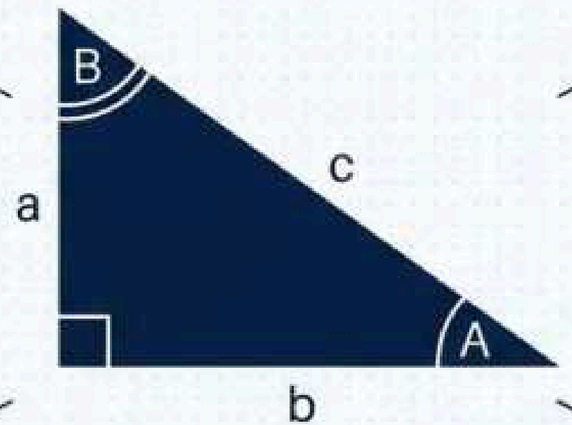
Solving Right Triangles and Their Applications

¶ 2.1

The Routine
(SOH CAH TOA)

Real-World Architectures
(Ladders, Cliffs, Roofs)



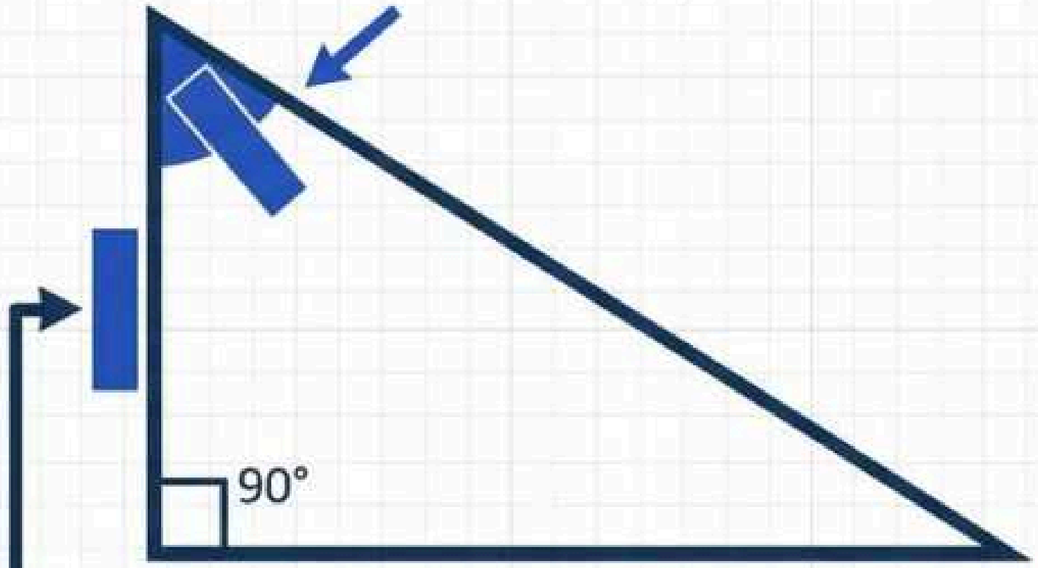
Reversing the Math
(Inverse Functions)

Quality Control
(Avoiding Common Traps)

What It Means to "Solve" a Triangle

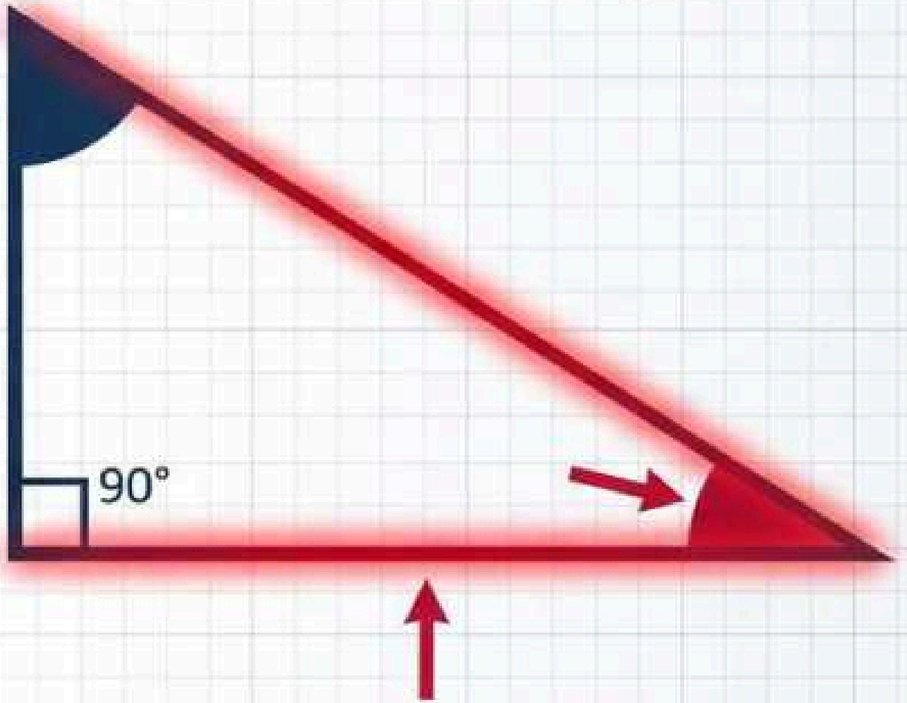
¶ 2.2

State 1: The Inputs



Requires **exactly TWO** pieces of information, at least **ONE** must be a side.

