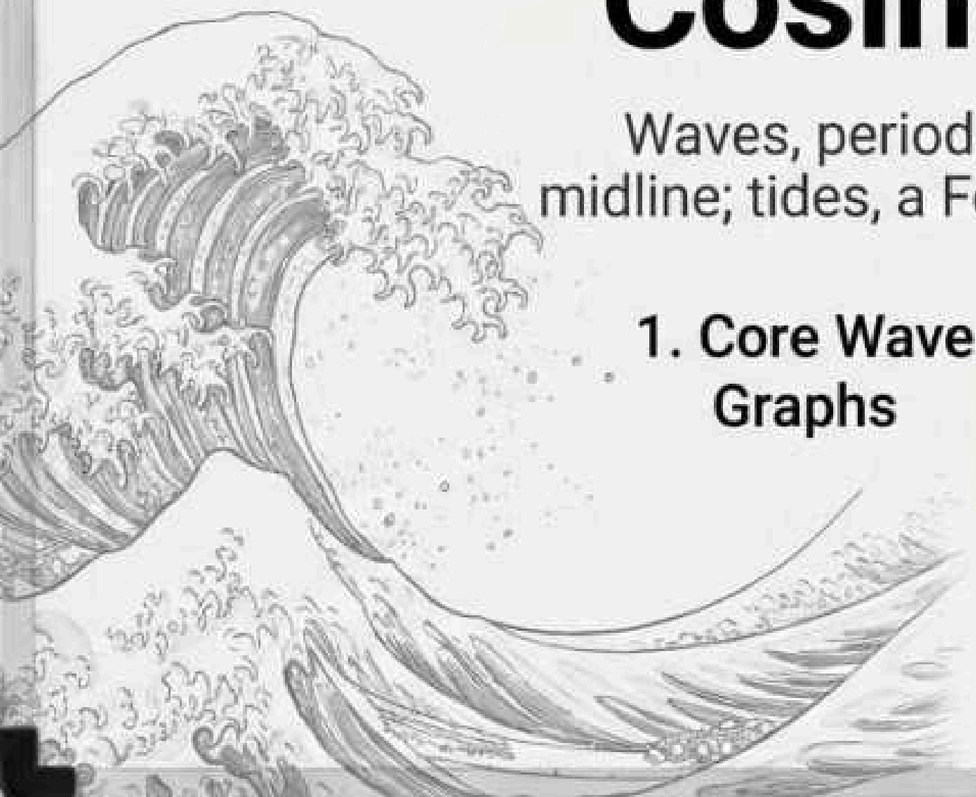
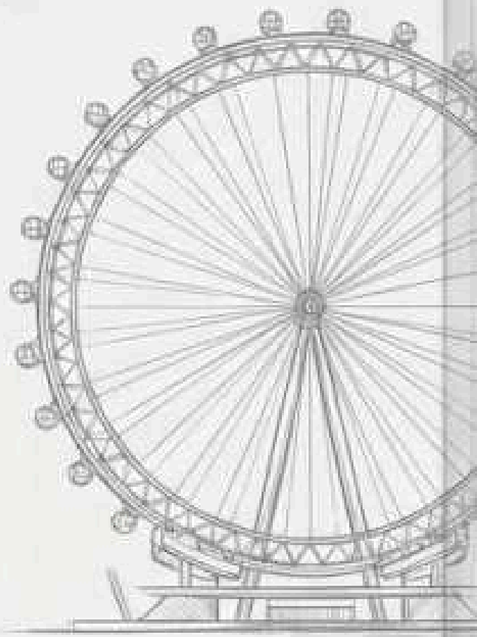
¶ 4.1

The Graphs of Sine, Cosine, and Tangent.

Waves, period, amplitude, frequency, phase shift, and midline; tides, a Ferris wheel, alternating current, and sound.



