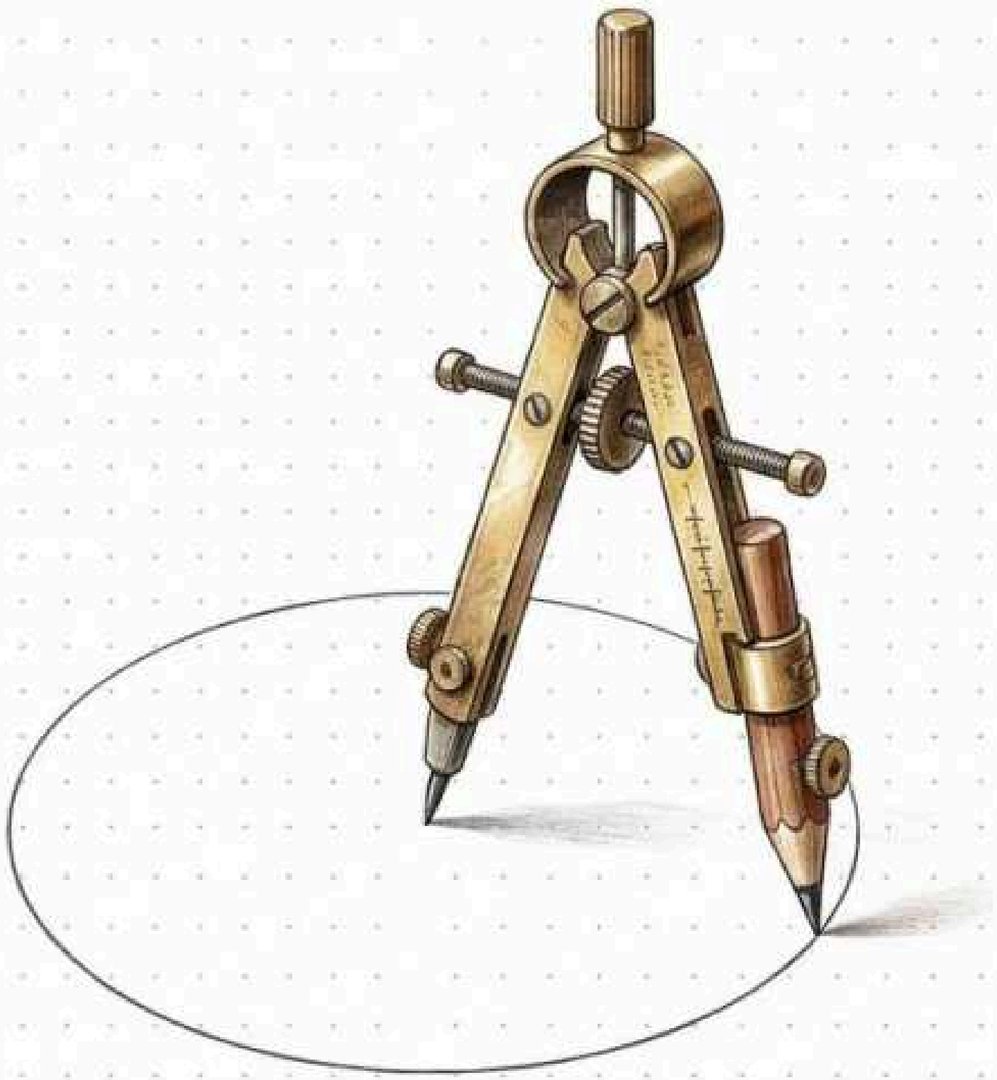
¶ 3.1



# Lecture 3: Radians and the Unit Circle

Why Radians?

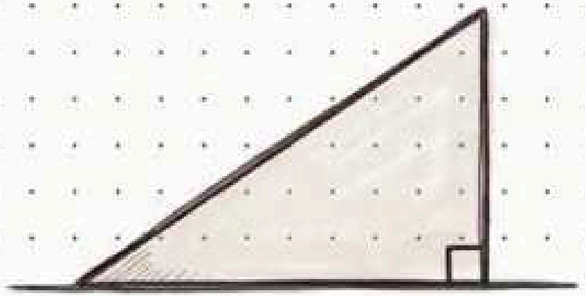
Converting Degrees ↔ Radians

Arc Length & Sector Area

Angular Speed

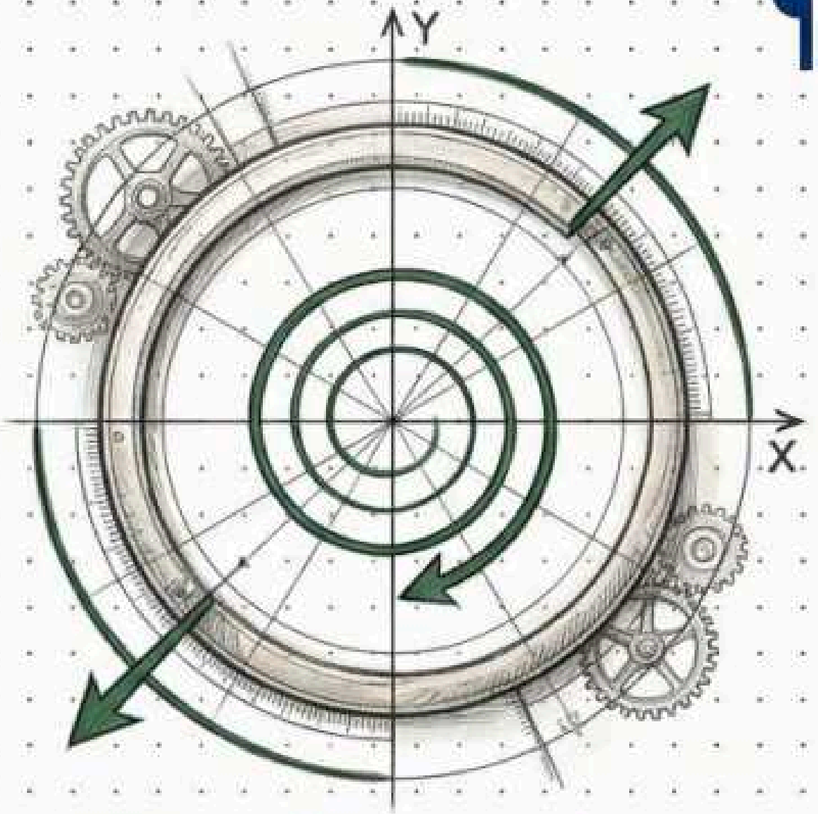
The Unit Circle (Quadrants, Reference Angles, Coterminal Angles)

# ¶ 3.2



## The Old Way

Degrees
(Acute Angles Only)

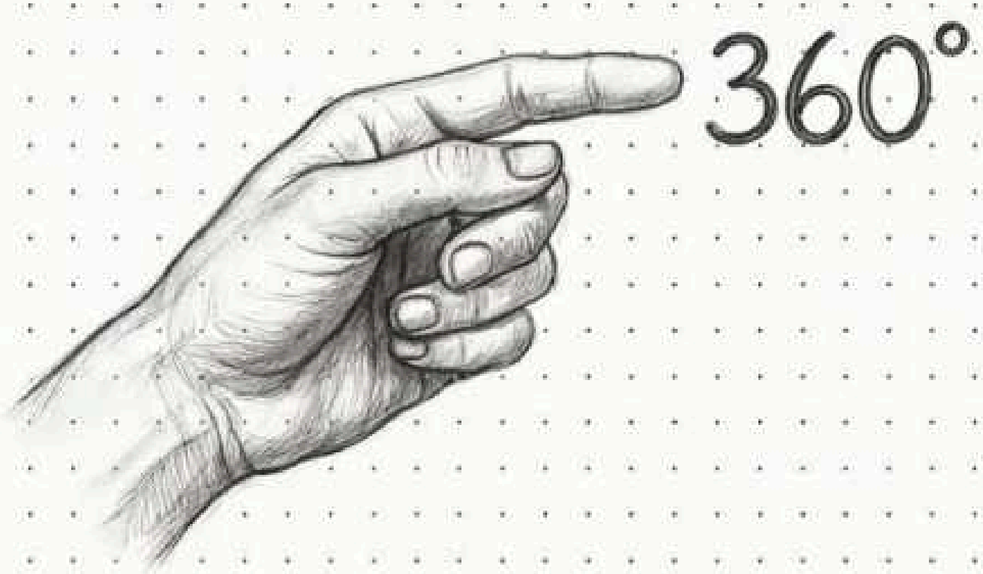



## The New Way

Radians & The Unit Circle
(Every Angle Possible)

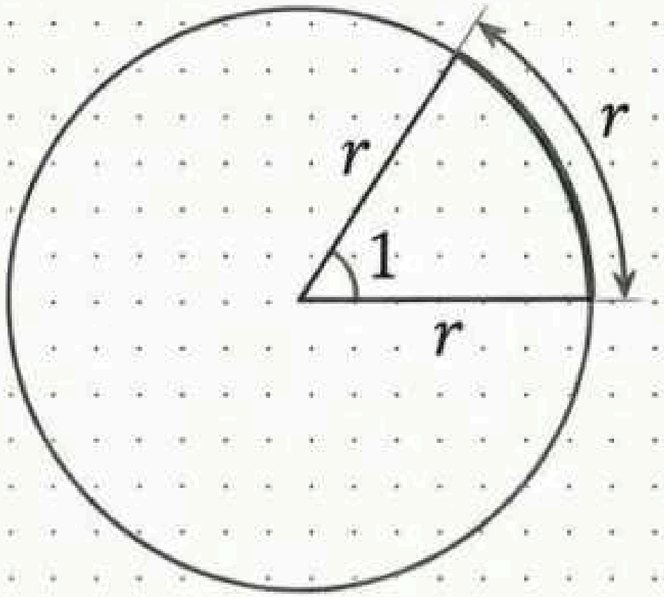
This lecture introduces a more natural unit (the radian) and the unit circle, enabling sine and cosine to work for every angle.