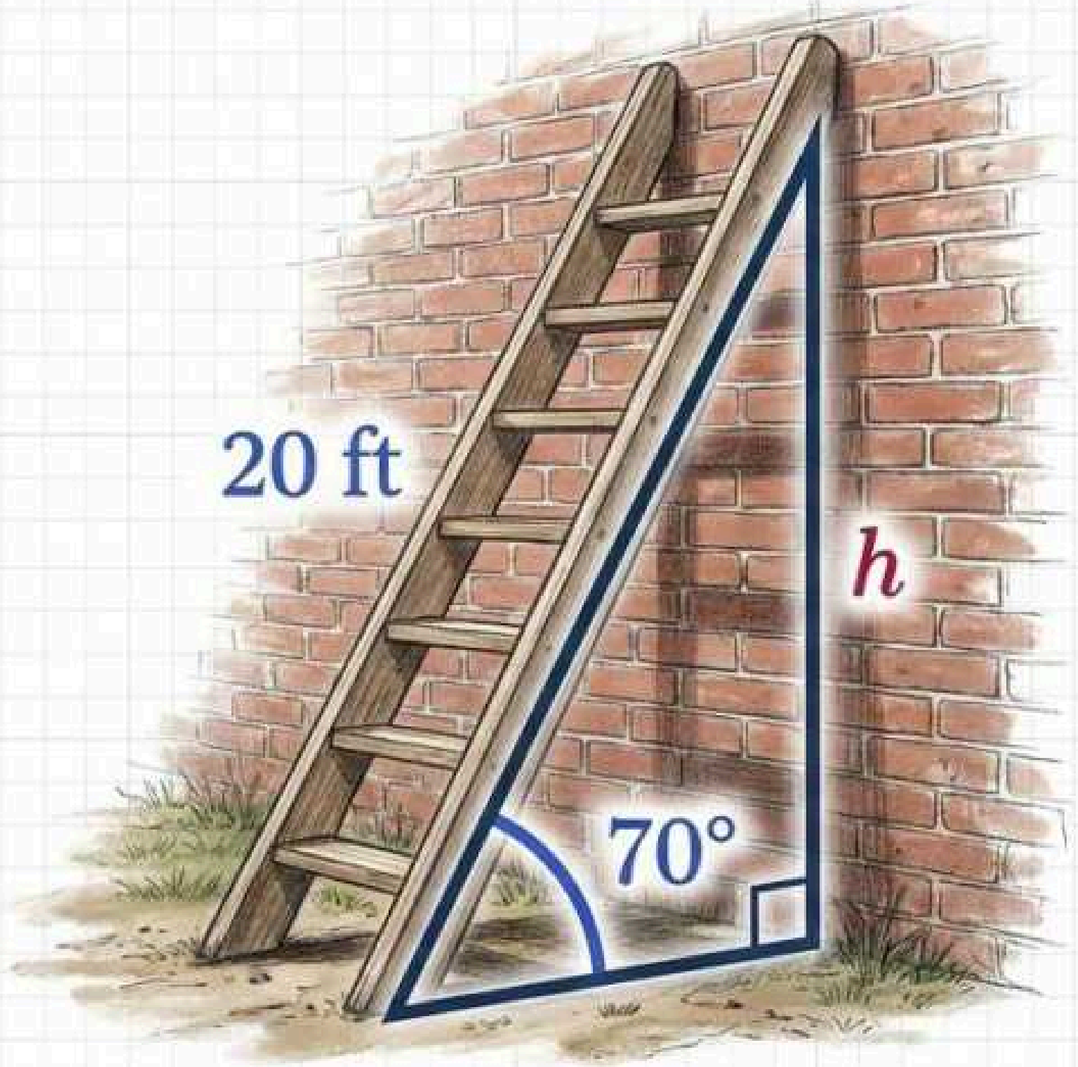
State 2: The Outputs



The **method is always the same:** Choose the ratio that contains the **ONE thing you want** and the **TWO things you know.**

