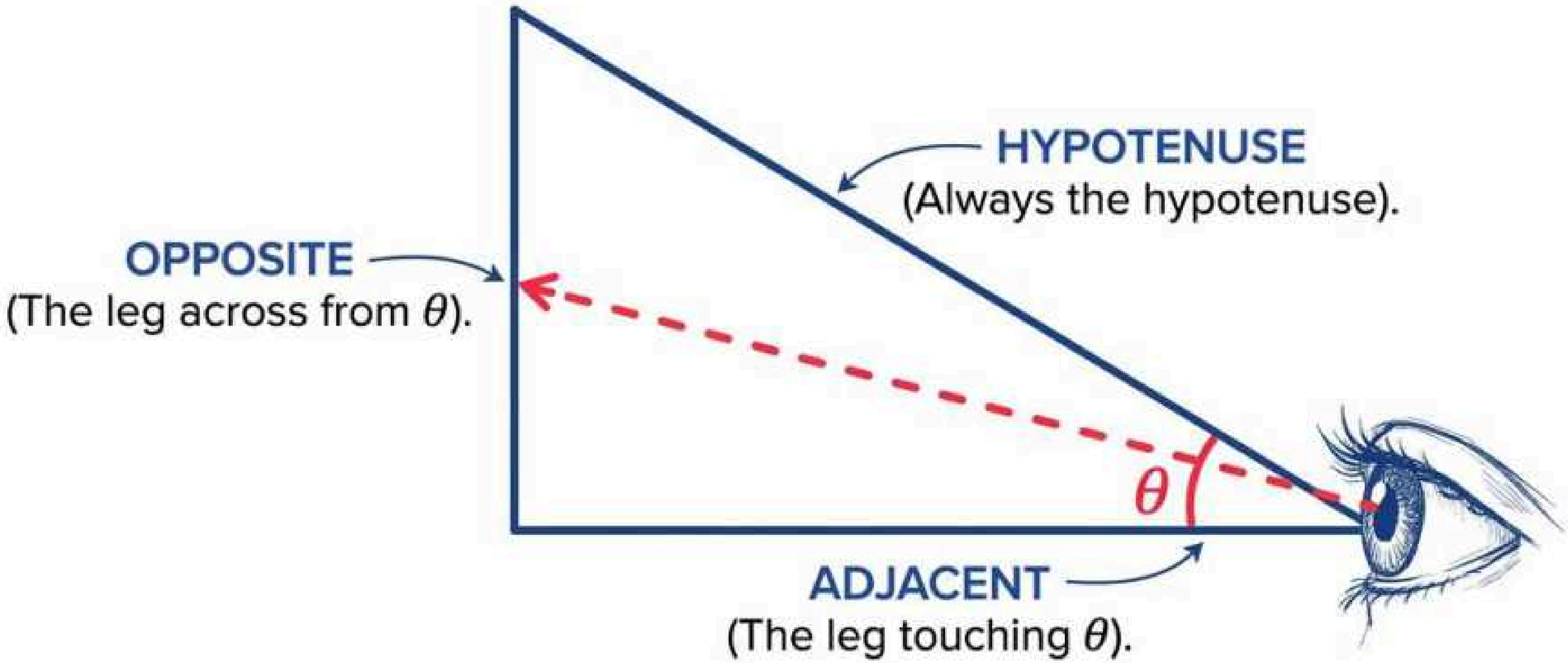


$$9 + 16 = 25 \rightarrow \sqrt{25} = 5$$



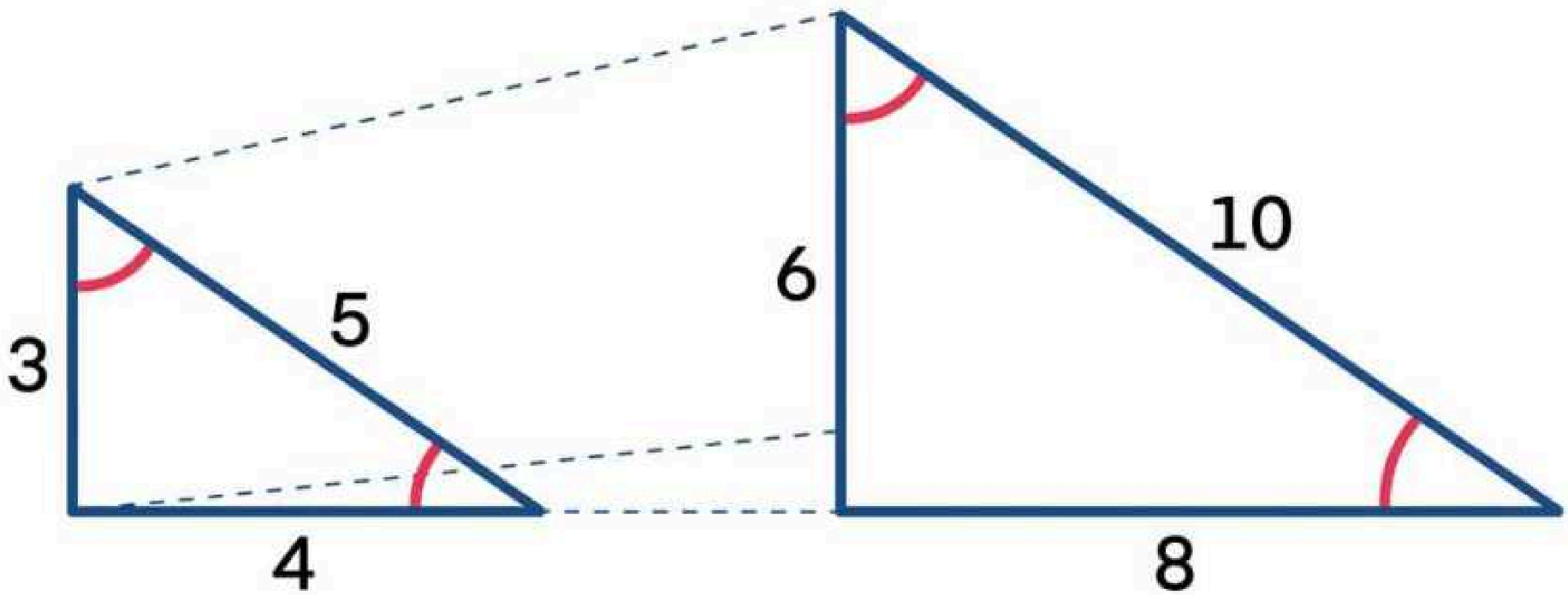
$$25 + 144 = 169 \rightarrow \sqrt{169} = 13$$