## 360°



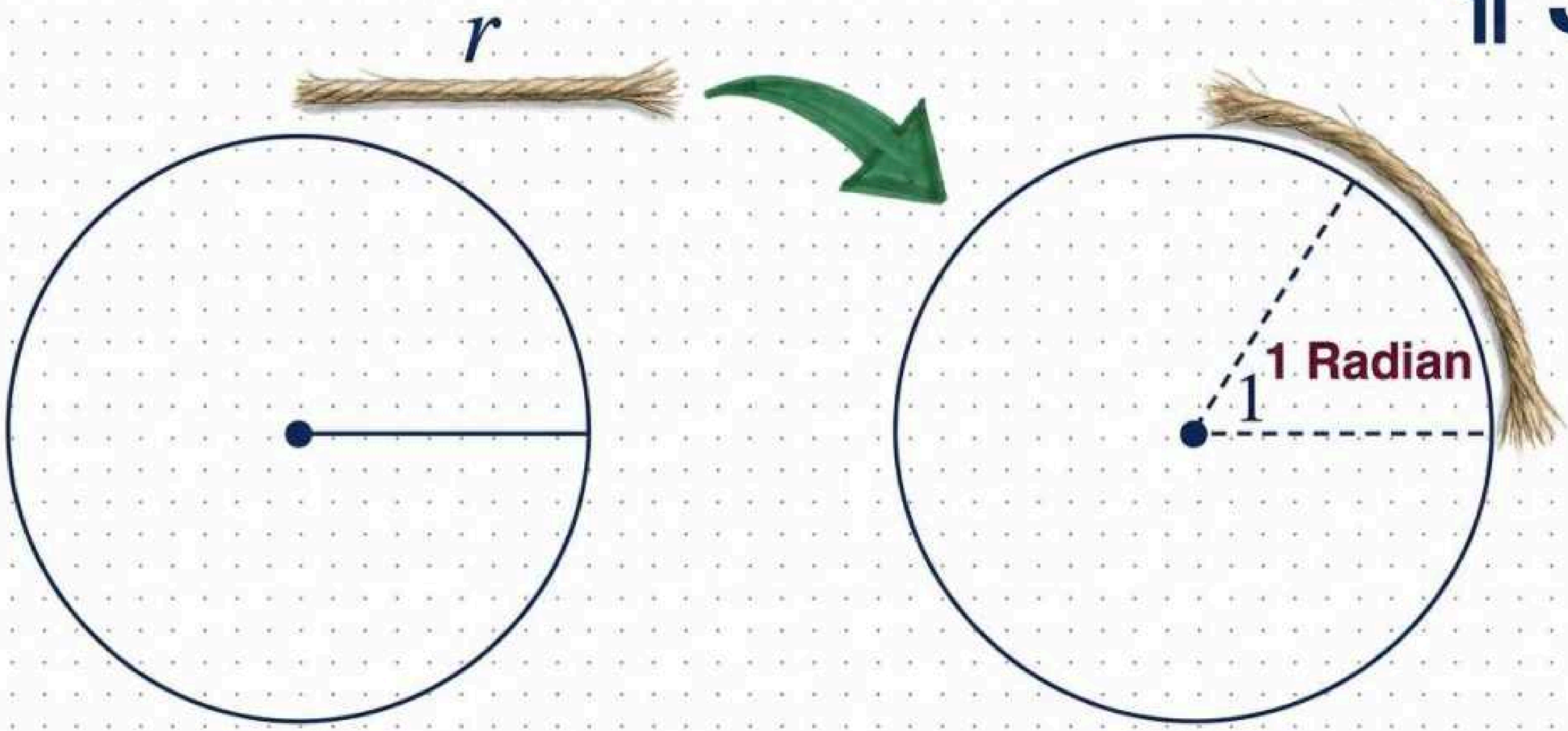

Handy divisors, but nothing to do with circles.

## Radians

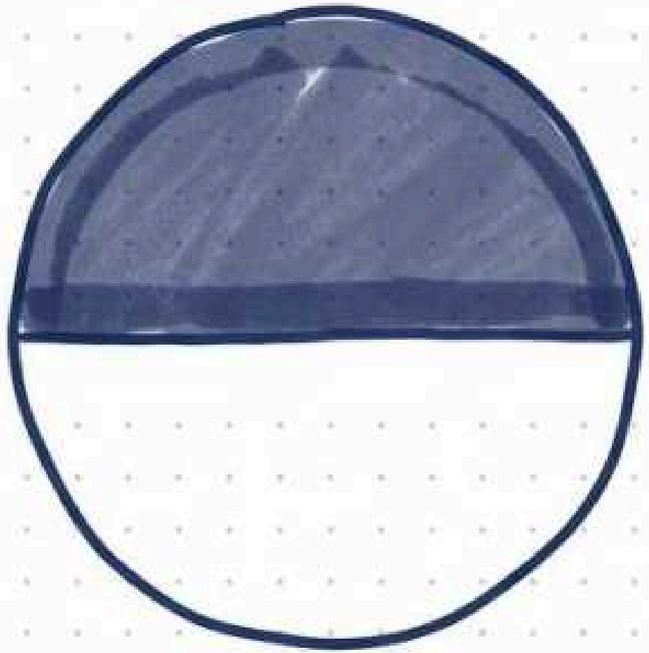



Derived from the circle itself. The standard for mathematicians and physicists.

# ¶ 3.4




The size of the circle does not matter — a bigger circle yields a proportionally bigger arc and radius.



Half Turn = $\pi$ rad (≈ 3.1416)



Full Turn = $2\pi$ rad (≈ 6.2831)

Derived from $C = 2\pi r$

**$180° = \pi$ rad**

## Degrees → Radians

Multiply by $\frac{\pi}{180}$

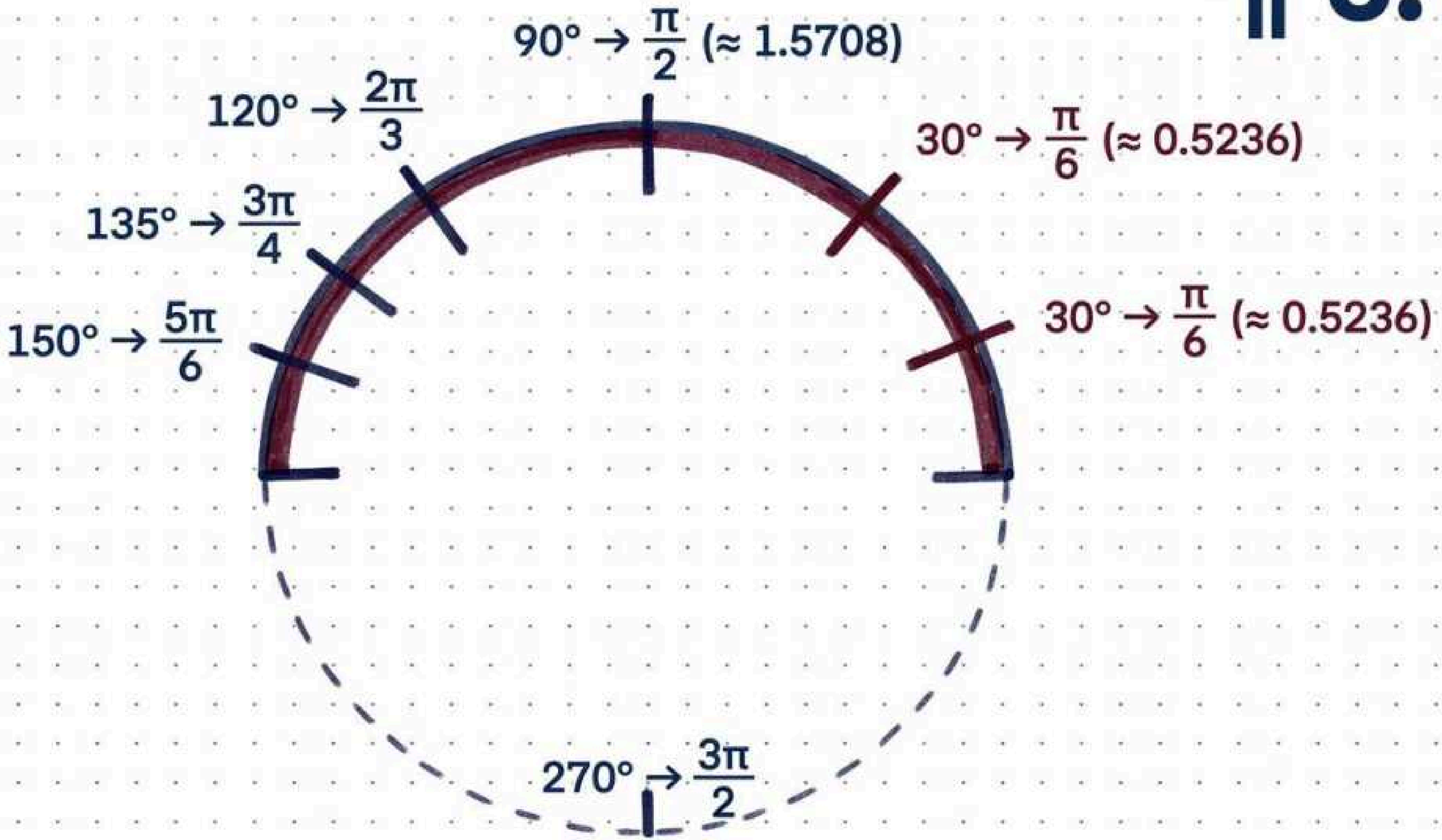
| | |
|---|---|
| 60° | $= \frac{\pi}{3} \approx 1.0472$ |
| 45° | $= \frac{\pi}{4} \approx 0.7854$ |