Starting With an Unknown Side: The Sine Ratio

¶ 2.3



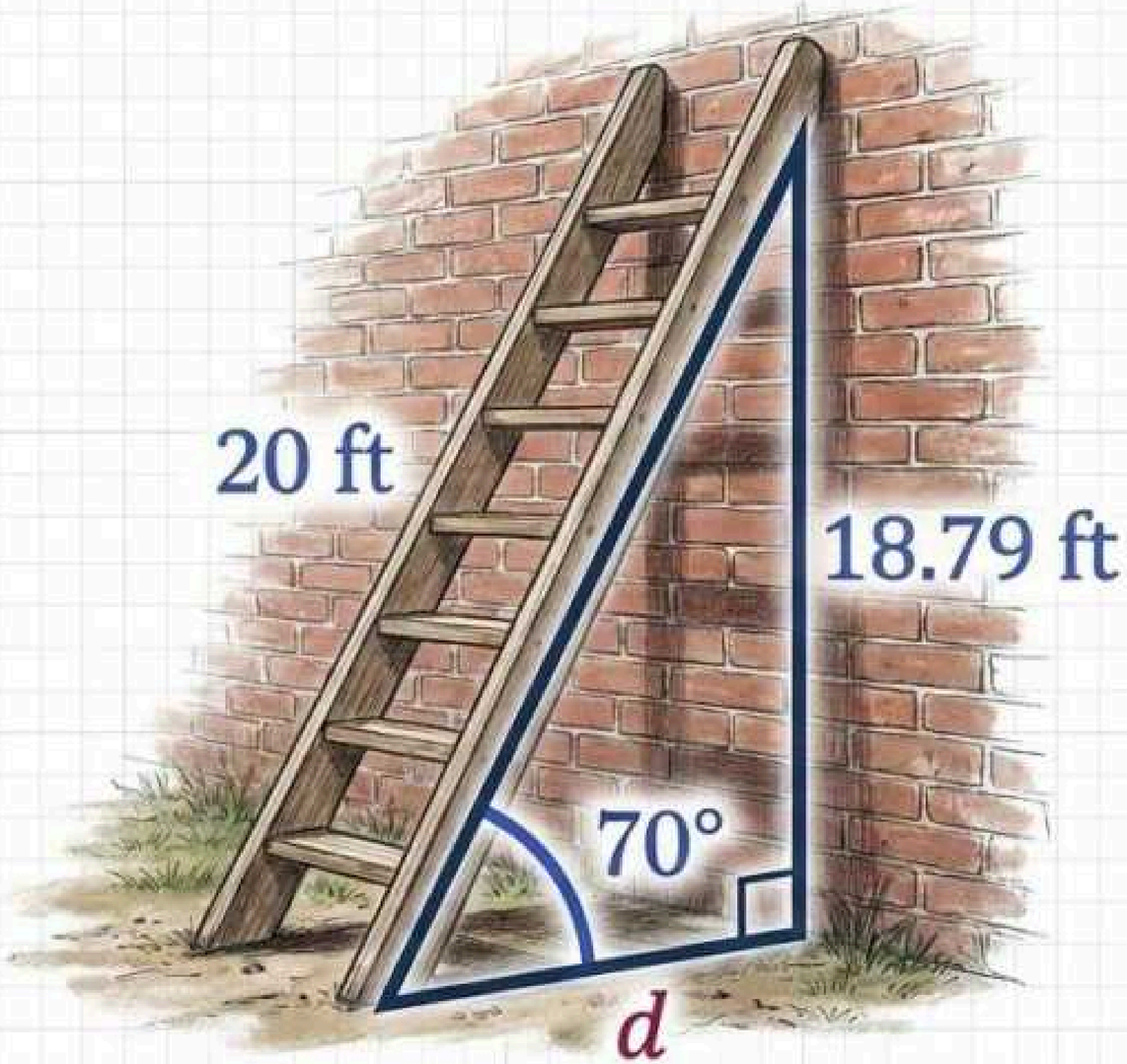
Step 1: $\sin(70°) = \dfrac{h}{20}$

Step 2: $h = 20 \cdot \sin(70°)$

Result: $h \approx 18.79$ ft

Finding the Adjacent Side & Proving the Math

¶ 2.4



$$\cos(70°) = \frac{d}{20}$$

$$d = 20 \cdot \cos(70°) \approx 6.84 \text{ ft}$$

Check it with the Pythagorean Theorem

$$18.79^2 + 6.84^2 \approx 400$$

$$400 = 20^2$$

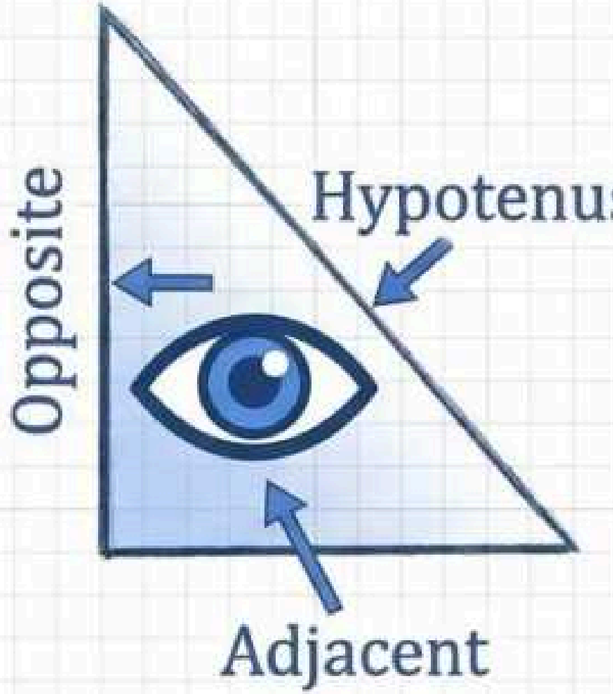
✓ (The answers agree perfectly)