The Observer's Point of View (θ)



Warning: If you switch to the other acute angle, Opposite and Adjacent swap places!

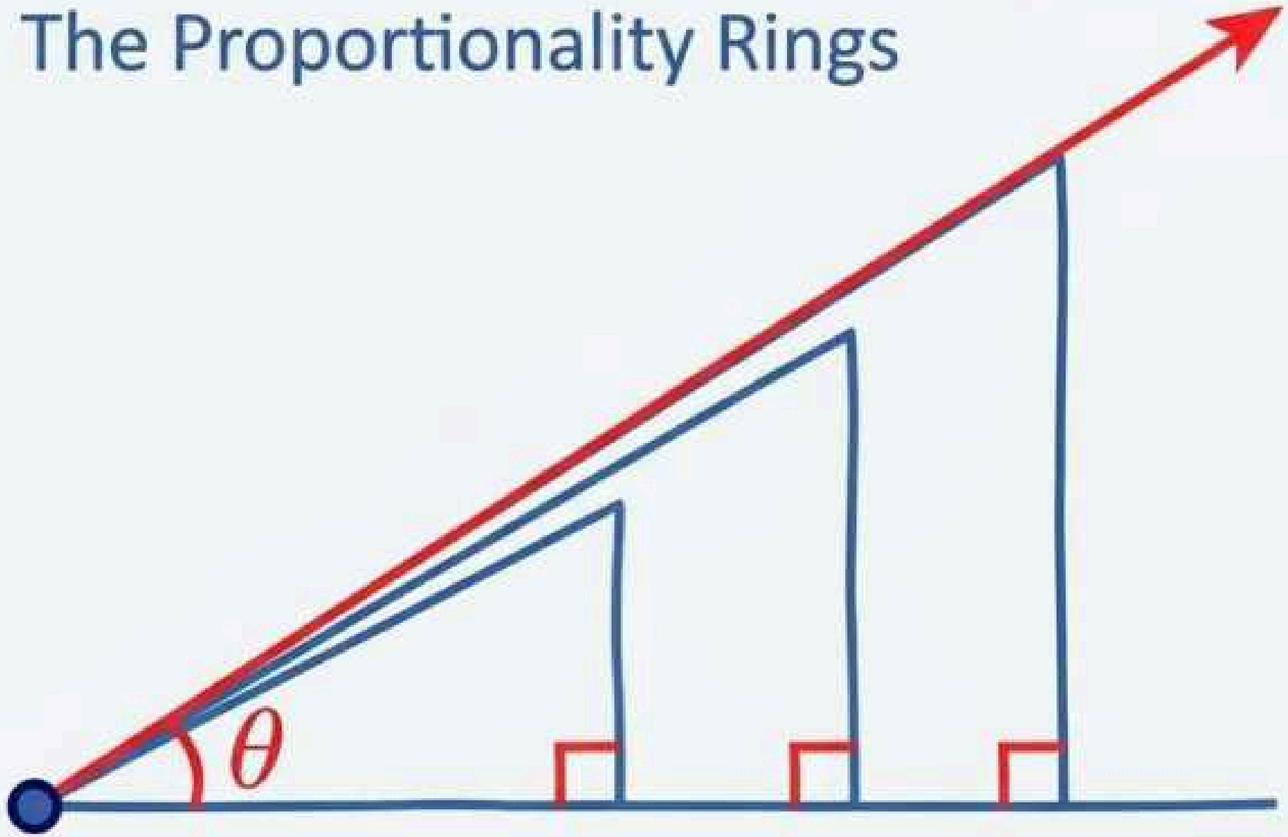
Similar Triangles: Same Shape, Different Size



- Same angles = Similar triangles.
- Matching sides are in the same proportion.
- Ratio check: In both, Short Leg = 3/5 of Hypotenuse (3/5 and 6/10).