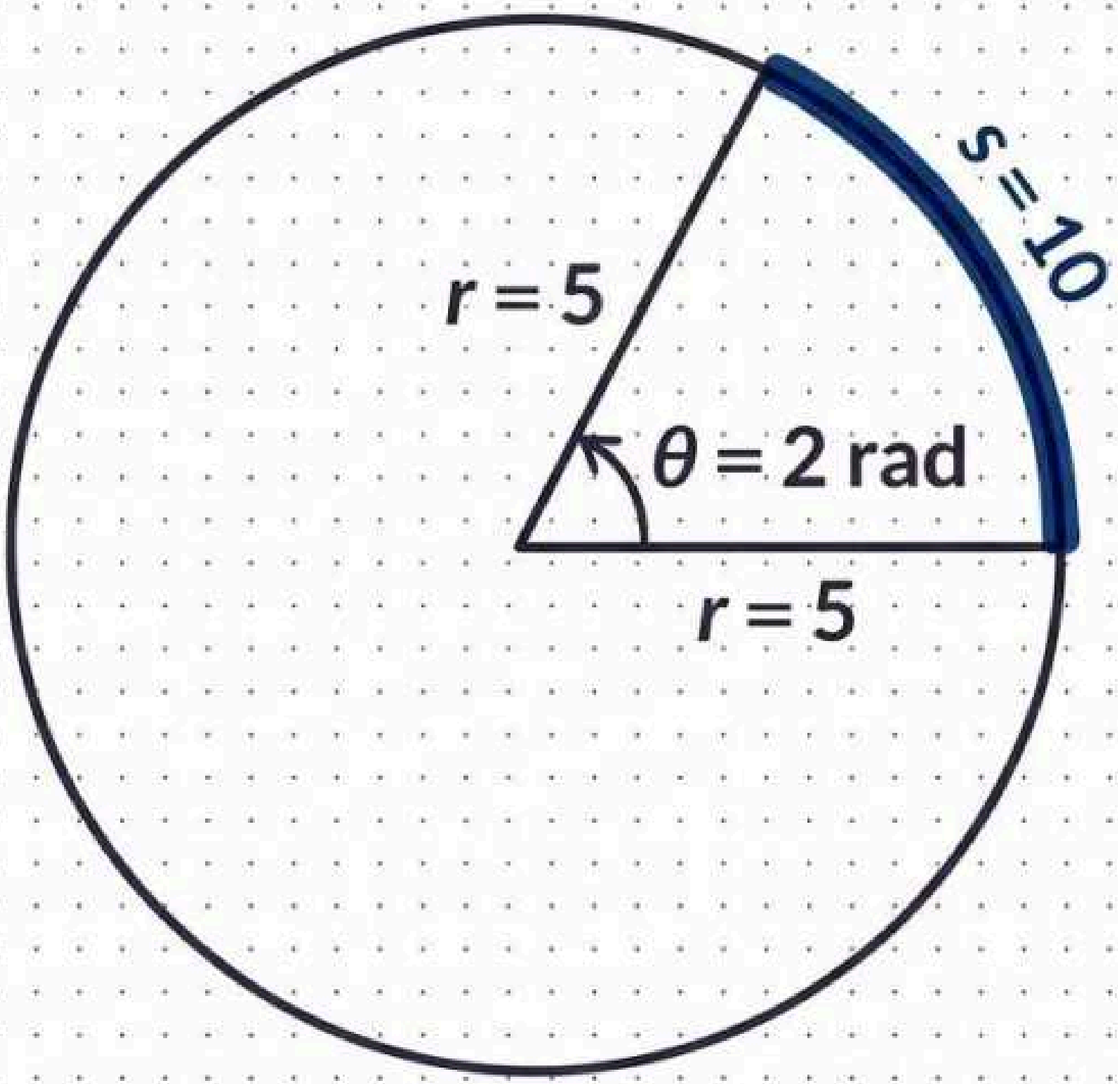
## Radians → Degrees

Multiply by $\frac{180}{\pi}$

| | |
|---|---|
| 1 rad | $\approx 57.2958°$ |

# ¶ 3.7


90° → π/2 (≈ 1.5708)
120° → 2π/3
30° → π/6 (≈ 0.5236)
135° → 3π/4
150° → 5π/6
30° → π/6 (≈ 0.5236)
270° → 3π/2



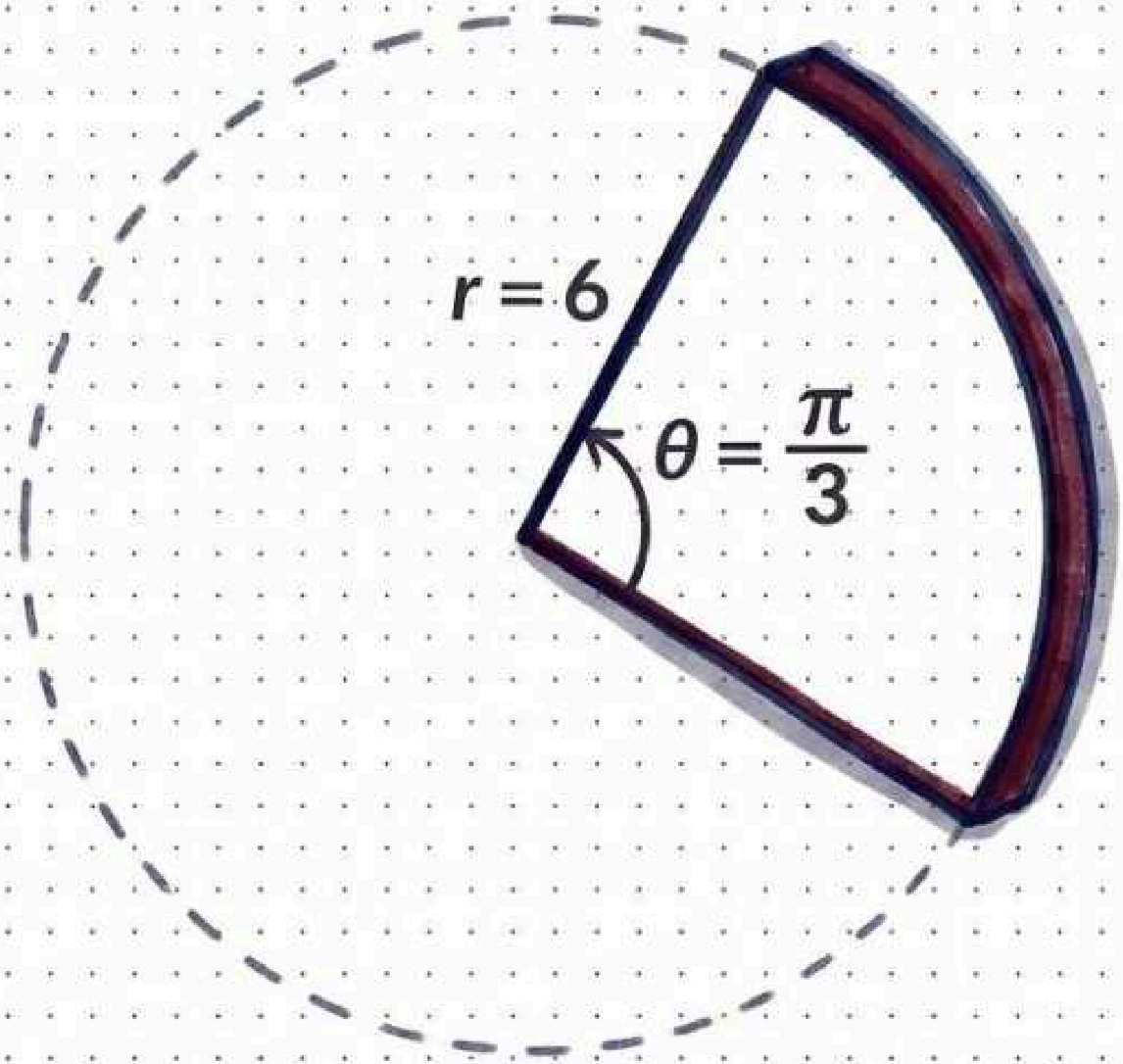

**Radians**

$s = r\theta \rightarrow 5 \times 2 = 10$

(Clean & direct)