The Universal Routine

¶ 2.5

1. Draw & Mark → 2. Label Sides → 3. Match Ratio





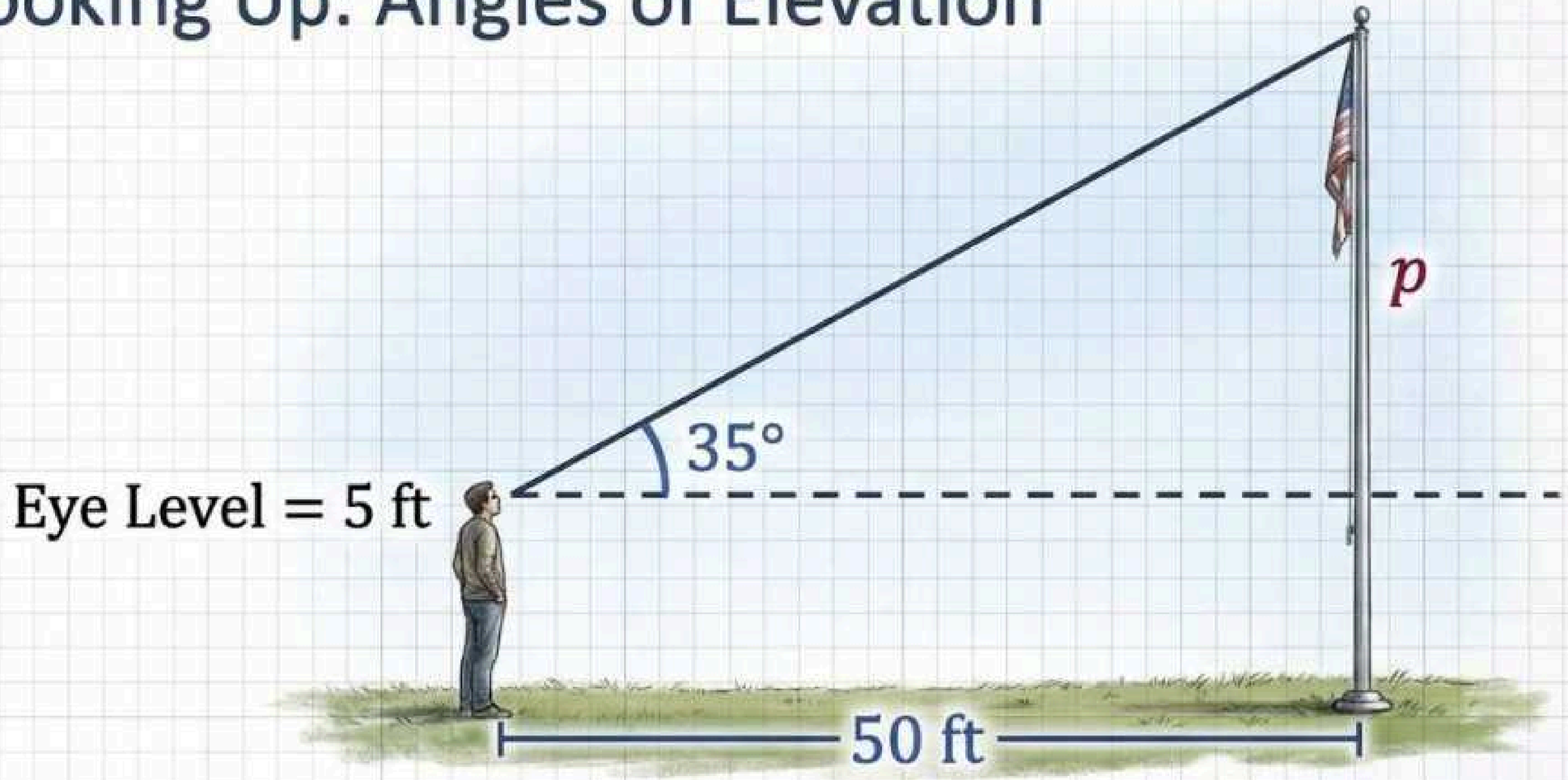
Opposite + Hypotenuse → **Sine**

Adjacent + Hypotenuse → **Cosine**

Opposite + Adjacent → **Tangent**

Looking Up: Angles of Elevation

¶ 2.6

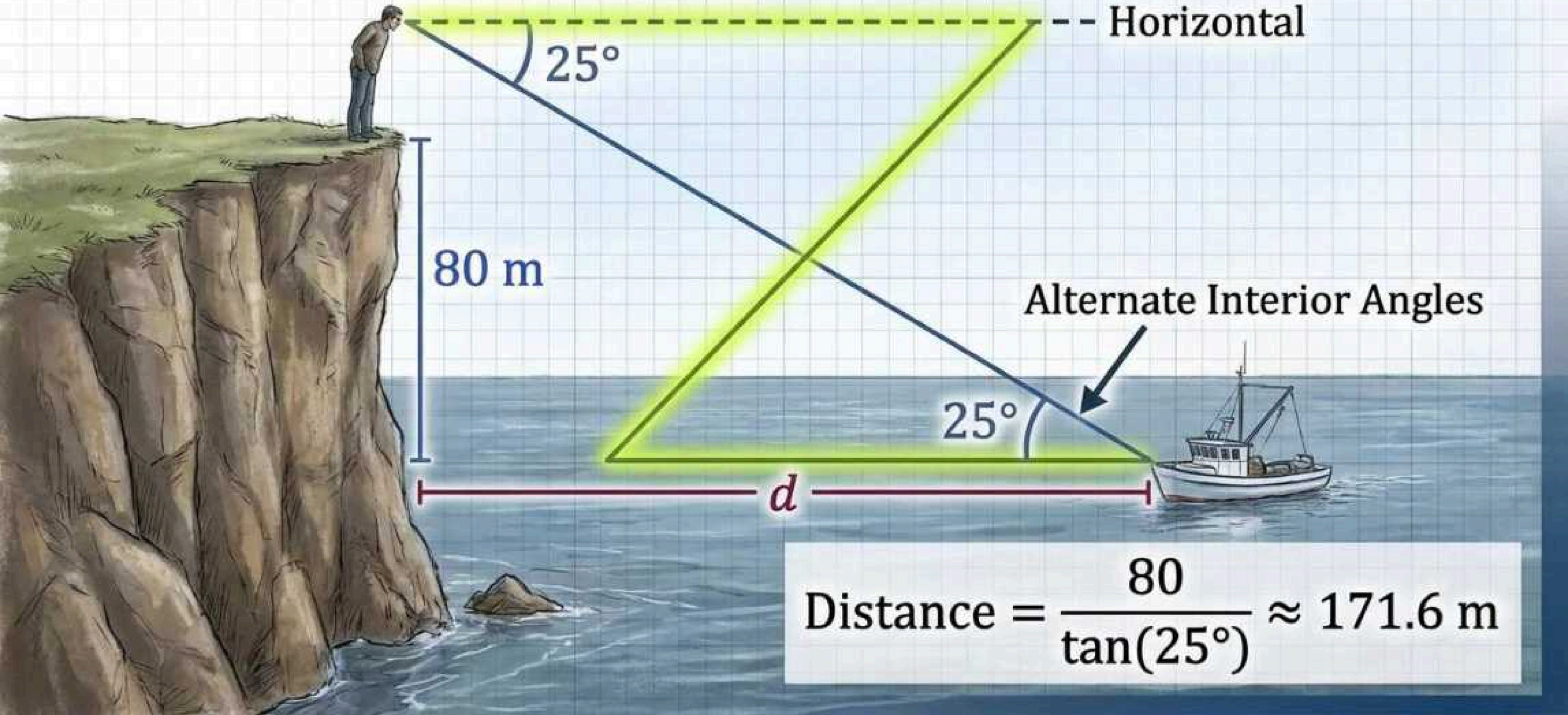


$$p = 50 * \tan(35°) \approx 35.01 \text{ ft}$$

$$\text{Total Height} = 35.01 \text{ ft} + 5 \text{ ft (Eye Level)} = 40 \text{ ft}$$