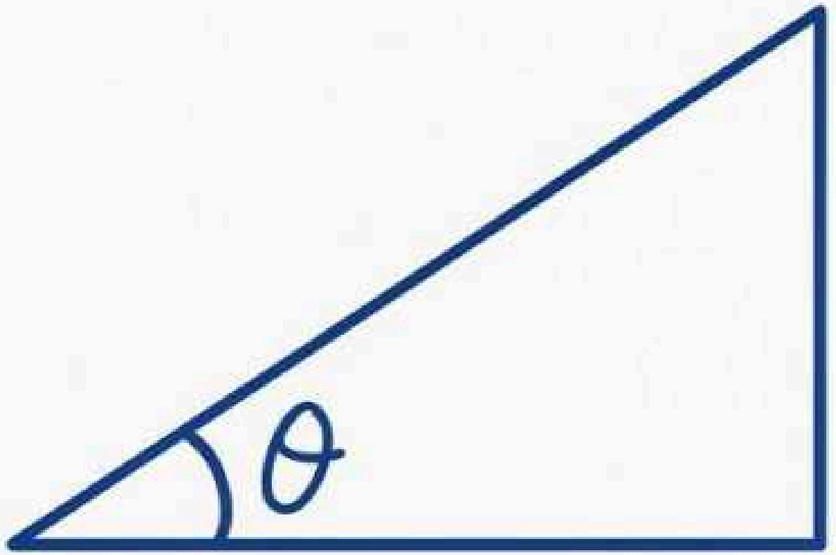


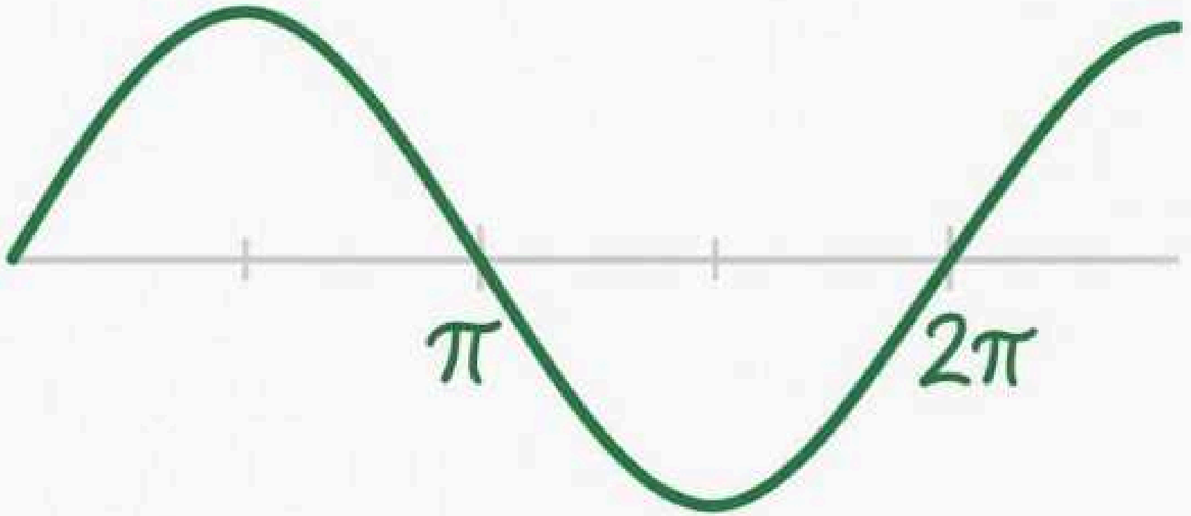


1. Core Wave Graphs
2. Transforming Waves
3. Real-World Applications

¶ 4.2

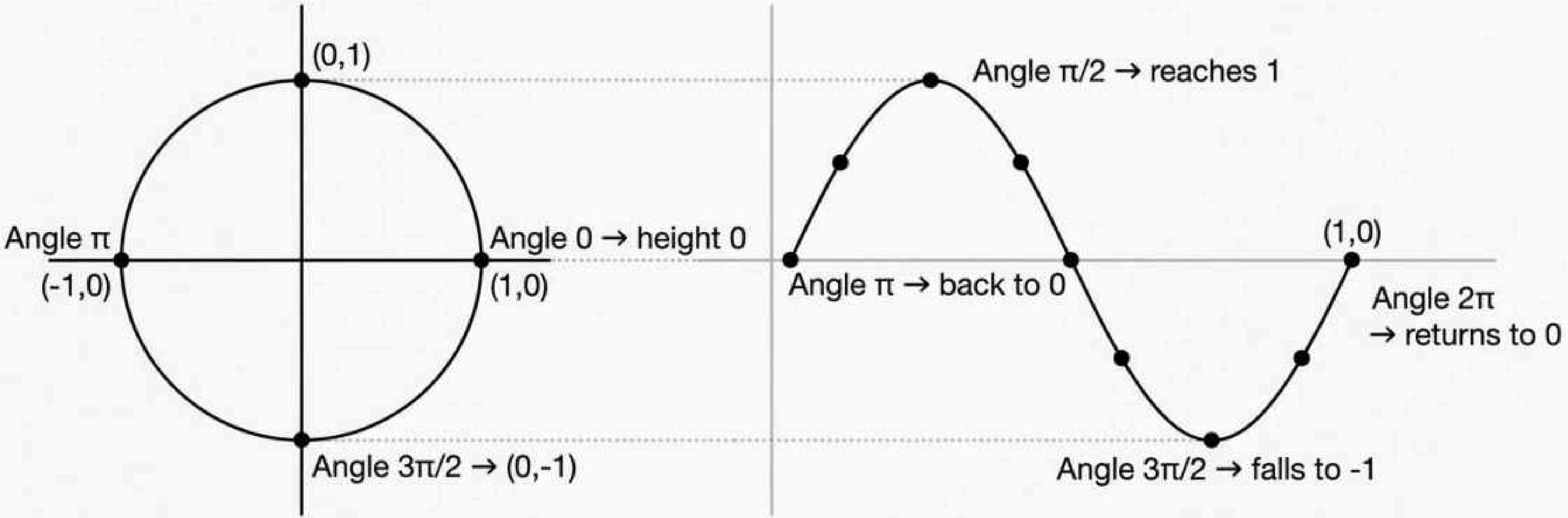
Trigonometry is not only about triangles.
Its other great use is describing things that repeat.





All graphs use radians on the horizontal axis to measure position.