**Degrees**

$s = r \times \left(\theta \times \frac{180}{\pi}\right)$




**Formula:**

$$A = \frac{1}{2} r^2 \theta$$

**Calculation:**

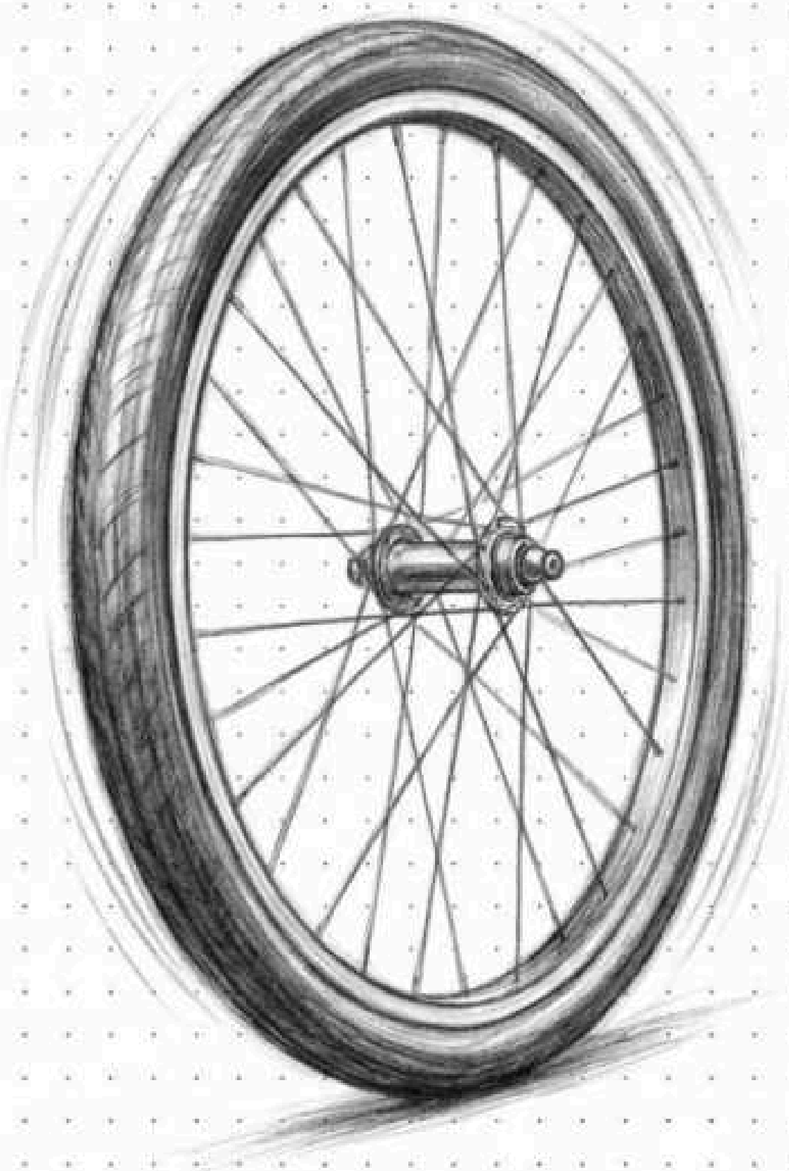
$$A = \frac{1}{2} \times 36 \times 1.0472$$

**Result:**

$$A \approx 18.85 \text{ square units}$$

Exactly 1/6 of the full circle's area.