The Key Observation: Ratio Depends Only on the Angle

The Proportionality Rings



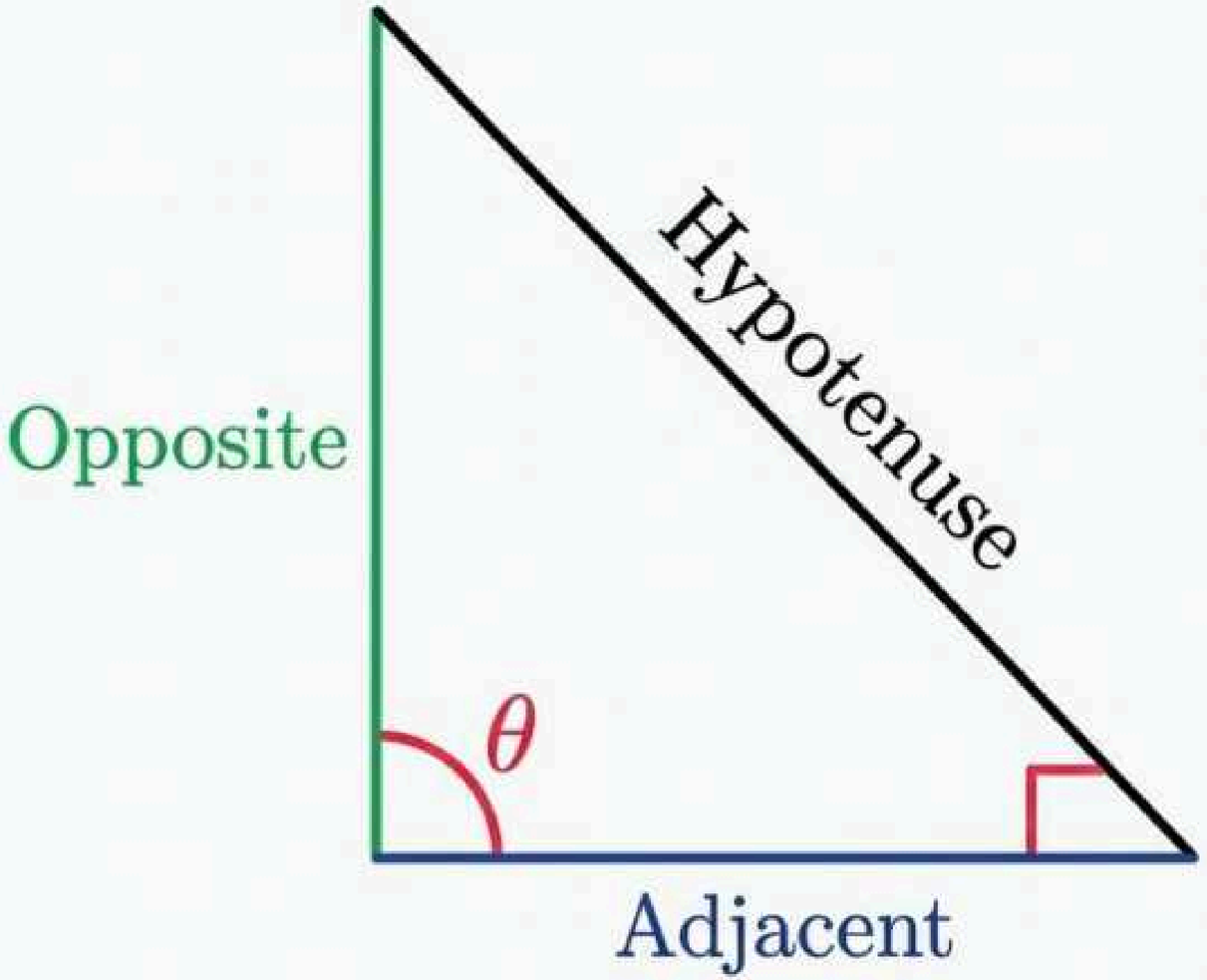
Every right triangle with the same acute angle θ is similar.

The ratio of any two sides remains identical, no matter the size of the triangle.

Conclusion: Each ratio is a function of the angle.

The Three Ratios

¶ 1.9



$$\sin(\theta) = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\cos(\theta) = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\tan(\theta) = \frac{\text{Opposite}}{\text{Adjacent}}$$

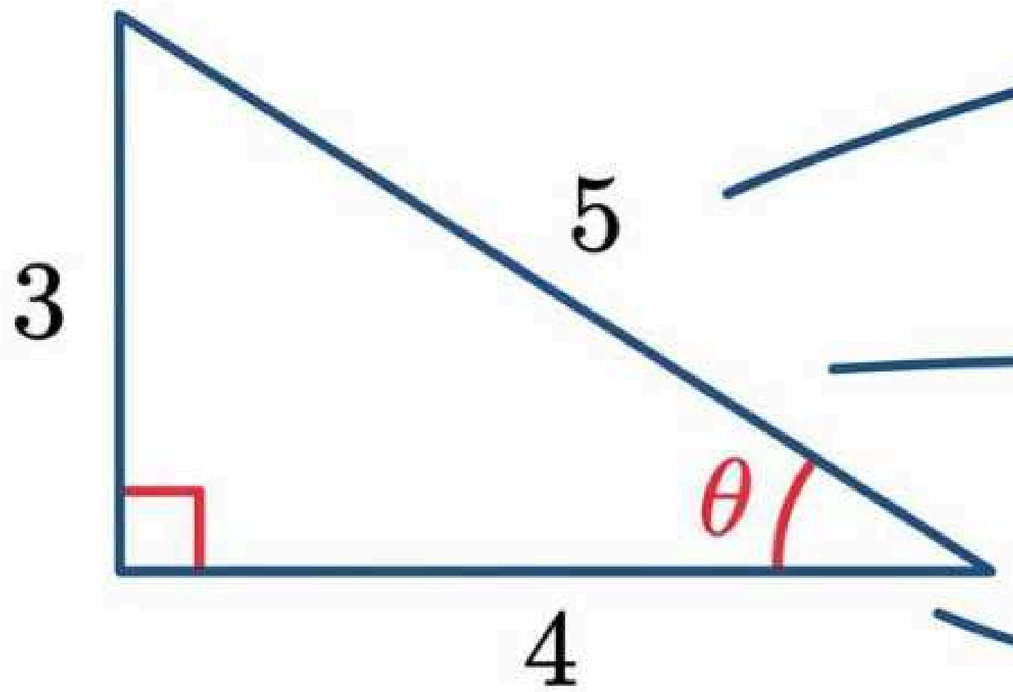
The Ultimate Rescue Tool

$$\underbrace{\text{SOH}}\quad\underbrace{\text{CAH}}\quad\underbrace{\text{TOA}}$$

$$\text{Sine} = \frac{\text{Opposite}}{\text{Hypotenuse}} \qquad \text{Cosine} = \frac{\text{Adjacent}}{\text{Hypotenuse}} \qquad \text{Tangent} = \frac{\text{Opposite}}{\text{Adjacent}}$$

Note: Write it at the top of every page of work. It will rescue you for years.