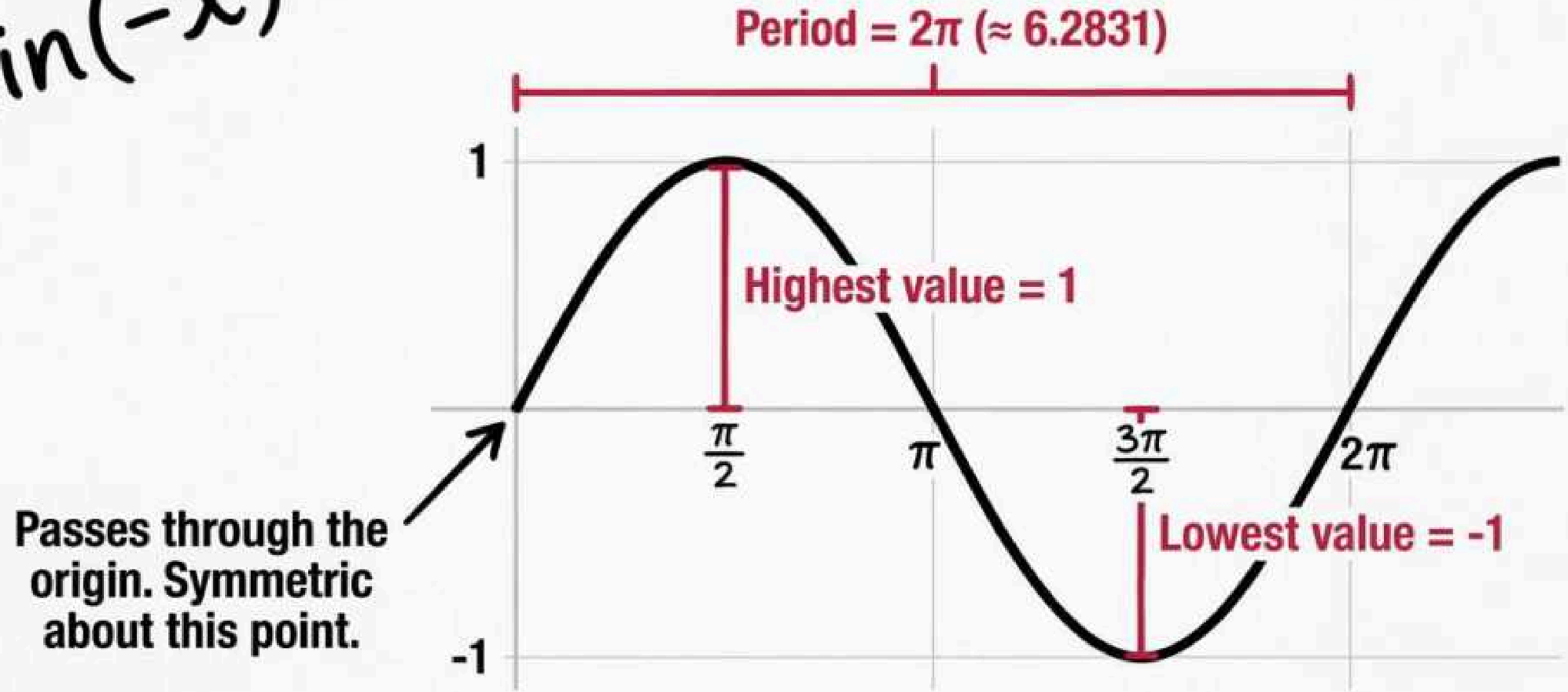
¶ 4.3

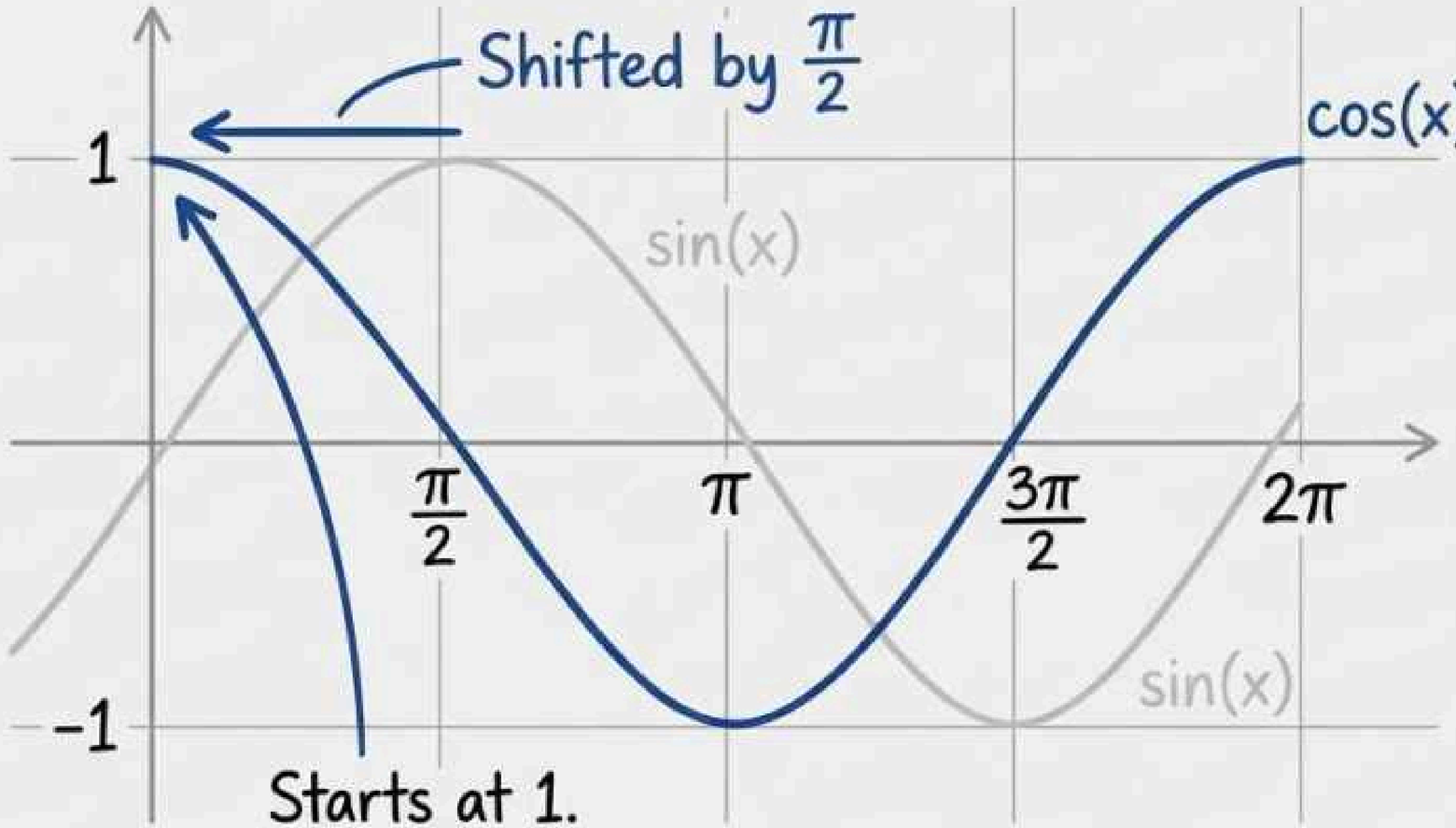


Drawn out, this is a smooth wave.

$$\sin(-x) = -\sin(x)$$

¶ 4.4


Period = 2π (≈ 6.2831)
1
-1
Highest value = 1
Lowest value = -1
π/2
π
3π/2
2π
Passes through the origin. Symmetric about this point.



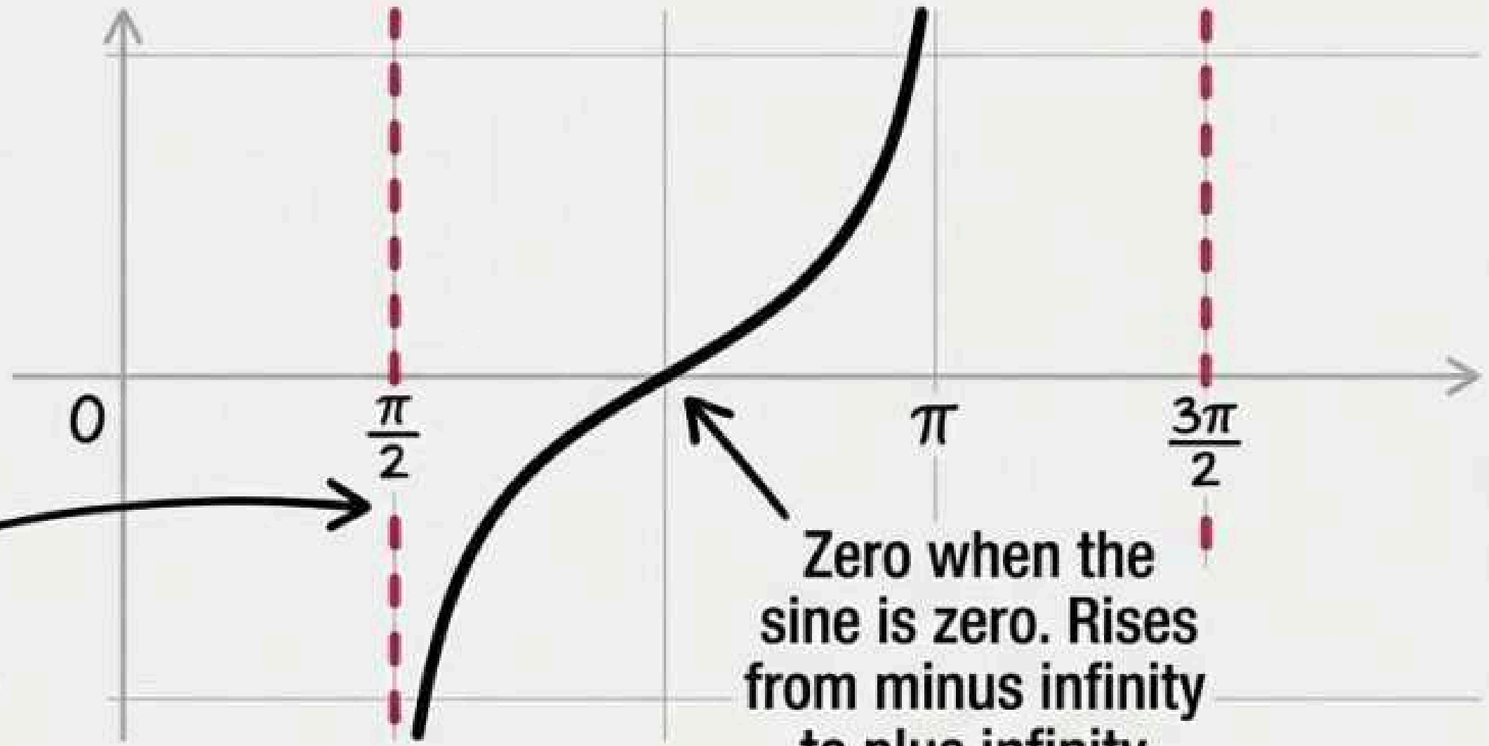
Starts at 1.
Symmetric about the vertical axis.