# ¶ 3.10



**Input:**
$2\ ^{\text{revs}}/_{\text{sec}}$

→

**Angular Speed:**
$\omega = 4\pi\ \frac{\text{rad}}{\text{s}} \approx 12.566$

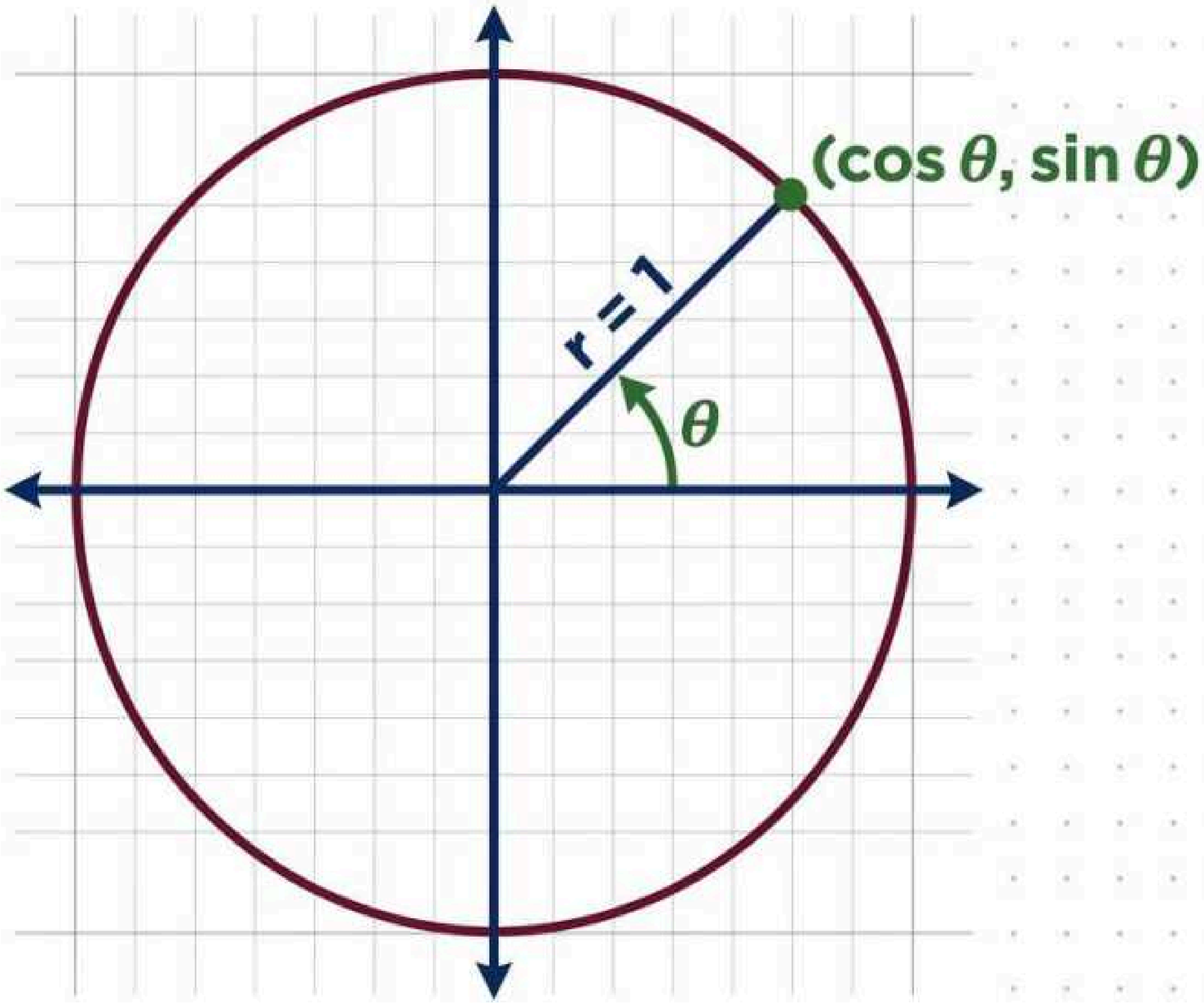
→

**Radius:**
$r = 0.3\ \text{m}$

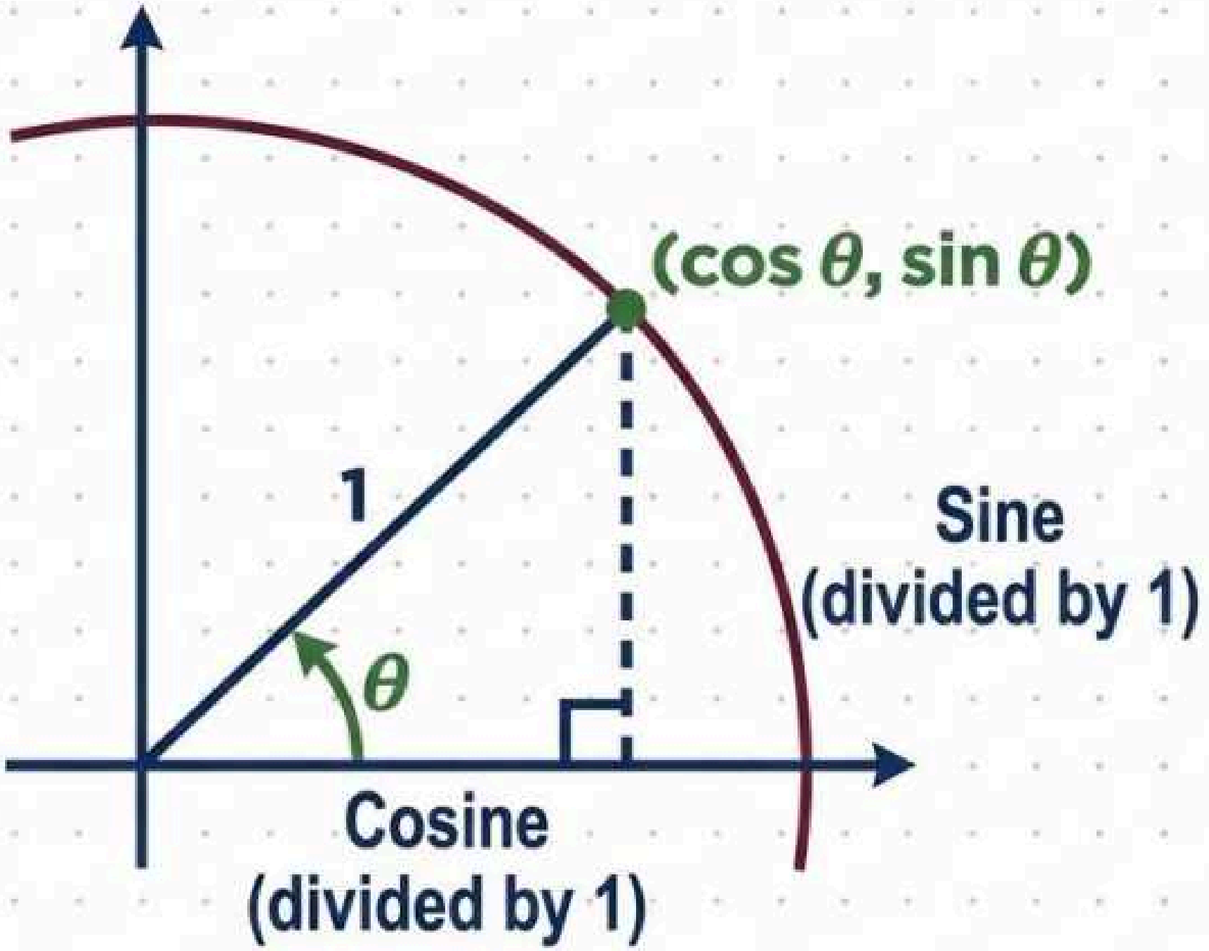
→

**Linear/Rim Speed:**
$v = r\omega \rightarrow v \approx 3.77\ \frac{\text{m}}{\text{s}}$

# ¶ 3.11

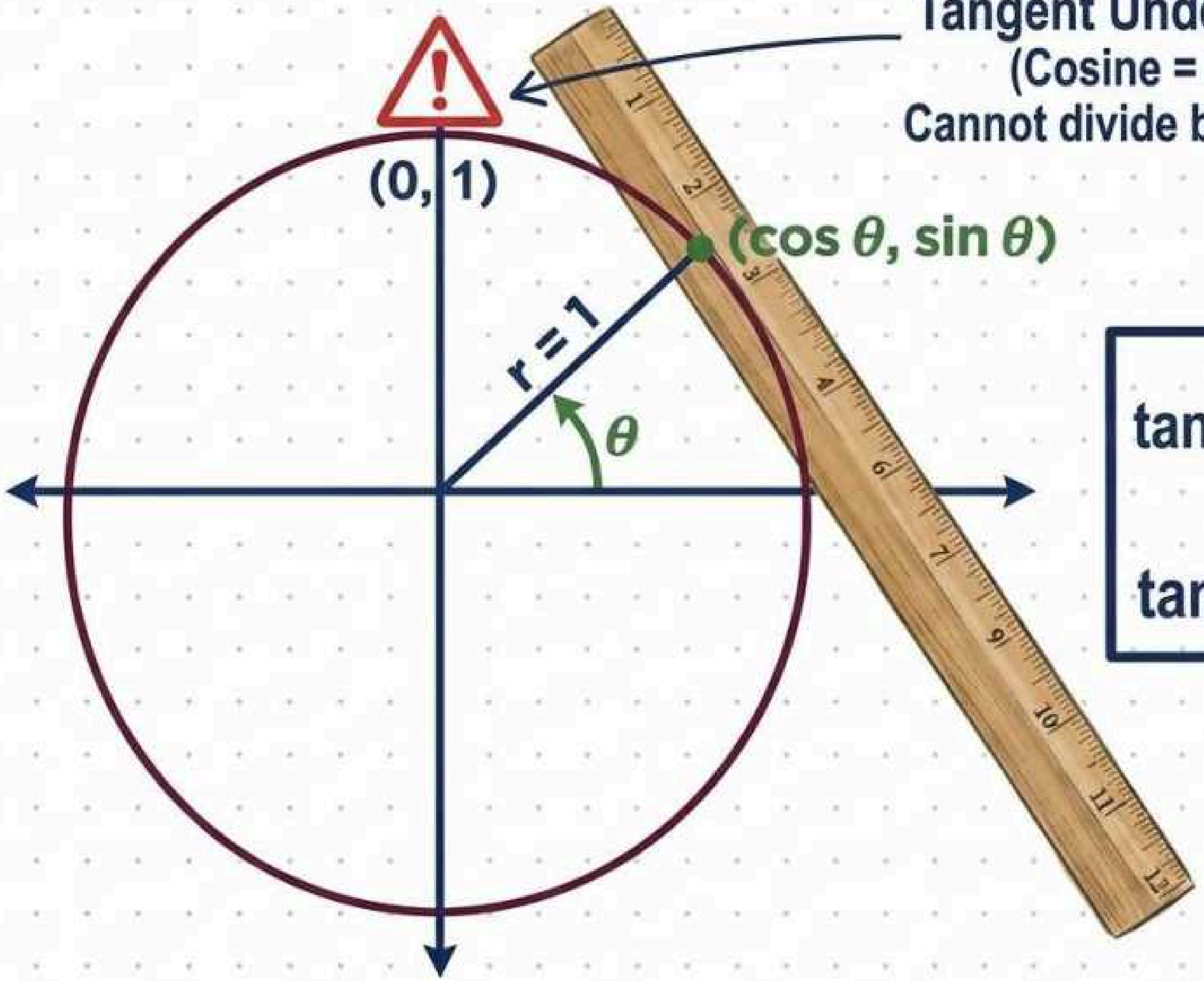


# ¶ 3.12




**The new circular definition perfectly agrees with SOH CAH TOA for acute angles, but expands to work for every angle.**

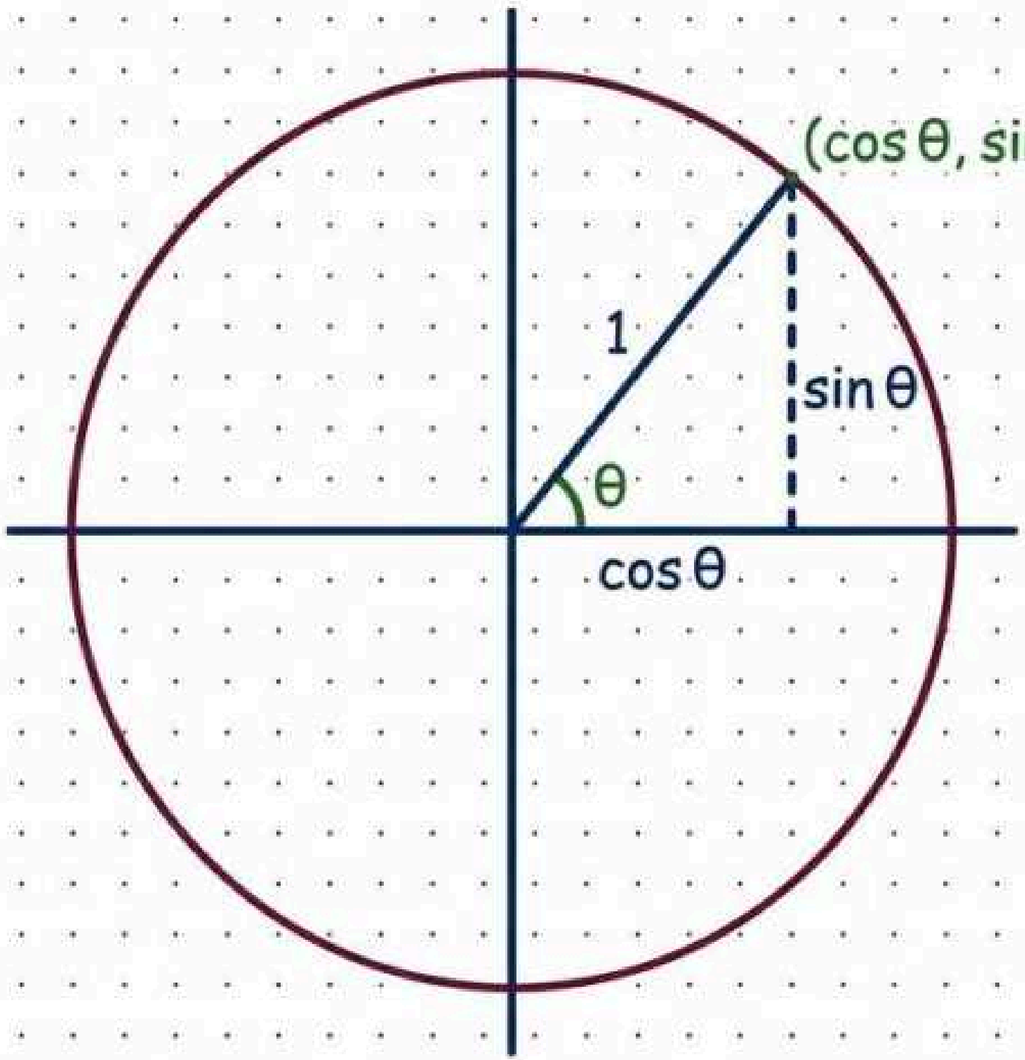
# ¶ 3.13




$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\text{vertical}}{\text{horizontal}}$$