Looking Down: Angles of Depression

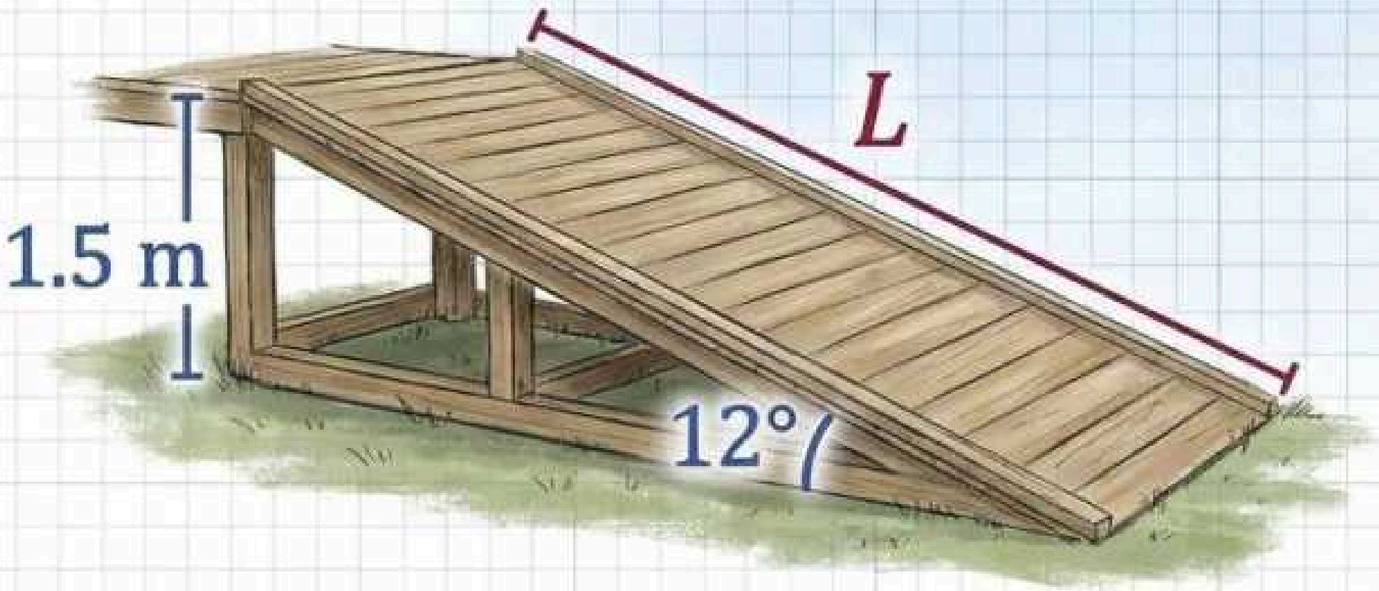
¶ 2.7



$$\text{Distance} = \frac{80}{\tan(25°)} \approx 171.6 \text{ m}$$

Algebraic Hurdles: Unknowns in the Denominator

¶ 2.8



$$\sin(12°) = \frac{1.5}{L}$$

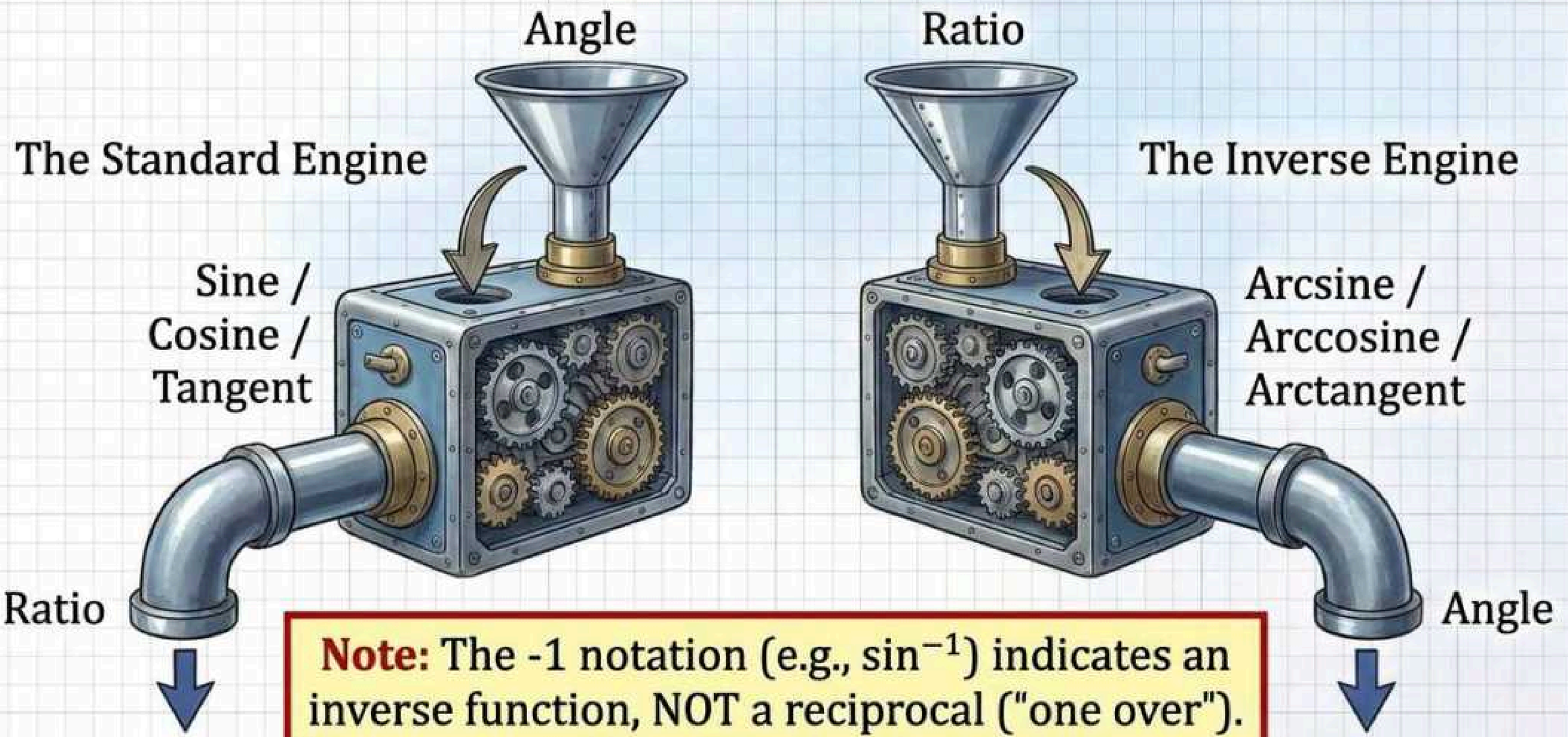
$$L = \frac{1.5}{\sin(12°)}$$

Result: $L \approx 7.21$ m

That is fine. It is just algebra.

Running the Ratio Backwards

¶ 2.9

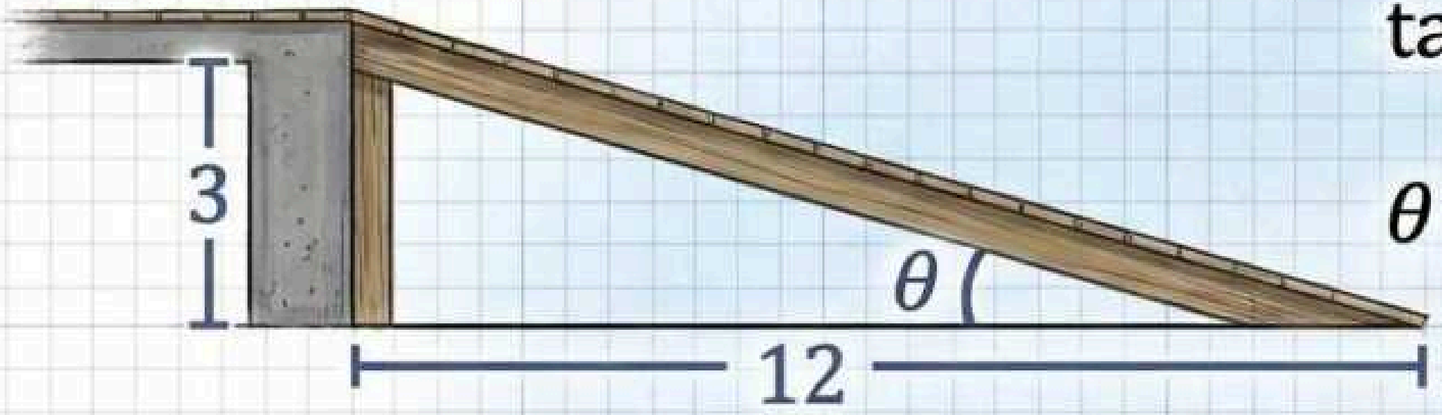


Note: The -1 notation (e.g., $\sin^{-1}$) indicates an inverse function, NOT a reciprocal ("one over").

Calculating Real-World Slopes (Inverse Tangent)