Calculating the Ratios

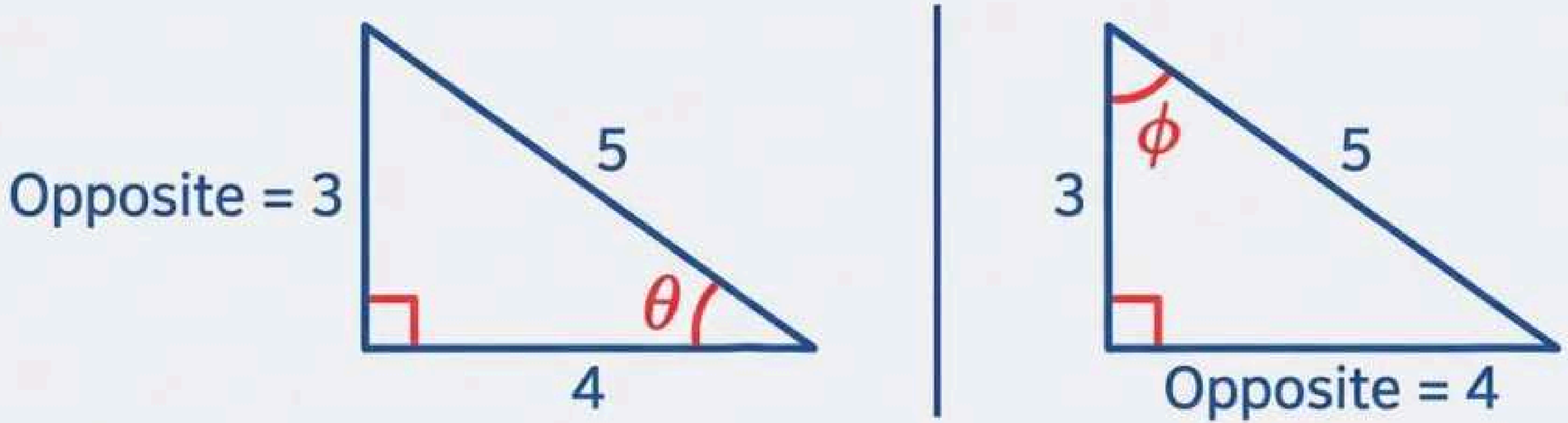


$$\sin(\theta) = \frac{3}{5} = 0.6$$

$$\cos(\theta) = \frac{4}{5} = 0.8$$

$$\tan(\theta) = \frac{3}{4} = 0.75$$

The Complementary Swap

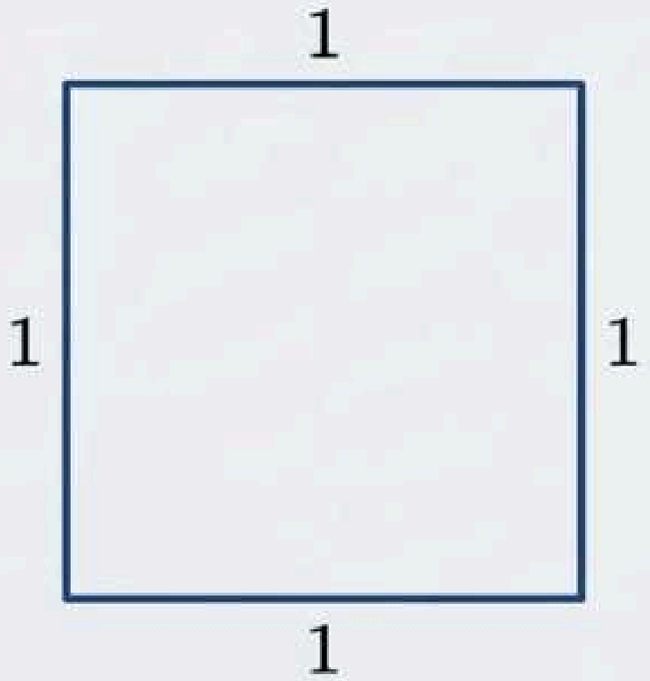


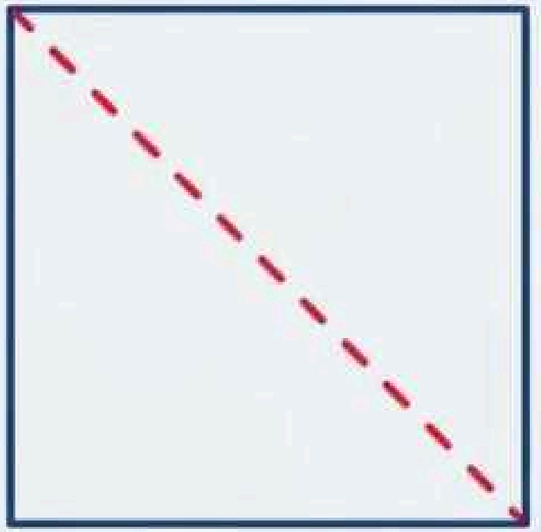
$$\text{Sine} = \frac{3}{5} \qquad \text{Sine} = \frac{4}{5}$$

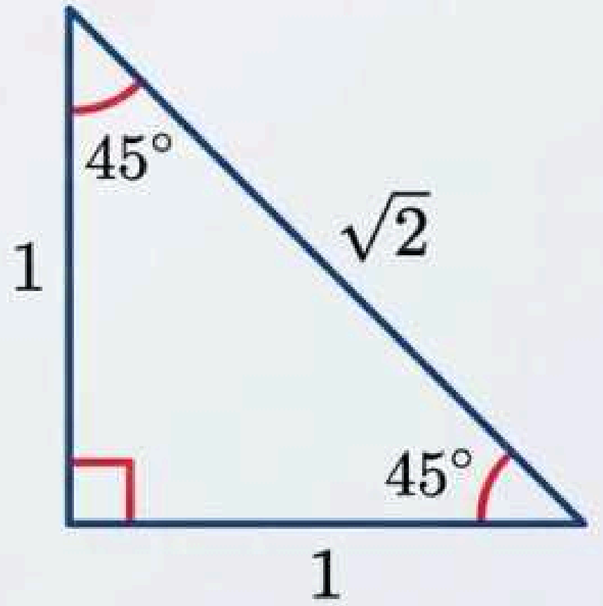
$$\text{Cosine} = \frac{4}{5} \qquad \text{Cosine} = \frac{3}{5}$$

- The sine of one angle equals the cosine of the other.
- The angles add to 90° (they are complements).
- Co-Sine literally means: The sine of the complement.