$$\cos(x) = \sin\left(x + \frac{\pi}{2}\right)$$

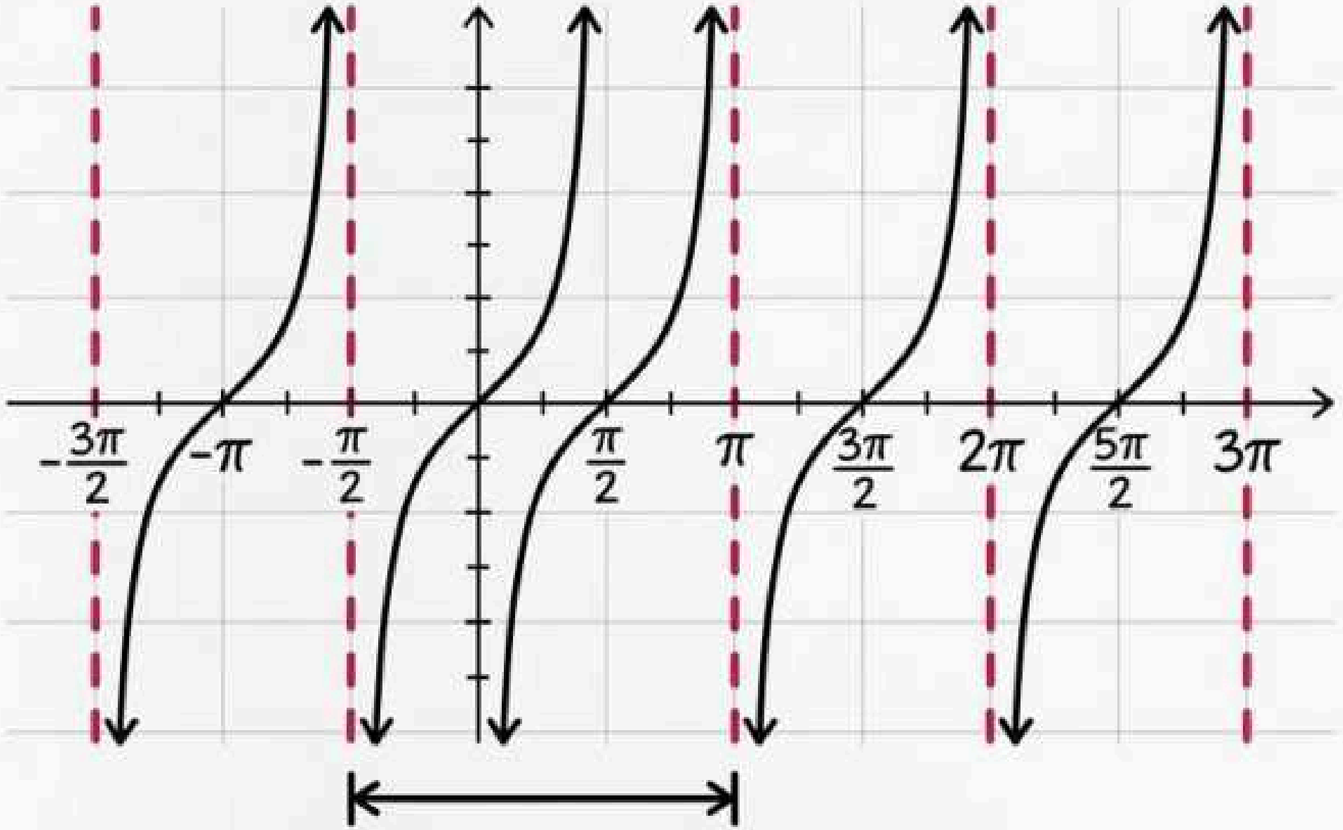
$$\cos(-x) = \cos(x)$$

¶ 4.6

$$\tan x = \frac{\sin x}{\cos x}$$



¶ 4.7

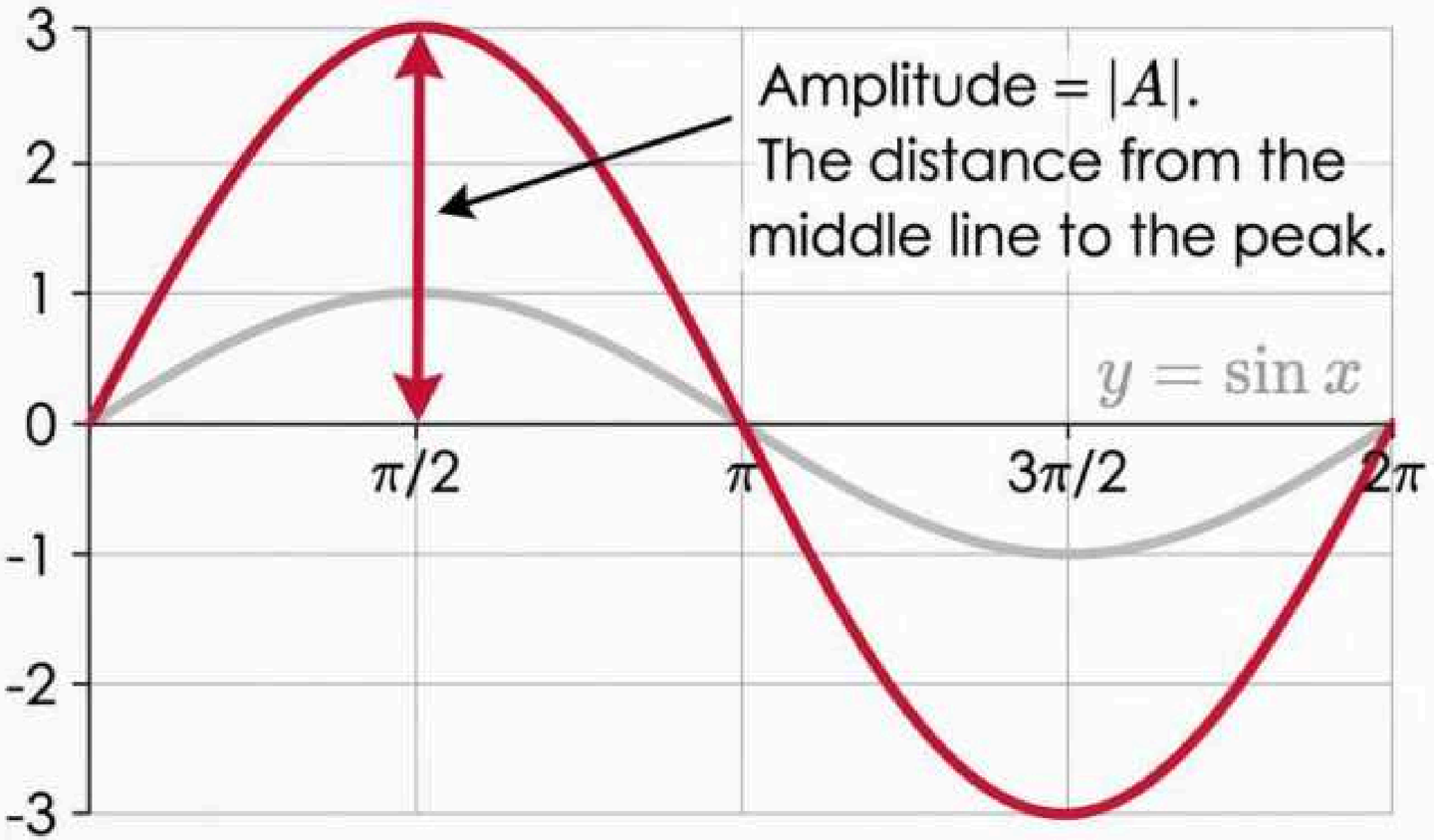


Period = π (≈ 3.1416).
Repeats every π, not 2π.

Sine / Cosine	Oscillate between limits (max 1, min –1). Suits physical things that oscillate.
Tangent	No highest or lowest value. Range is all real numbers. Suits calculating slopes.

¶ 4.8

$y = A \sin x$

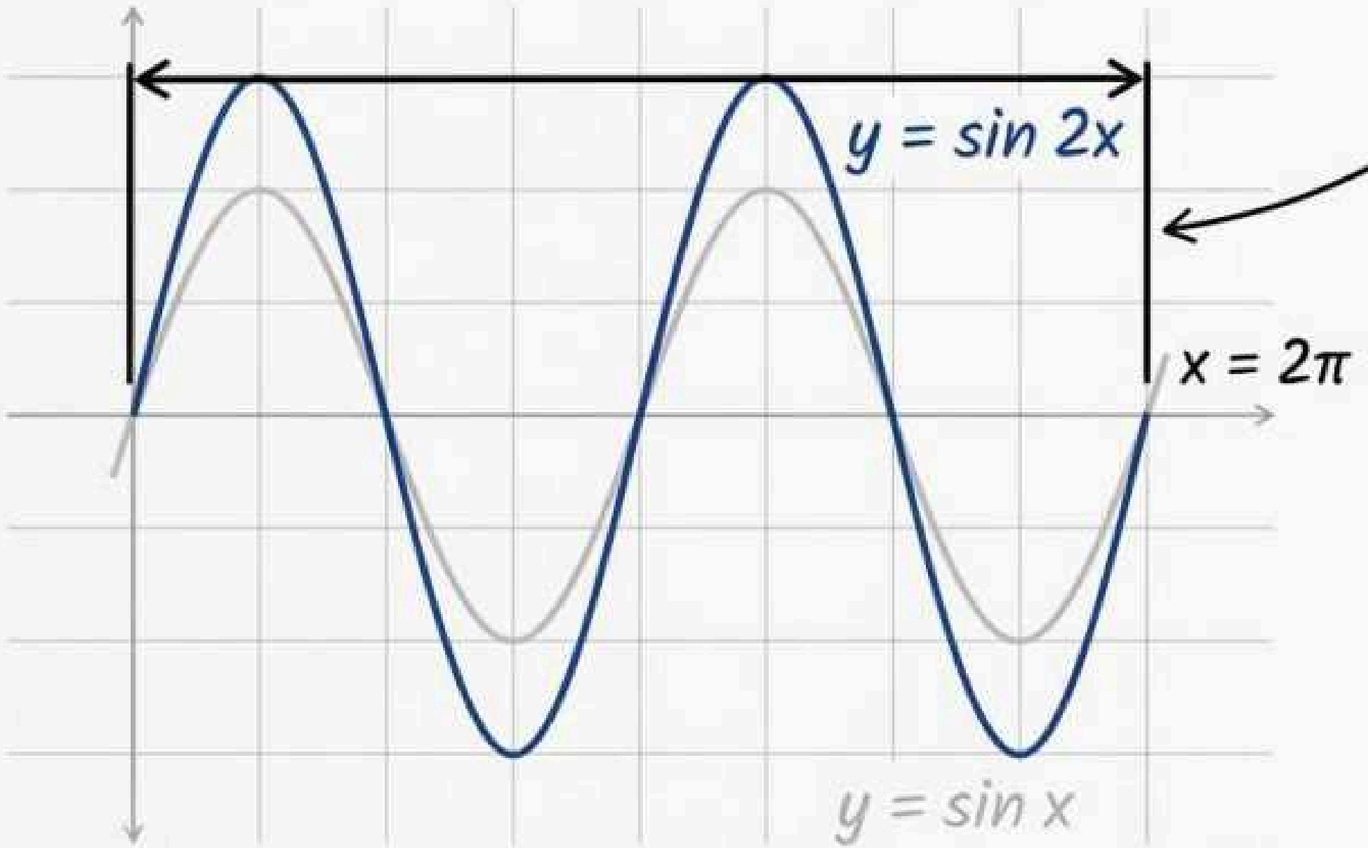


Example: $y = 3 \sin x$

Amplitude is 3, wave runs between -3 and 3. The number A stretches the wave vertically.