¶ 2.10

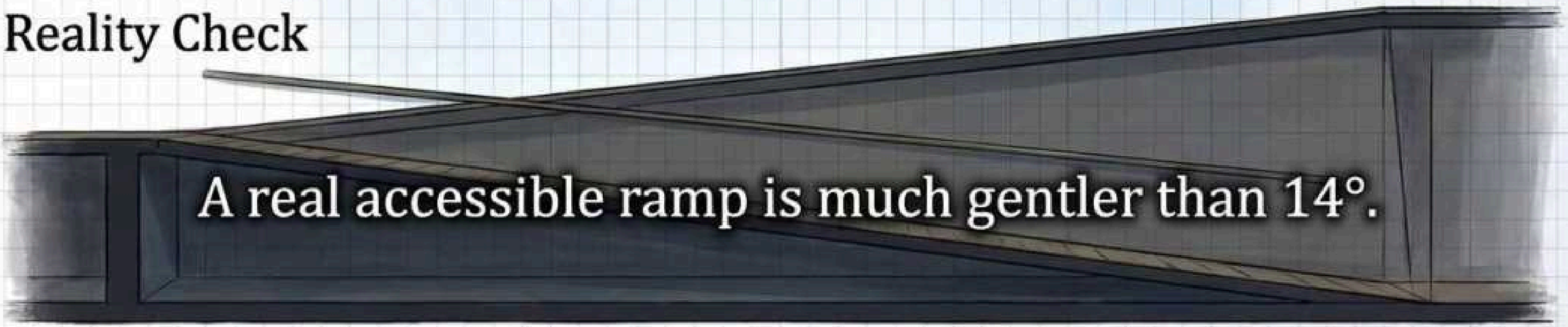
Math Example



$$\tan(\theta) = \frac{3}{12} = 0.25$$

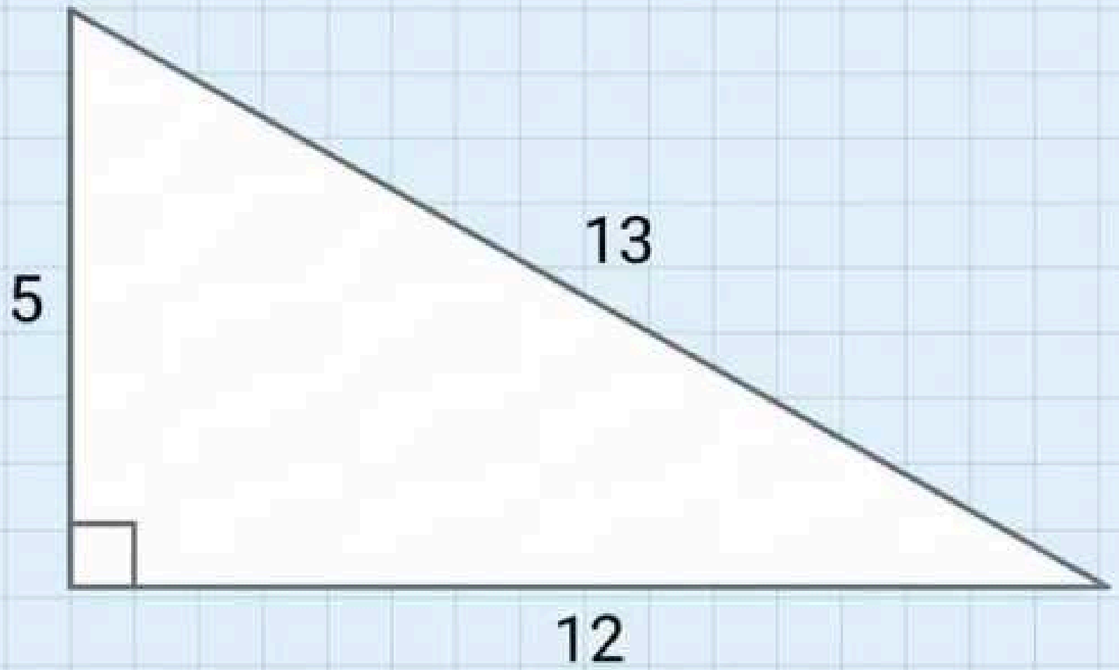
$$\theta = \tan^{-1}(0.25) \approx 14.04°$$

Reality Check



Finding All Acute Angles

¶ 2.11



Step 1: Finding the angle opposite to 5

$$\tan^{-1}\left(\frac{5}{12}\right) \approx 22.62^\circ$$

Step 2: Finding the remaining acute angle

$$90^\circ - 22.62^\circ = 67.38^\circ$$

Proof

Check the work with sine:

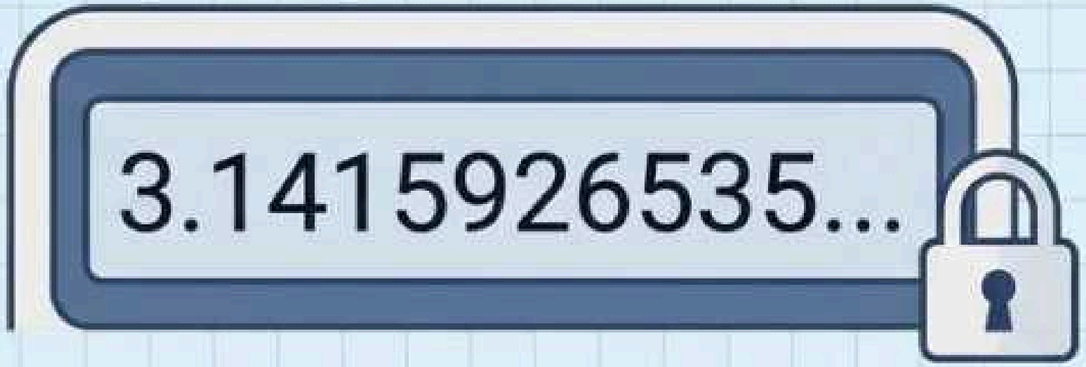
$$\sin^{-1}\left(\frac{5}{13}\right) = 22.62^\circ$$

Perfect match.

Quality Control: Accuracy and Logic Checks

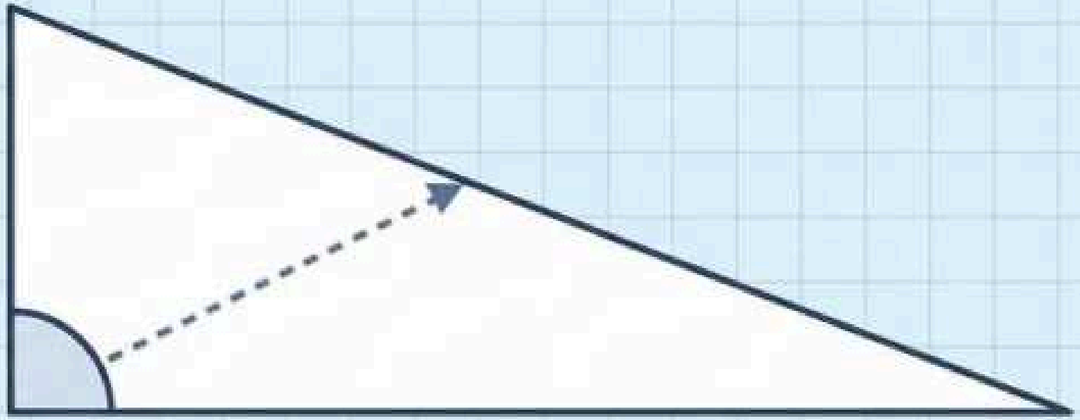
¶ 2.12

Calculator Rule



Keep ALL digits until the end.
Round only once at the last step.
Early rounding grows errors.

Logic Scales



1. The hypotenuse MUST be the longest side.
2. The largest angle MUST be opposite the longest side.

(If an answer breaks these rules, go back).

Carpentry Math in the Real World

¶ 2.13



Rise (Opposite)

$$= 12 * \tan(30°)$$

$$\approx 6.93 \text{ ft}$$

Rafter (Hypotenuse)

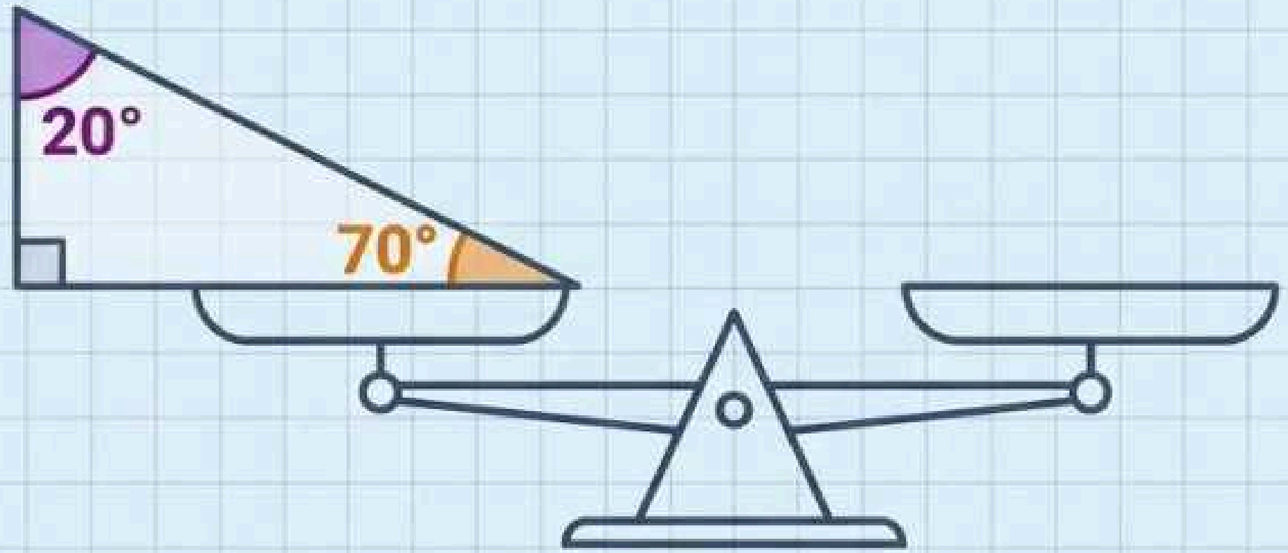
$$= \frac{12}{\cos(30°)}$$

$$\approx 13.86 \text{ ft}$$

A carpenter orders lumber from numbers like these.