$\tan\theta$ = Slope of the line

# ¶ 3.14




Pythagorean Theorem:

$$a^2 + b^2 = c^2$$

↓

Substituted for the Unit Circle:

$$\cos^2\theta + \sin^2\theta = 1^2$$

↓

The Universal Identity:

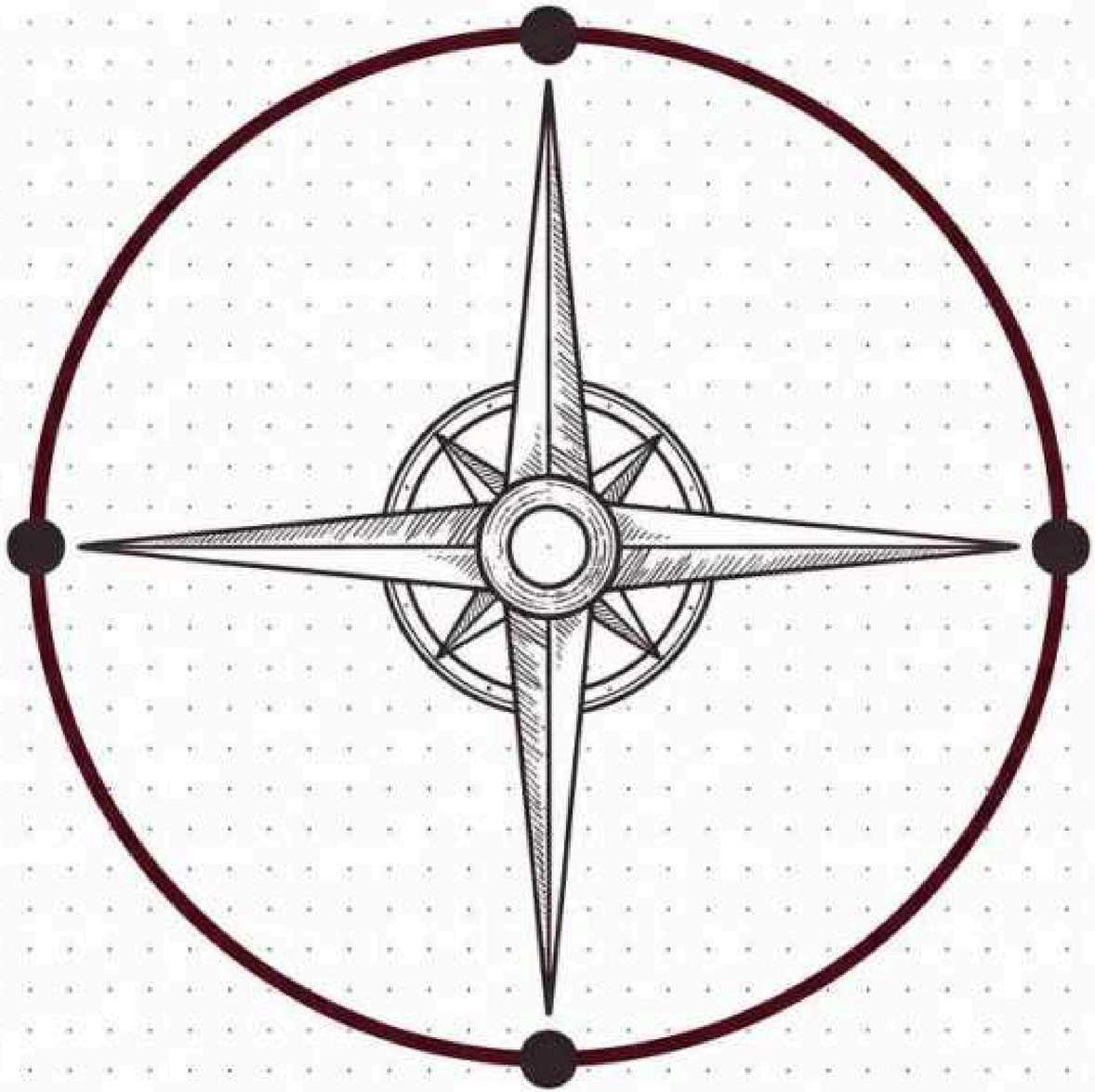
$$\cos^2\theta + \sin^2\theta = 1$$

**True for every angle.**

**Essential for Lecture 5.**

¶ 3.15

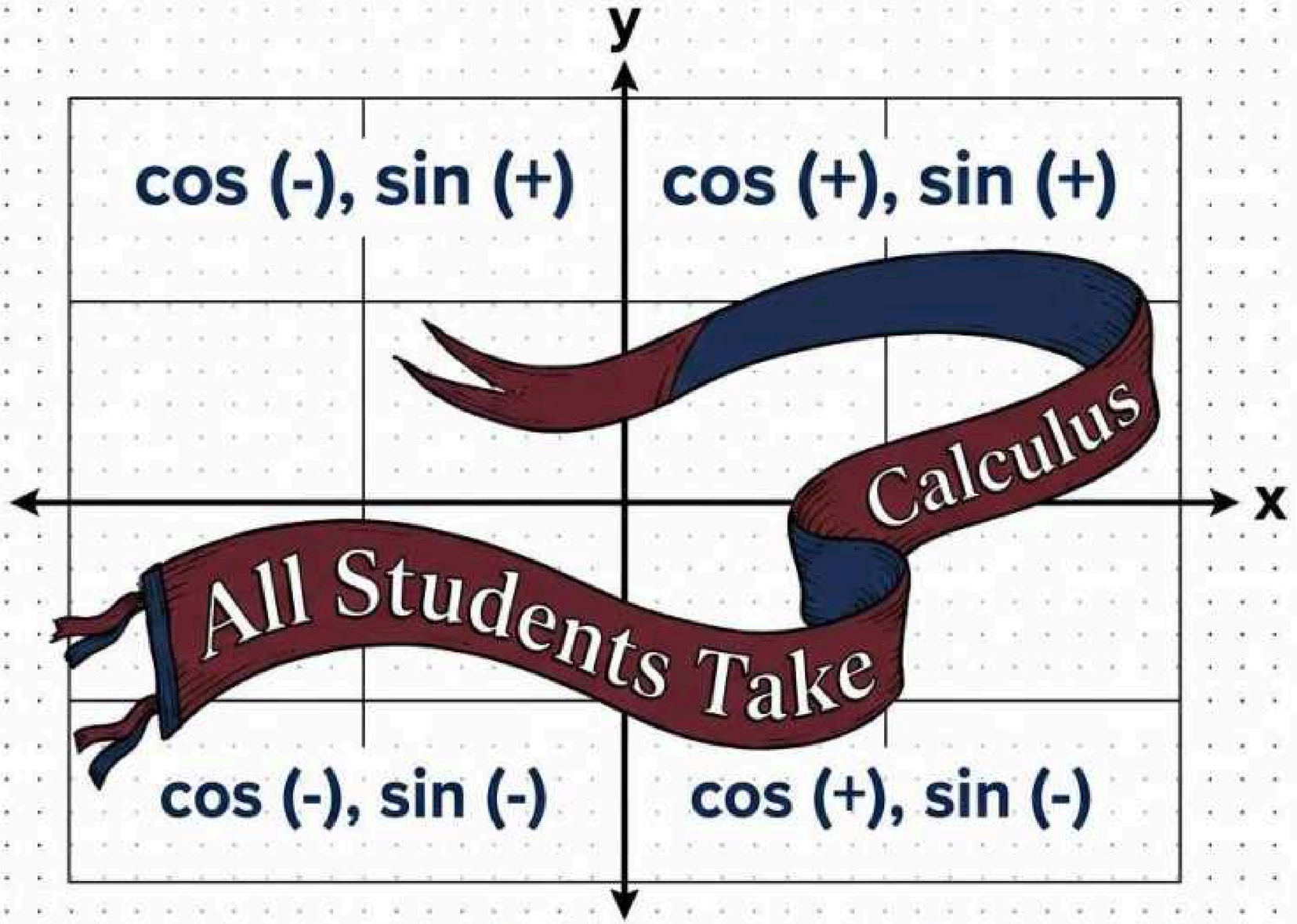
North (90°): (0, 1)