¶ 4.9

$y = \sin(B\,x)$



Period = $2\pi / B$.
The number B squeezes the wave horizontally.

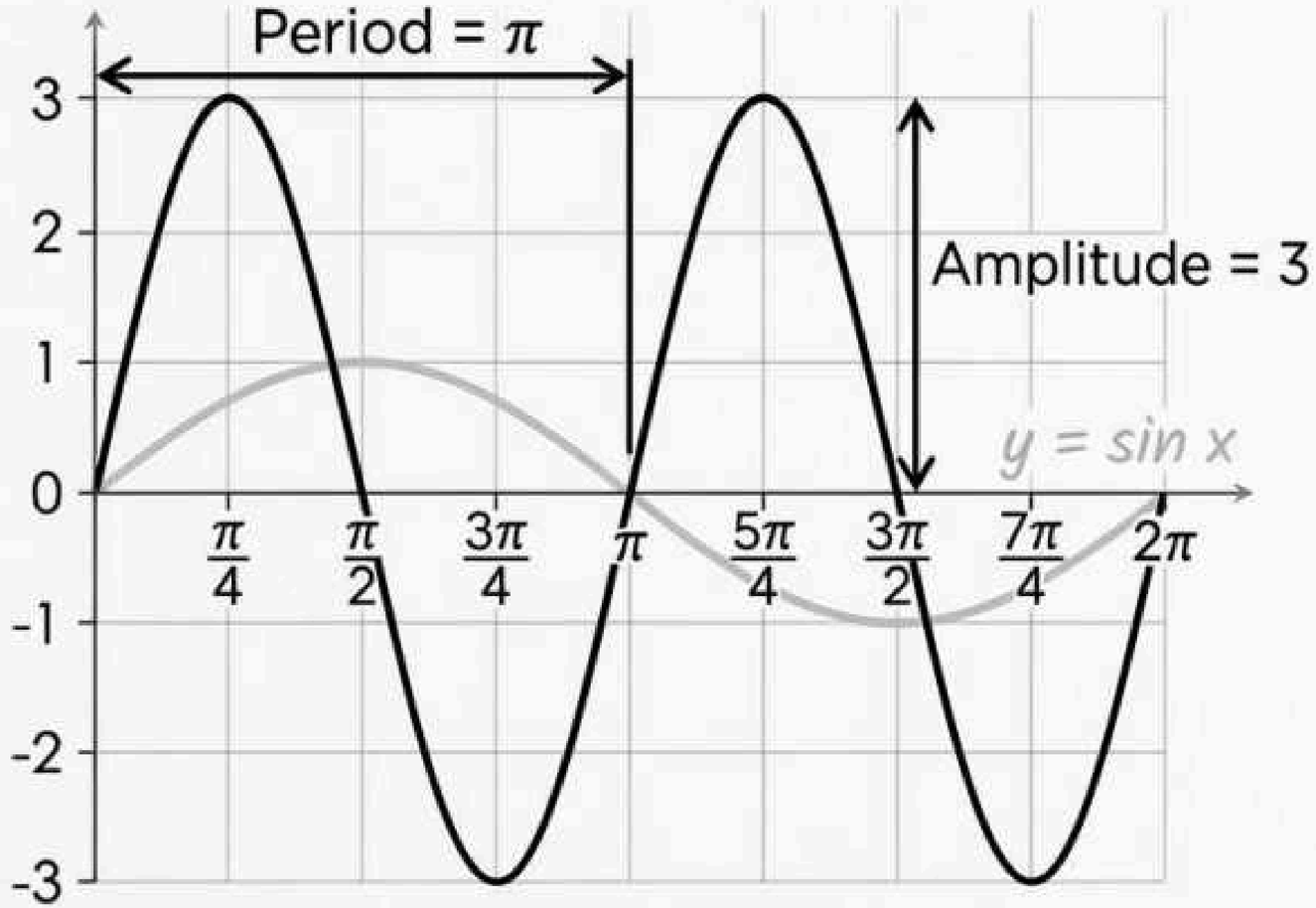
Example: y = sin(2x).
Period is π.

Insight: Completes two cycles in the time the basic wave completes one. A bigger B means a faster wave.

Combine them: Amplitude is 3, Period is π.

¶ 4.10

$$y = 3\sin(2x)$$

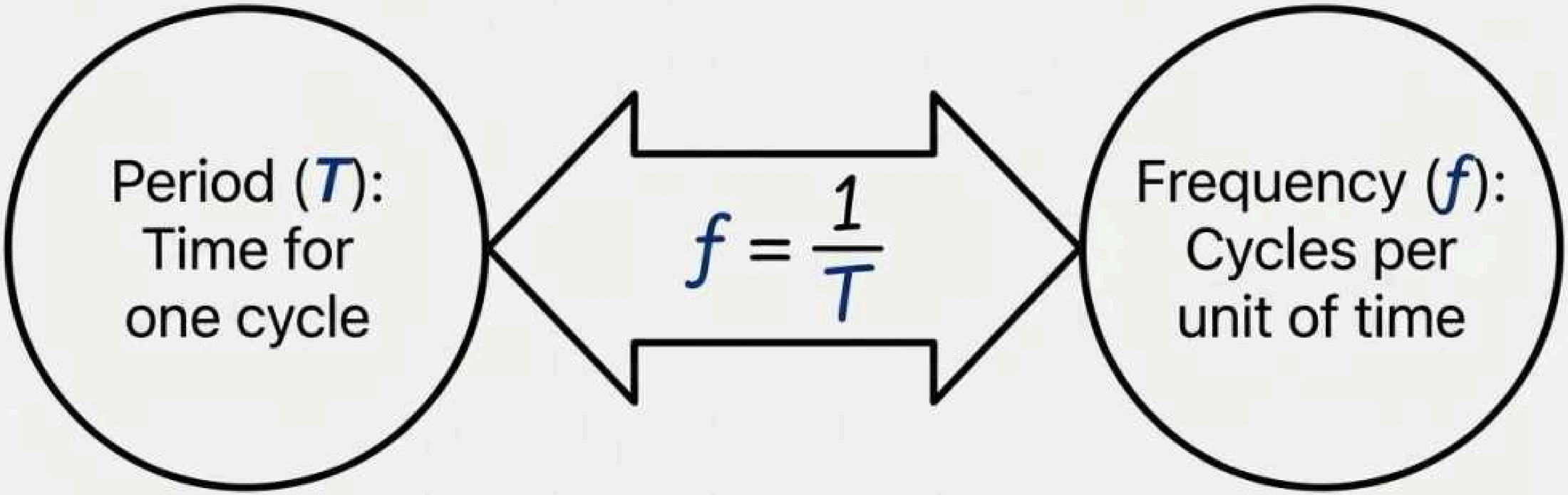


check your work

Plug in $x = \frac{\pi}{4}$:	$2x = \frac{\pi}{2} \rightarrow \sin(\frac{\pi}{2}) = 1$ $\rightarrow y = 3.$
Plug in $x = \pi$:	$2x = 2\pi \rightarrow y = 0.$

¶ 4.11

Frequency and period are two ways of describing the same speed.