Deriving Exact Values: $45°$





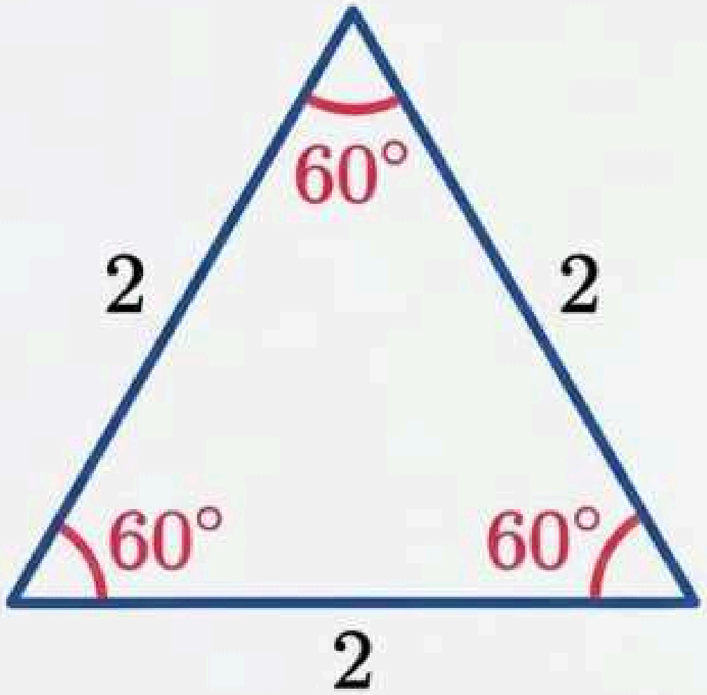


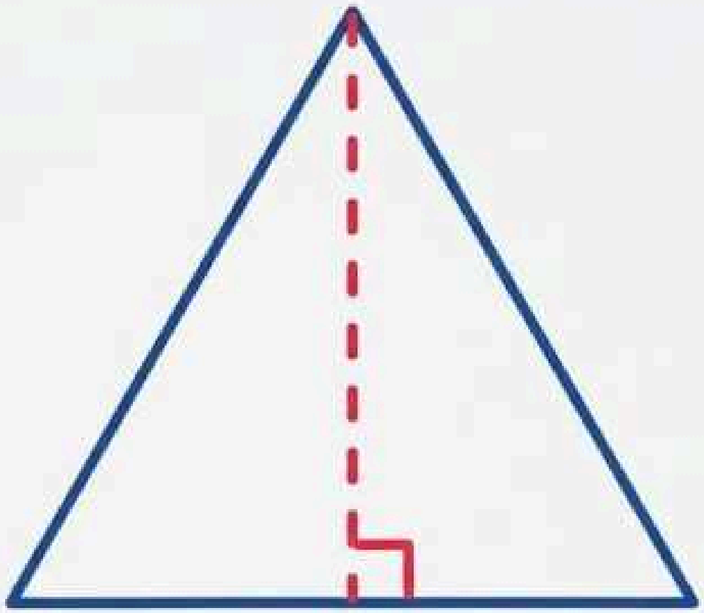
$$\sin(45°) = \frac{1}{\sqrt{2}} \approx 0.7071$$