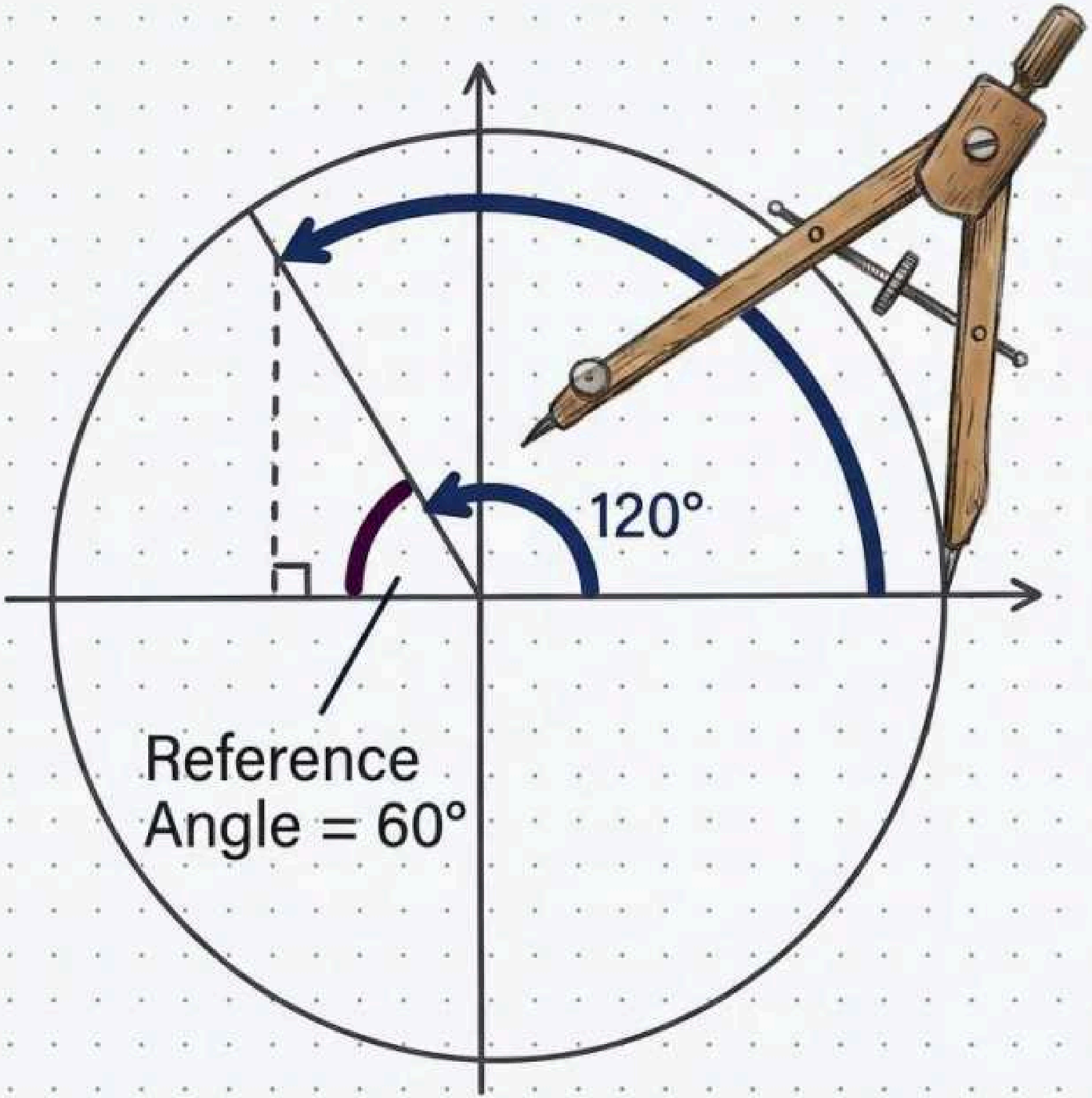
West (180°): (-1, 0)

East (0° / 360°):
(1, 0) → cos=1, sin=0

South (270°): (0, -1)




**All (I), Sine (II), Tangent (III), Cosine (IV)**



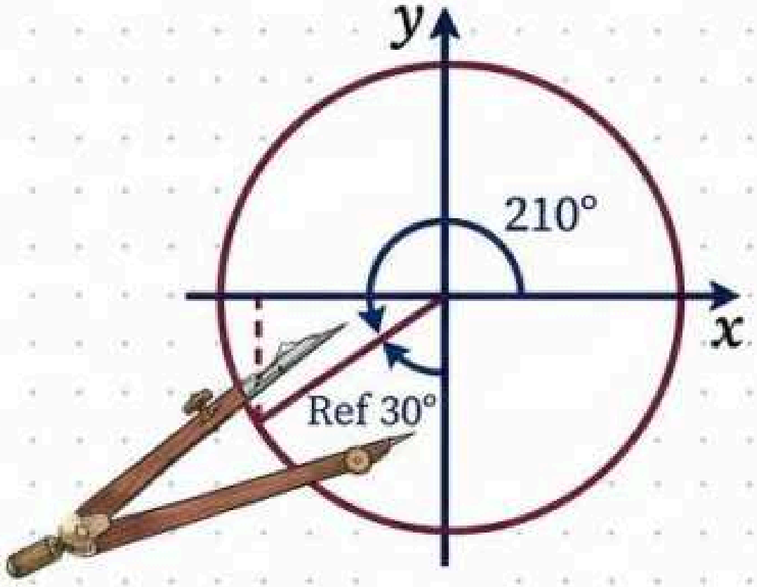

## ¶ 3.17

Values match 60°, signs determined by **Quadrant II.**

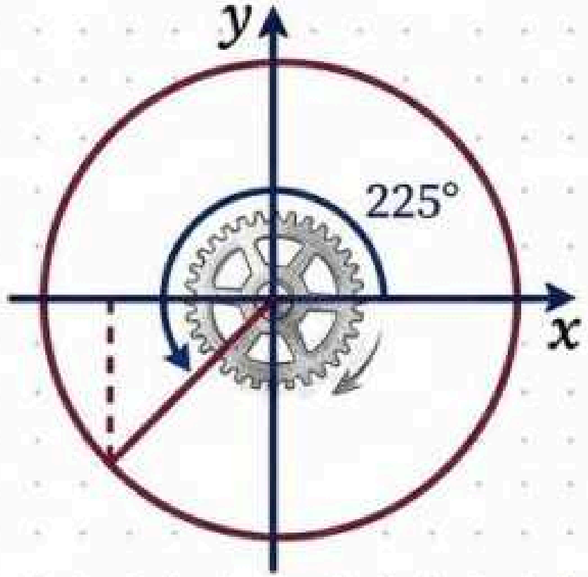
**sin(120°) = +0.8660**

**cos(120°) = -1/2**

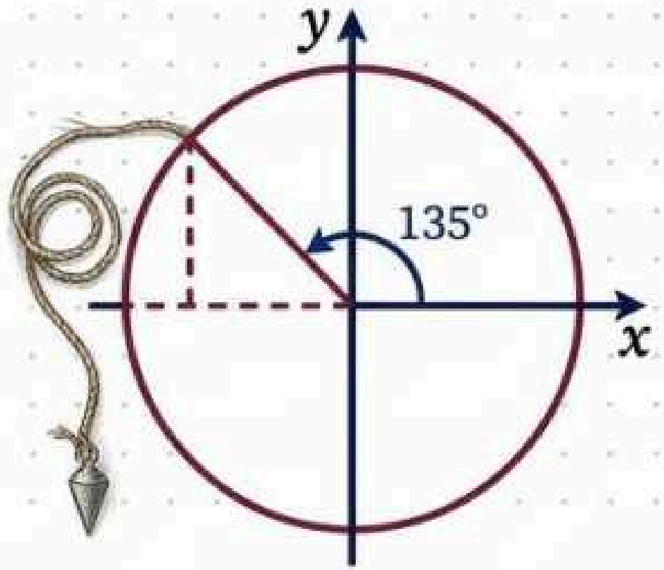
# ¶ 3.18



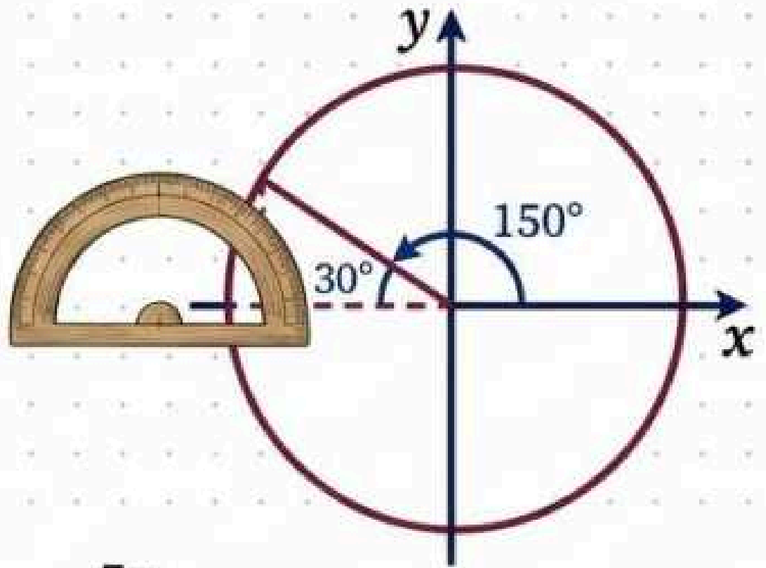

$210° \rightarrow \text{Ref } 30° \text{ (Q III)} \rightarrow \sin = -\frac{1}{2}$