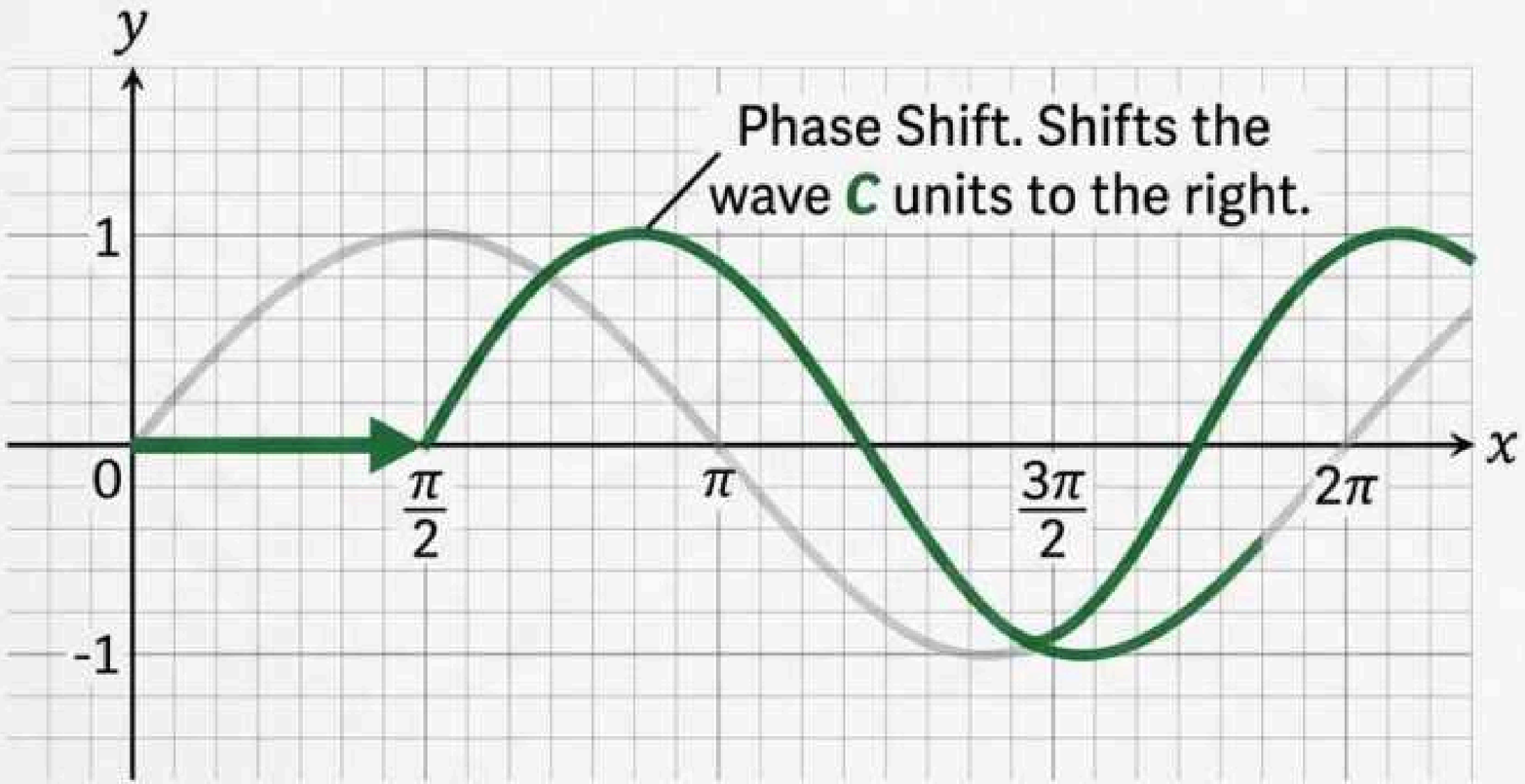


- If period is 0.5s → Frequency is 2 $^{cycles}/_{second}$ = 2 Hertz (Hz).

Musicians and engineers usually talk about frequency.

¶ 4.12

$y = \sin(x - C)$

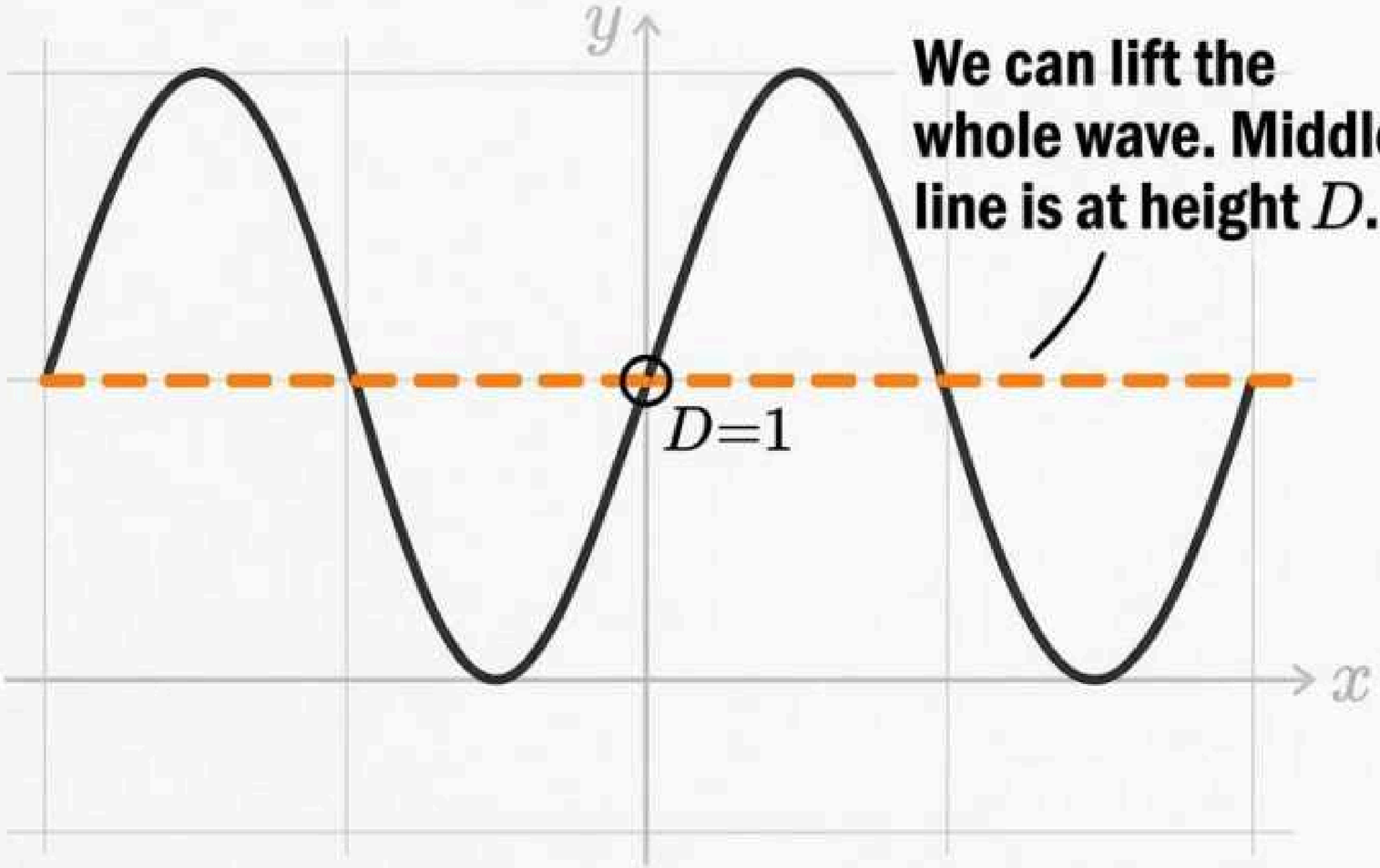


Example: $y = \sin\left(x - \frac{\pi}{3}\right)$.