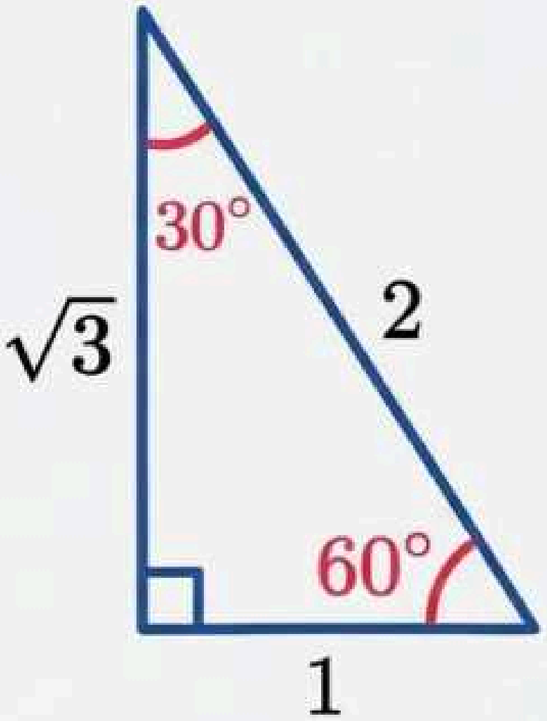
$$\tan(45°) = \frac{1}{1} = 1$$

Deriving Exact Values: 30° & 60°

¶ 1.14







$$\sin(30^\circ) = \frac{1}{2}$$

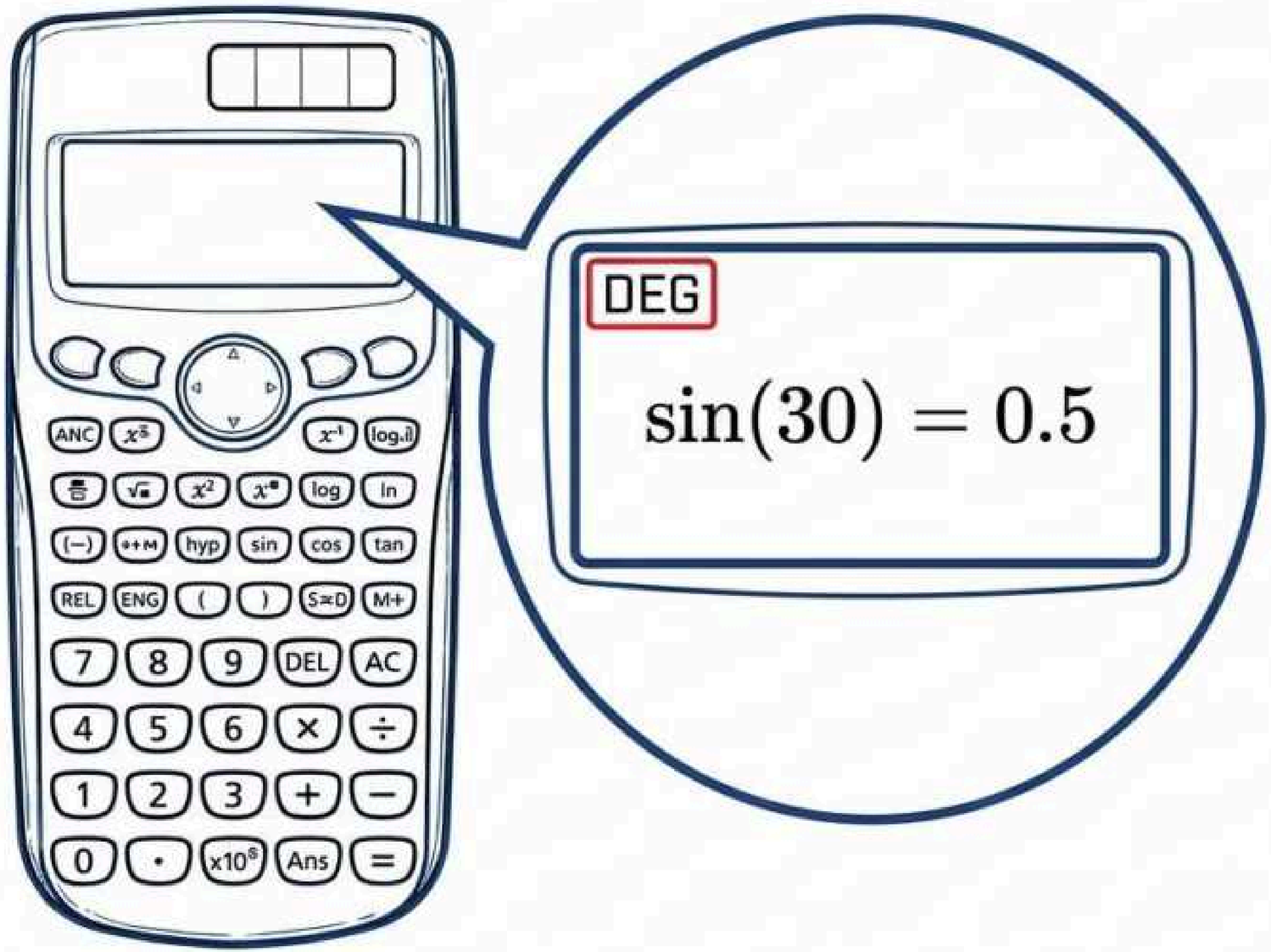
$$\cos(30^\circ) = \frac{\sqrt{3}}{2} \approx 0.8660$$

The Essential Exact Values

	Sine	Cosine	Tangent
30°	$\frac{1}{2}$ 0.5	$\frac{\sqrt{3}}{2}$ 0.8660	$\frac{1}{\sqrt{3}}$ 0.5774
45°	$\frac{1}{\sqrt{2}}$ 0.7071	$\frac{1}{\sqrt{2}}$ 0.7071	1
60°	$\frac{\sqrt{3}}{2}$ 0.8660	$\frac{1}{2}$ 0.5	$\sqrt{3}$ 1.7321

Learn these five facts, and you will meet them again and again.

For Other Angles: The Calculator



Step 1: Set mode to Degrees.

Step 2: Test by asking for $\sin(30°)$. If it does not equal exactly 0.5, the mode is wrong!

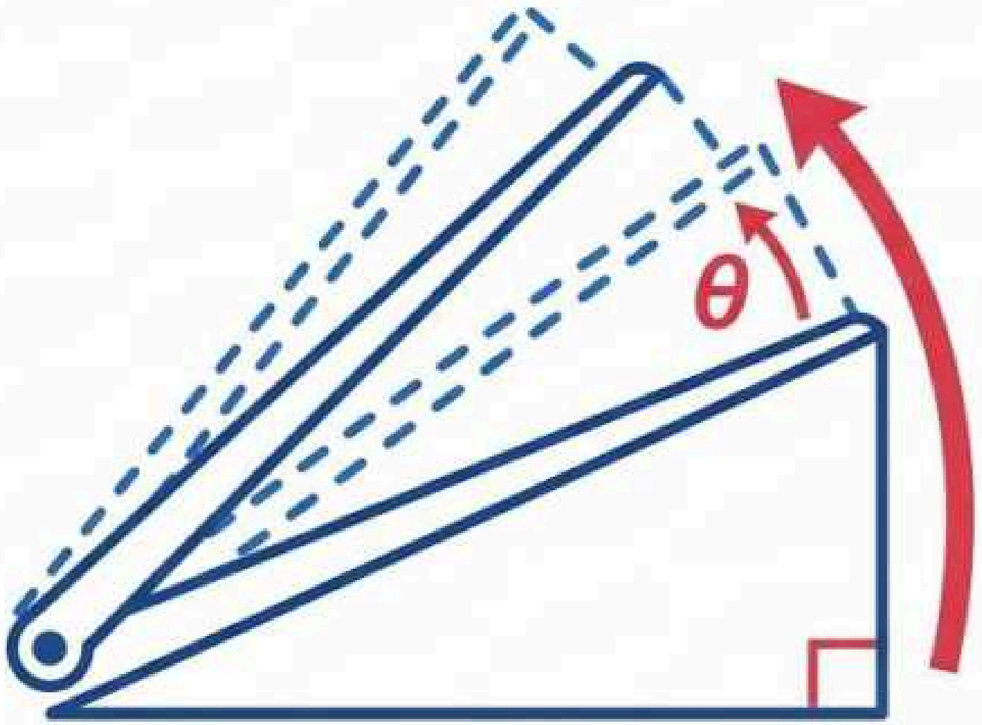
Examples:

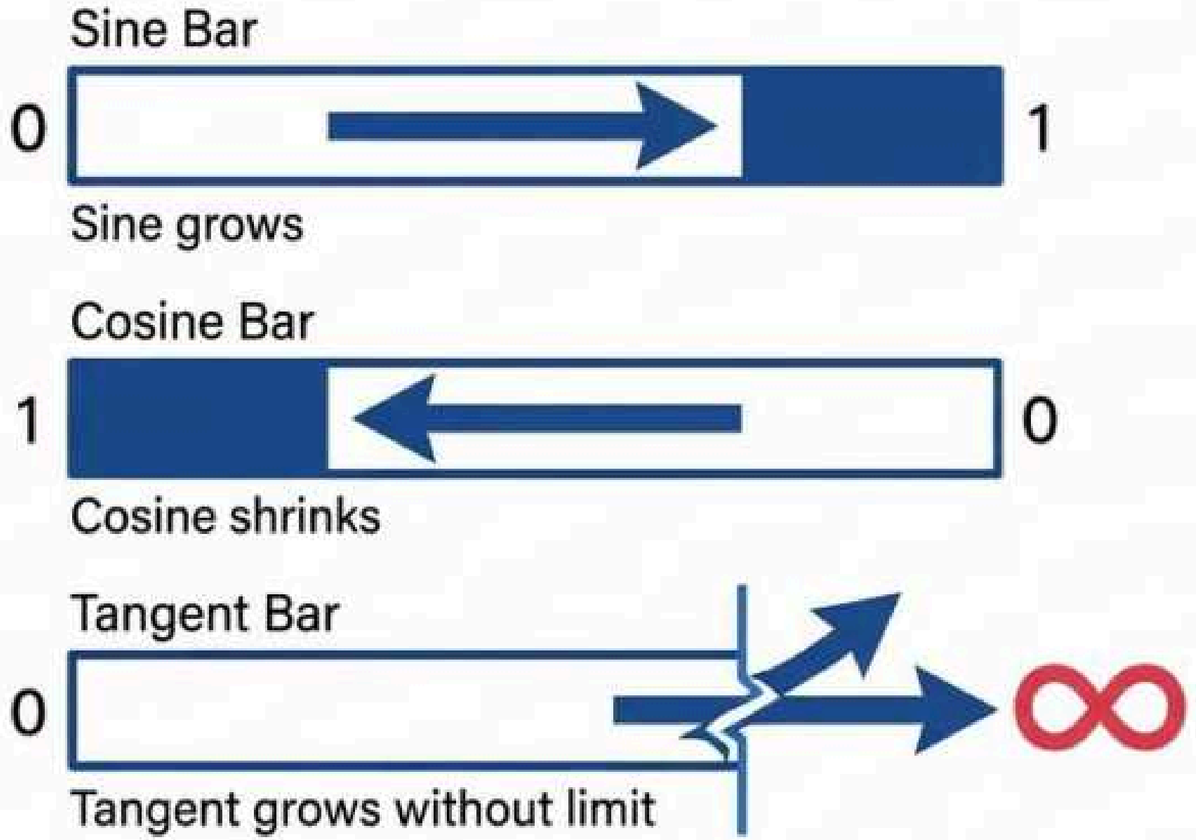
$$\sin(20°) \approx 0.3420$$

$$\cos(20°) \approx 0.9397$$

$$\tan(20°) \approx 0.3640$$

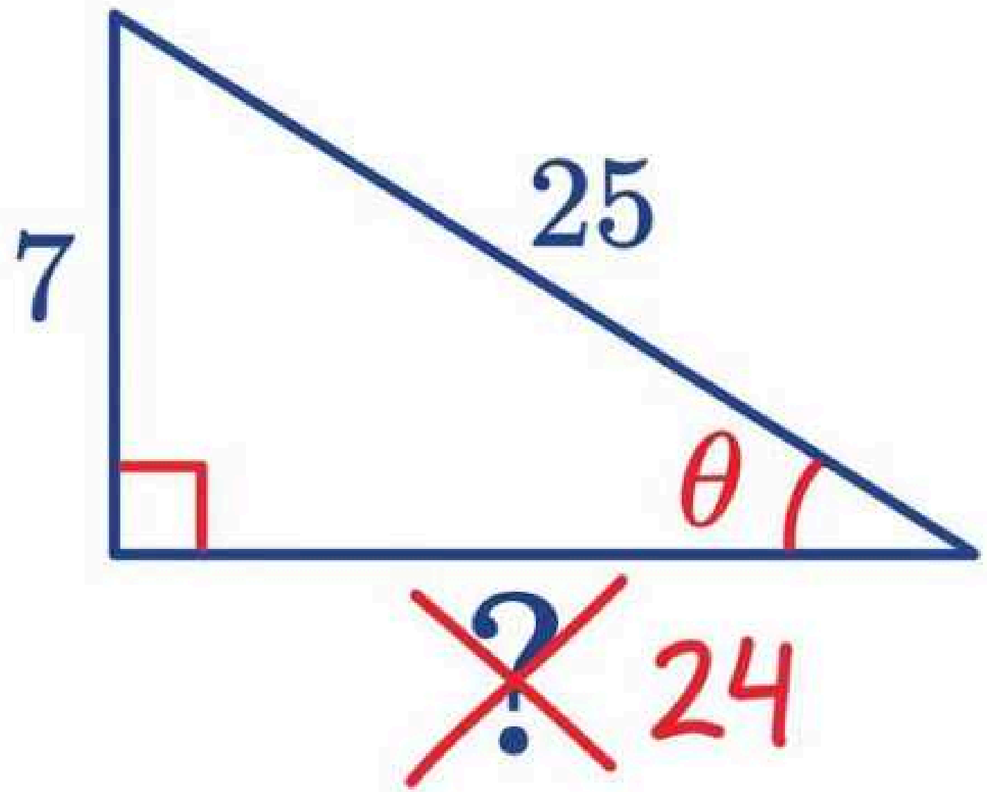
Dynamic Patterns (0° → 90°)