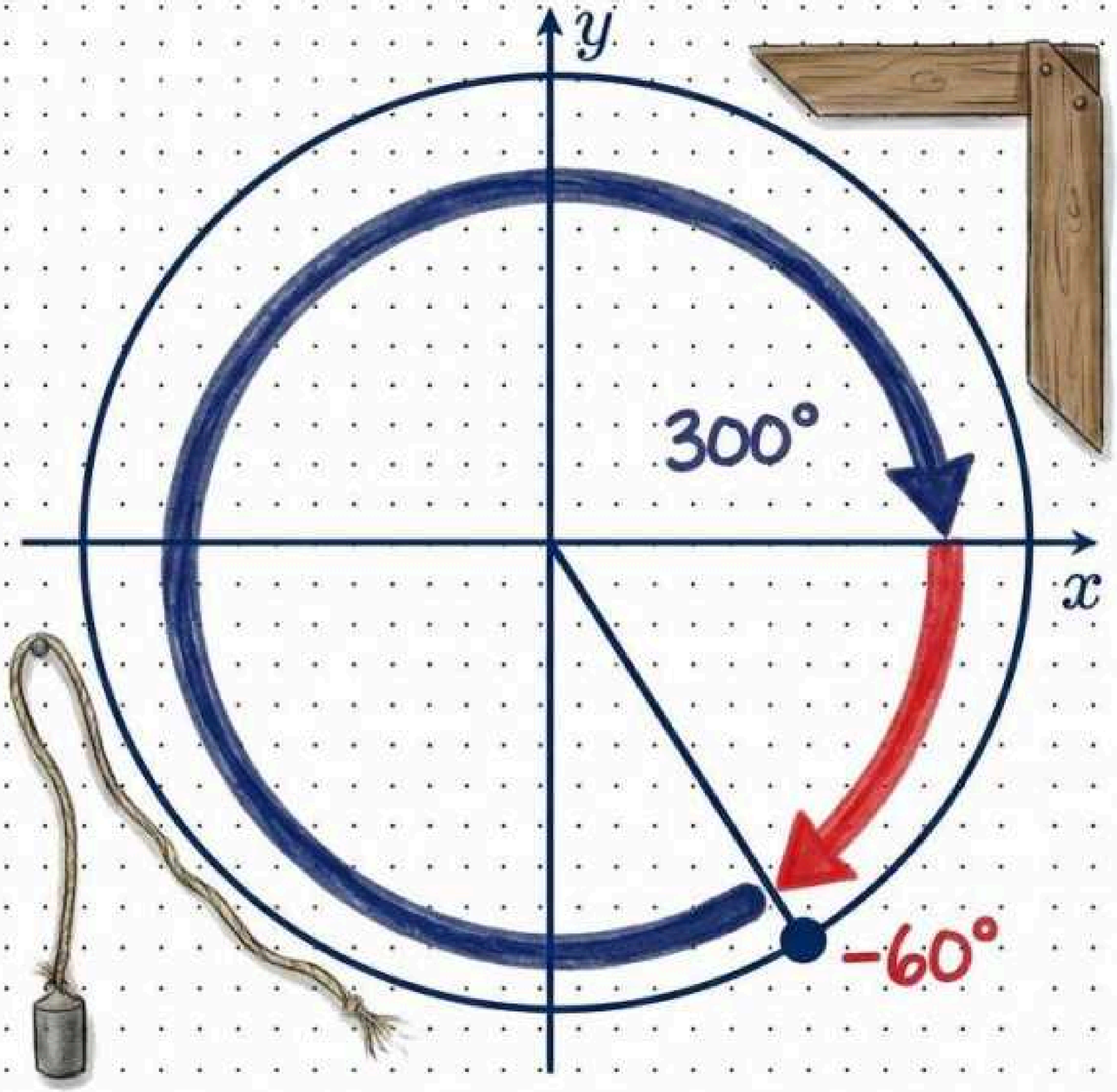

$225° \rightarrow \text{(Q III)} \rightarrow \cos \approx -0.7071$




$135° \rightarrow \text{(Q II)} \rightarrow \tan = -1$




$\frac{5\pi}{6}$ rad $(150°) \rightarrow \text{(Q II)} \rightarrow \sin = \frac{1}{2}$

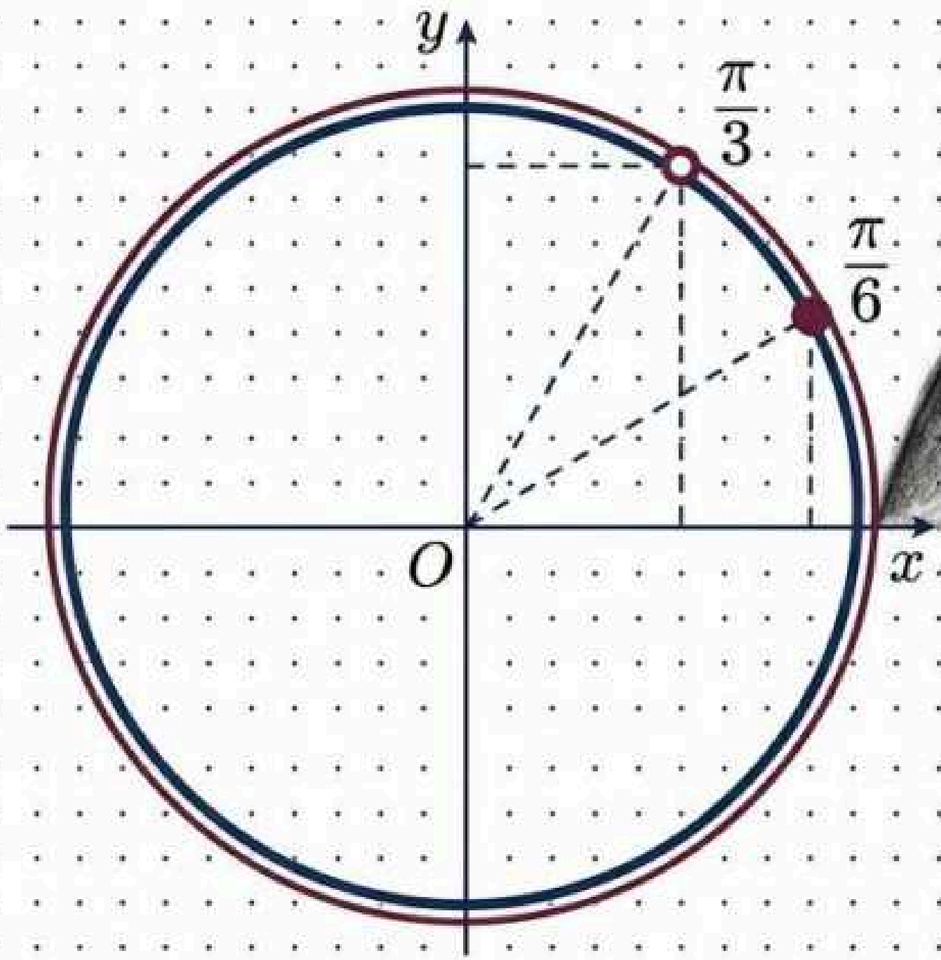


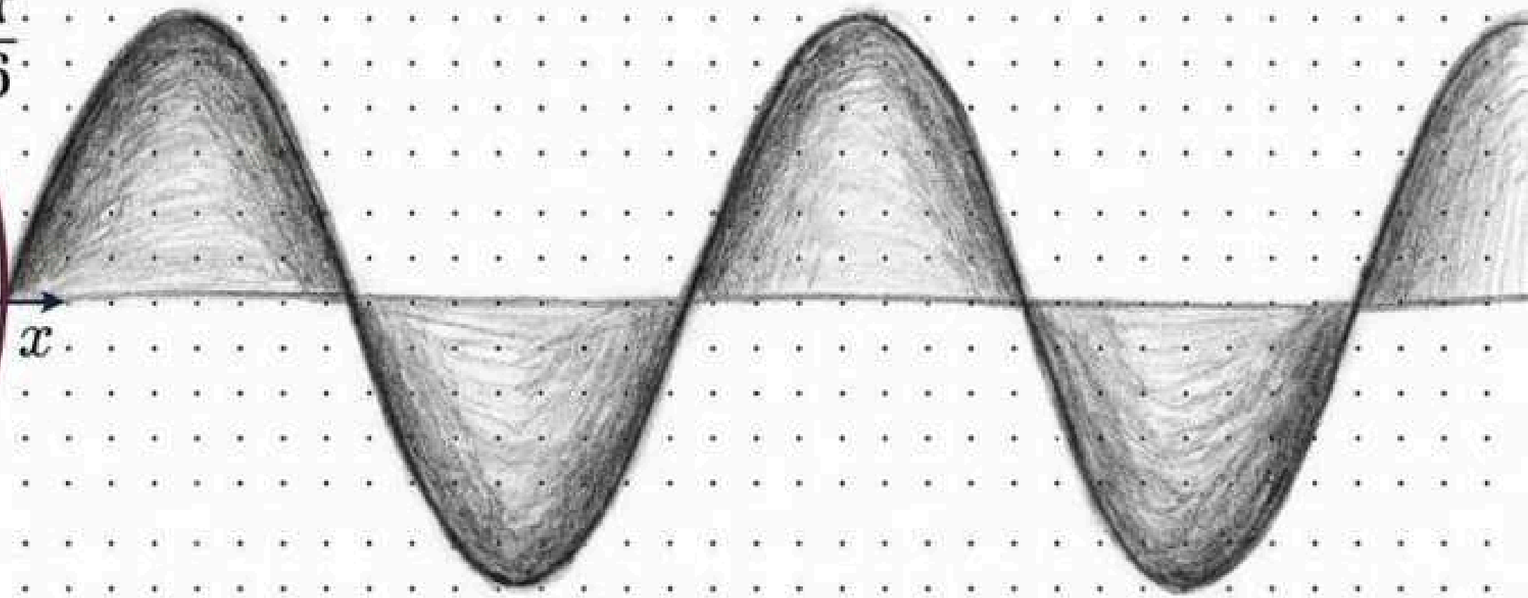

$-60°$ is coterminal with $300°$.

$$\cos(-60°) = \frac{1}{2}$$

$$\sin(-60°) \approx -0.8660$$

# ¶ 3.20



What is the sine of $\pi/6$? Why is it identical to the cosine of $\pi/3$?

Think about the picture. In Lecture 4, we unroll the circle... and we get waves.