- Every point moves right by about 1.0472. The starting zero is now at $\pi/3$.

¶ 4.13

$$y = A\sin(B(x - C)) + D$$



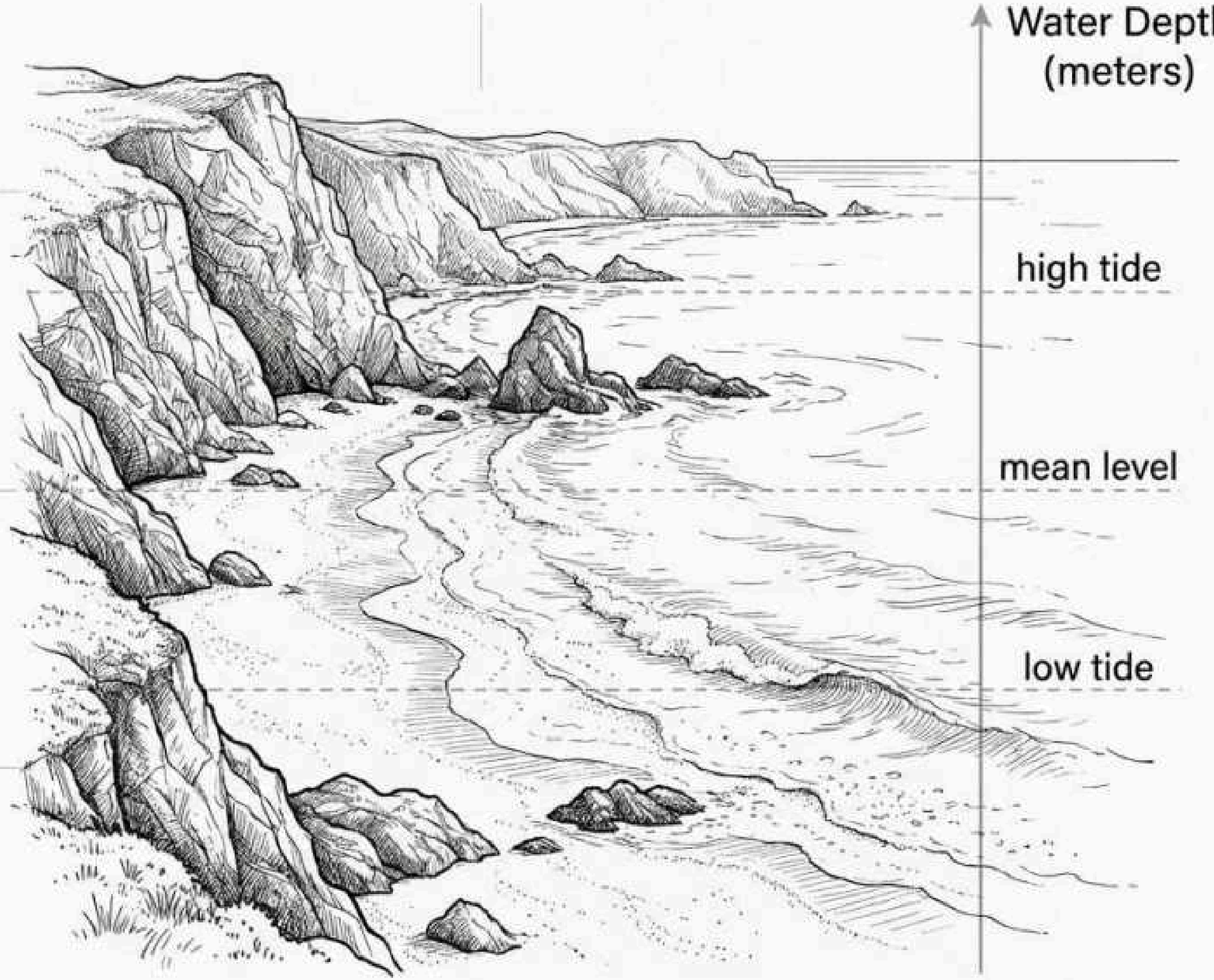
We can lift the whole wave. Middle line is at height D.

Example:

$$y = 2\sin x + 1$$

- **Amplitude is 2, middle line is 1, middle line is 1. The wave runs between -1 and 3.**

¶ 4.14

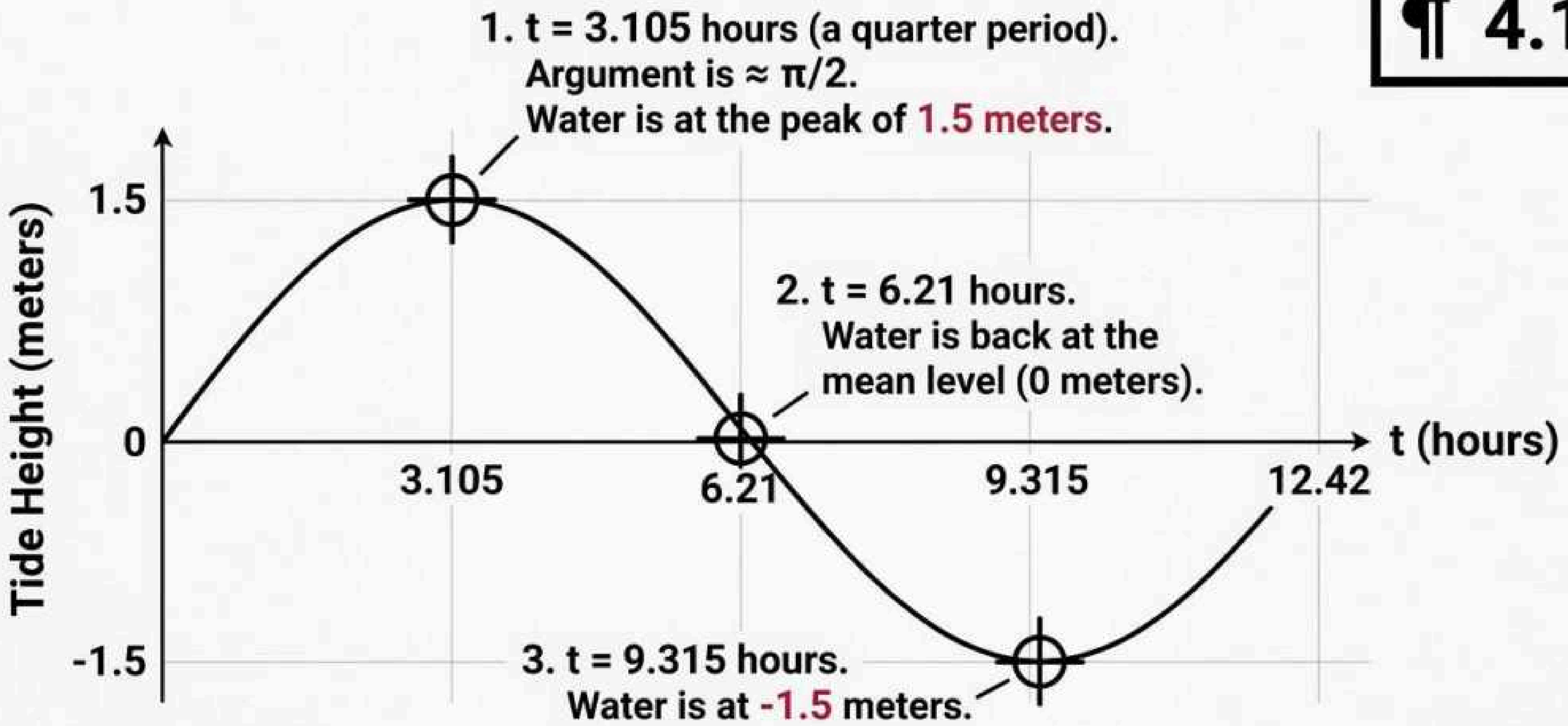


Tides rise and fall about every 12.42 hours. Water oscillates around the mean level with an amplitude of 1.5 meters.