- Sine and Cosine are always between 0 and 1 (because the hypotenuse is always the longest side).
- As θ grows to 90°: Sine grows, Cosine shrinks, Tangent explodes (adjacent side shrinks toward nothing).
- Crucial: Sine is a function, not a number multiplied by θ.

Bringing It Together: A Final Calculation



Step 1 (Pythagoras):

$$\begin{aligned}\text{Adjacent} &= \sqrt{25^2 - 7^2} \\ &= \sqrt{625 - 49} \\ &= \sqrt{576} = 24\end{aligned}$$

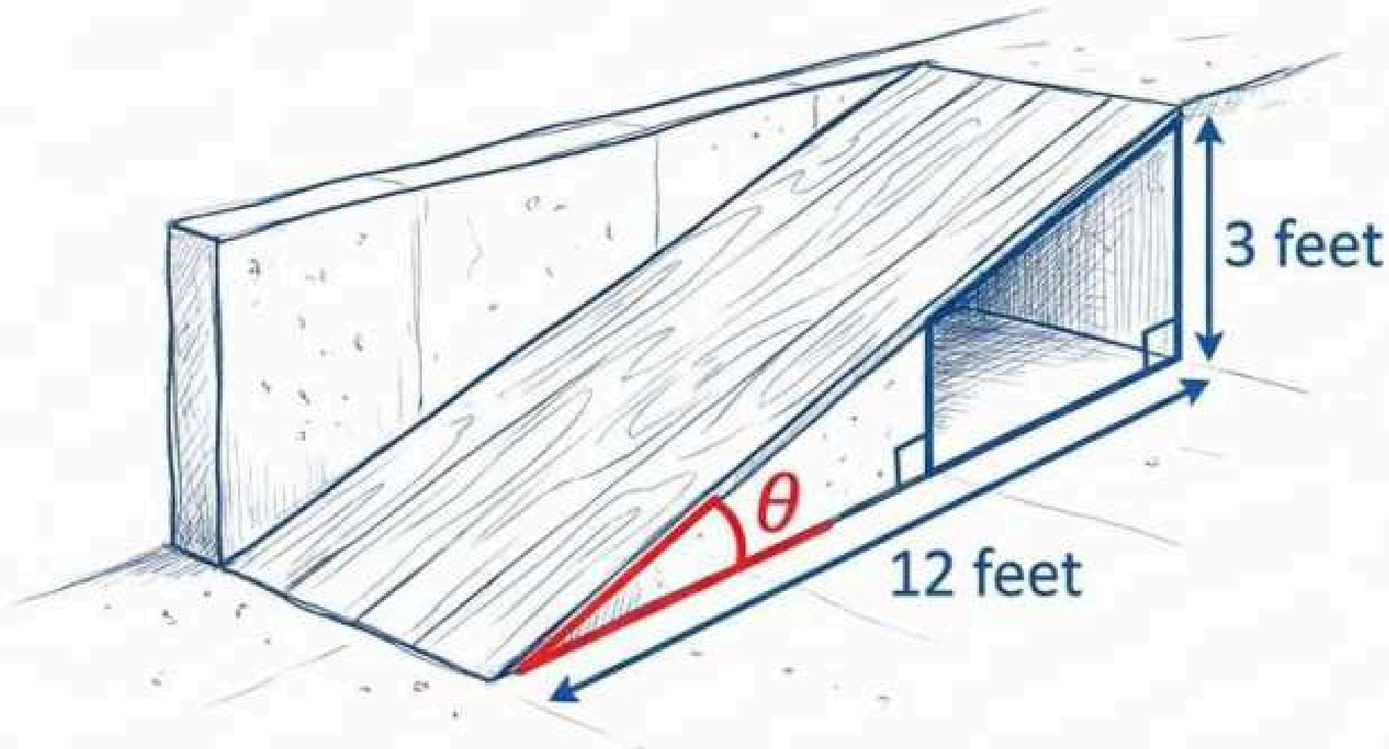
Step 2 (Cosine):

$$\cos(\theta) = \frac{24}{25} = 0.96$$

Step 3 (Tangent):

$$\tan(\theta) = \frac{7}{24} \approx 0.2917$$

The Takeaway Question



A ramp rises 3 feet over a horizontal run of 12 feet.

Question 1: Which ratio describes how steep it is?

Question 2: What does that ratio equal?

$$\text{Ratio}(\theta) = \left[\ ?\ \right]$$

Next time: Using the three ratios to solve triangles and measure the unreachable.
Try to answer this before Lecture 2!