The Power of Complements

¶ 2.14



$$\sin(70°) \leftrightarrow \cos(20°)$$

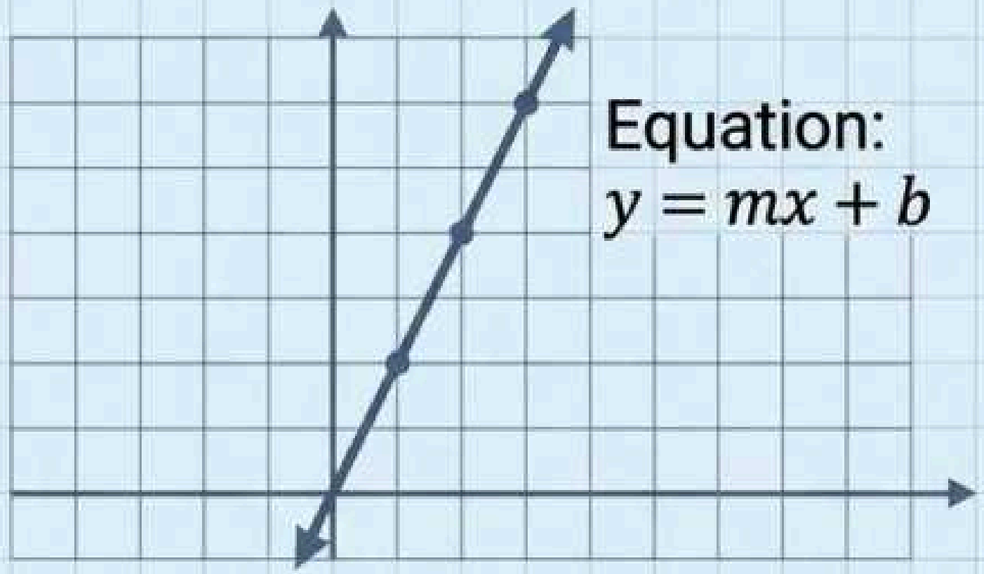
Both evaluate to ≈ 0.9397

Key Insight: The sine of an angle ALWAYS equals the cosine of its complement. This provides a free mathematical check and a tool to switch perspectives.

Translation: Algebraic Slope vs. Trigonometric Angle

¶ 2.15

Algebra (Pure)

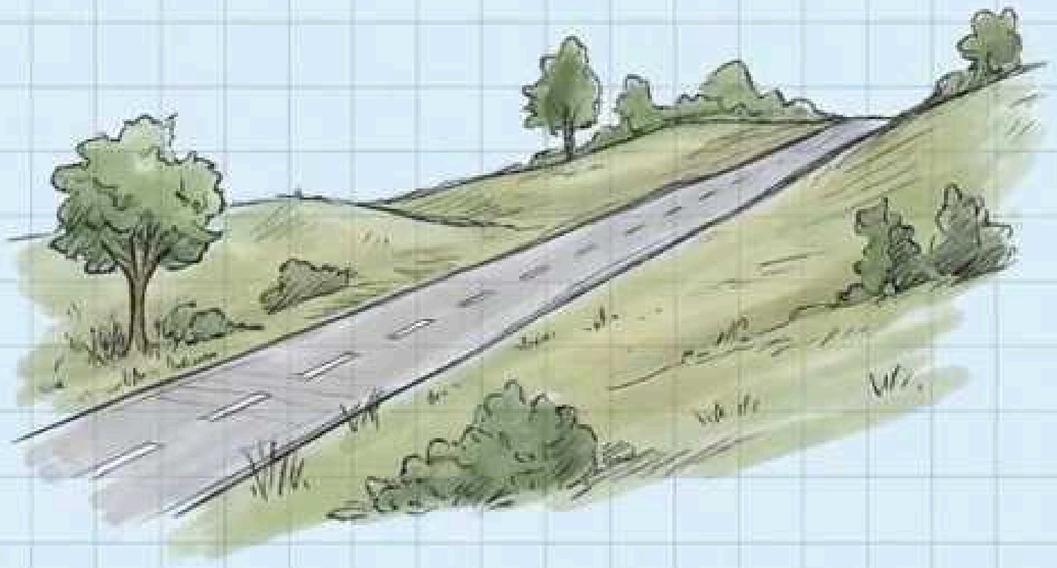


Equation:
$y = mx + b$

Slope = Rise / Run = 2

→ Translation: $\tan^{-1}(2) \approx 63.43°$

Civil Engineering (Applied)



Grade = 8% (Slope = 0.08)

→ Translation: $\tan^{-1}(0.08) \approx 4.57°$

Slope and **angle** are two ways to say the exact same thing.

The Troubleshooting Matrix: Common Mistakes

¶ 2.16

1. Wrong Calculator Mode

Fix: Always check degrees vs. radians before starting.



2. Misreading Inverse

Fix: The -1 power means inverse function, NOT a reciprocal.

$$\sin^{-1}(x) \neq \frac{1}{\sin(x)}$$

3. Wrong Side Label

Fix: Mixing up Opposite and Adjacent—always refer back to the observer's vantage point.



4. Forgetting Units

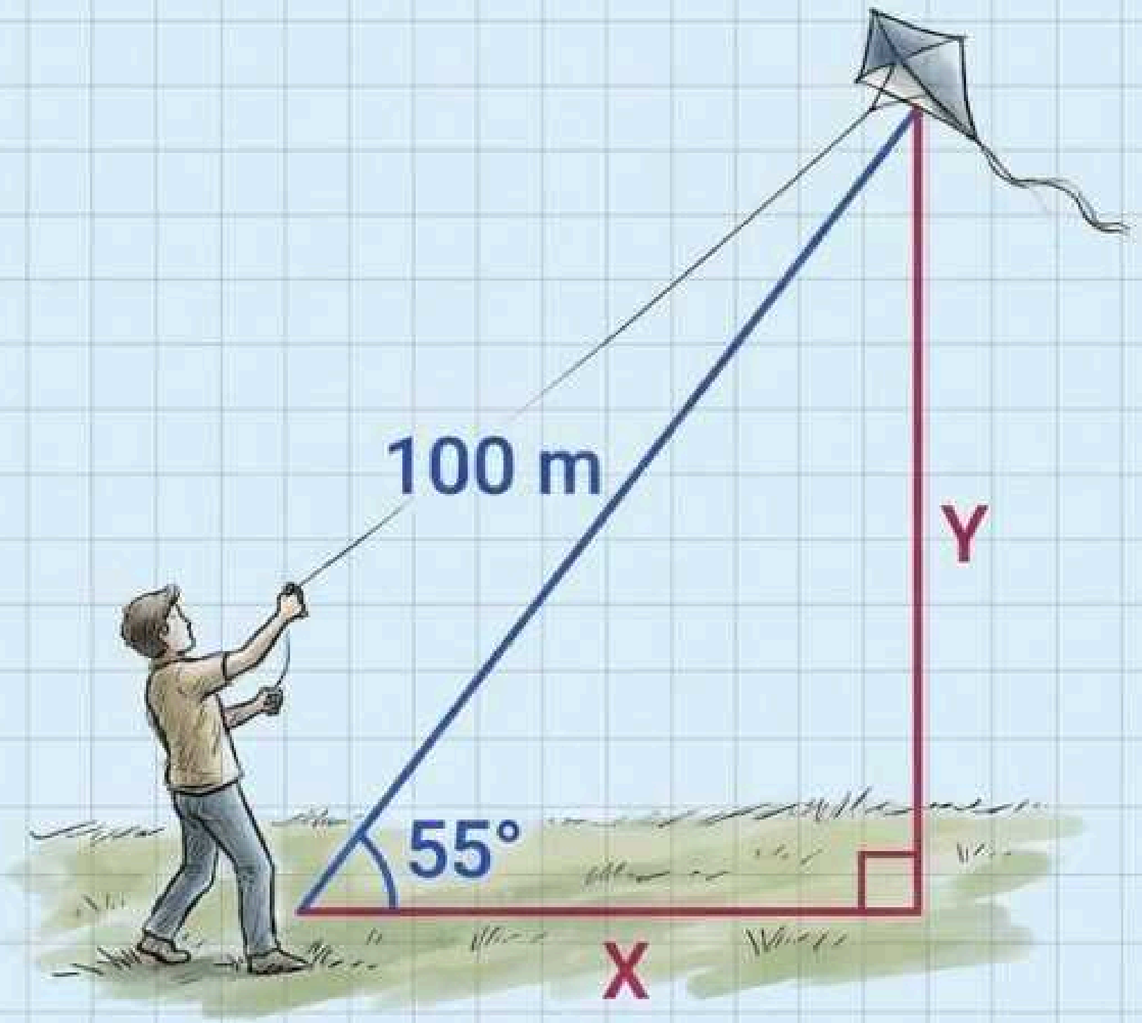
Fix: Always explicitly label feet, meters, or degrees.

10 ft, 3.5 m, 45°

Whenever you can, double-check with the Pythagorean theorem or a second ratio.

Resolving Two Dimensions Simultaneously

¶ 2.17



Vertical Component (Height):

$$Y = 100 * \sin(55°) \approx 81.92 \text{ m}$$

Horizontal Component (Distance):

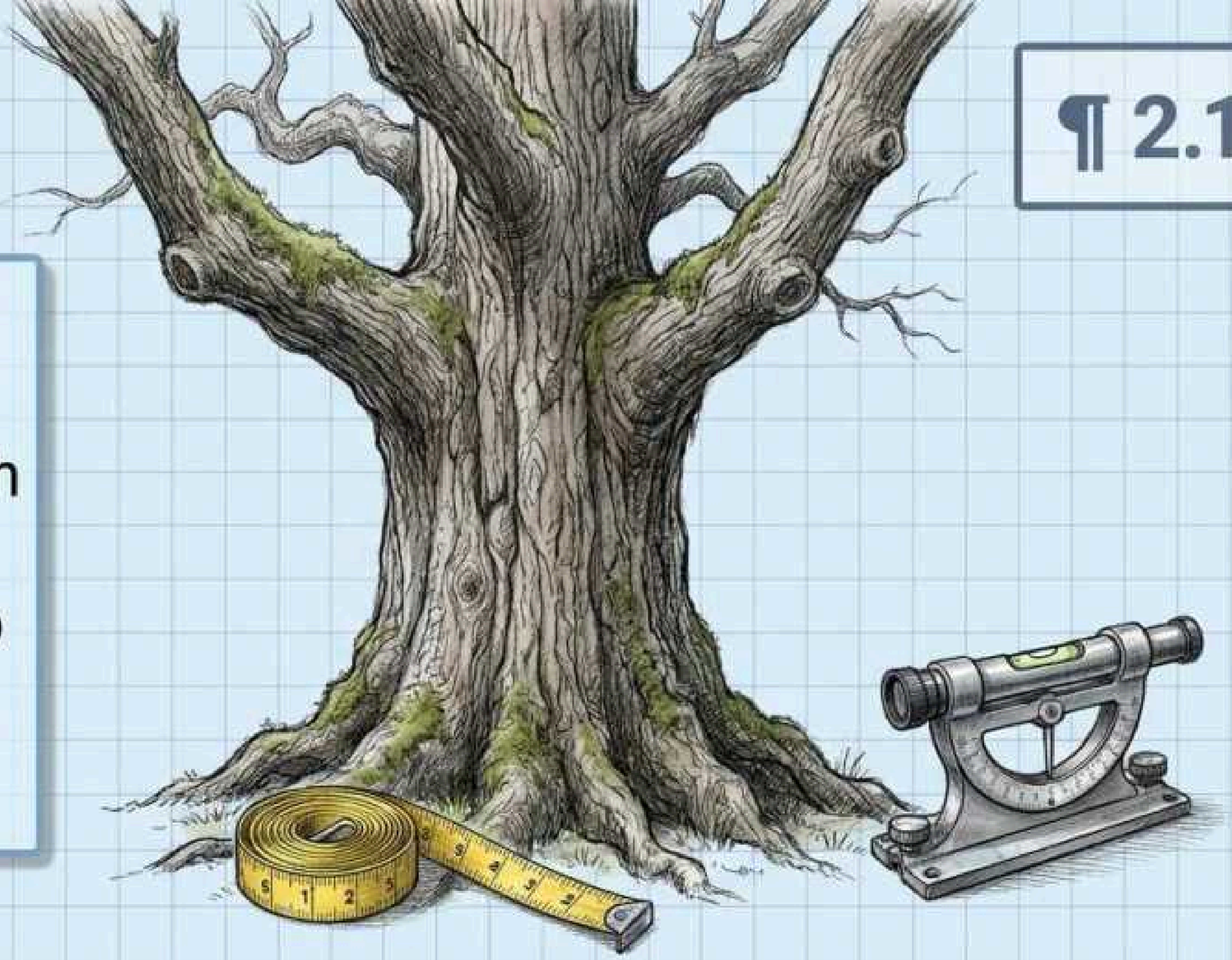
$$X = 100 * \cos(55°) \approx 57.36 \text{ m}$$

Your Field Assignment

¶ 2.18

Think of a tall object near you. How could you find its height with with only a tape measure and a way to measure an angle?

Write down your plan.



Coming up in Lecture 3: We introduce radians and the unit circle, breaking the 90° barrier.