$$\text{Height} = 1.5\sin(Bt)$$

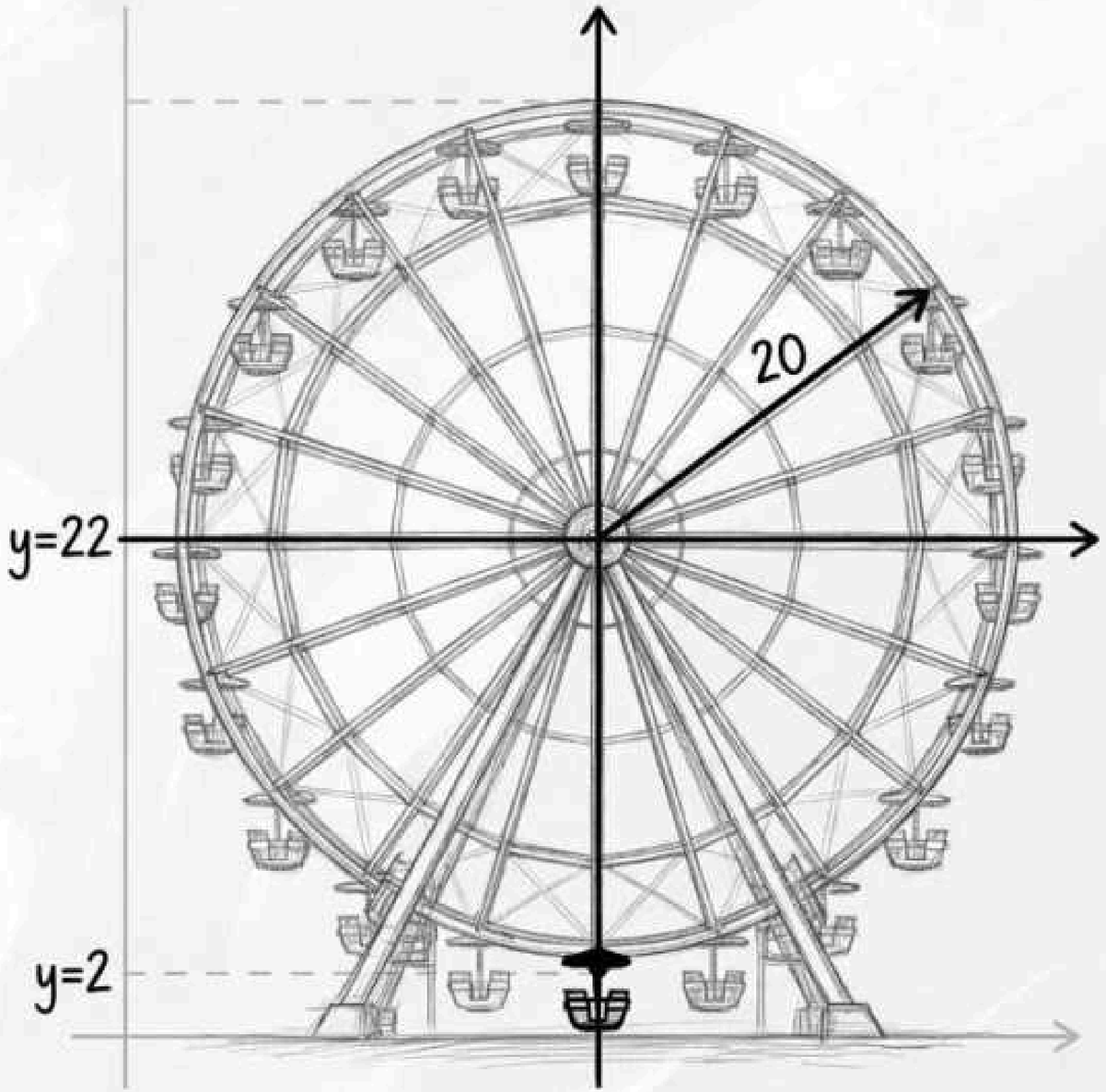
$$B = \frac{2\pi}{12.42} \approx 0.5059 \text{ per hour.}$$

¶ 4.15



The numbers confirm the picture. The model gives a sensible tide.

¶ 4.16



- Radius = 20m
- Center = 22m above ground
- Turns once every 8 minutes
- Rider starts at the bottom

$$\text{Height} = 22 - 20\cos\left(\frac{\pi}{4}t\right)$$

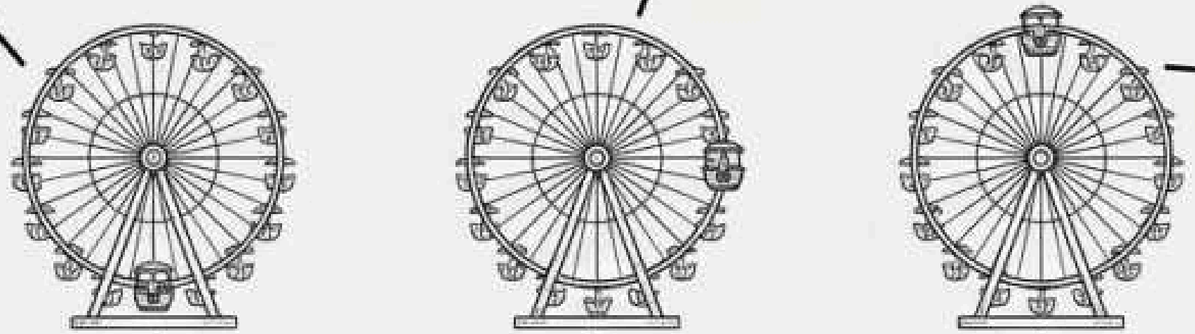
At $t = 0$, the height is 2 meters.

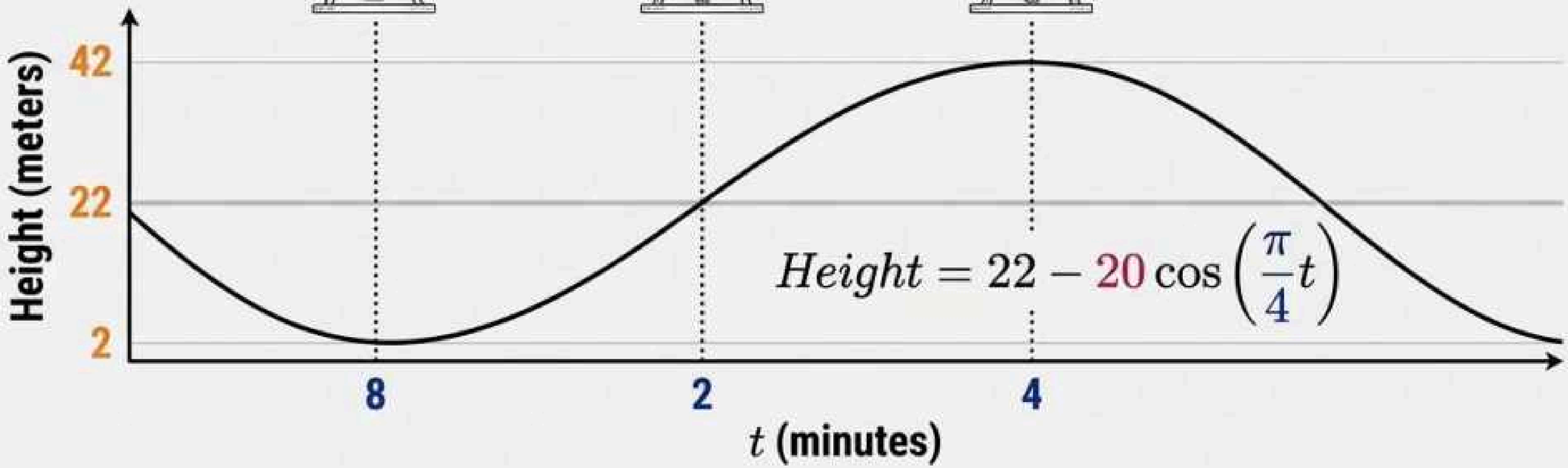
¶ 4.17

Check 1: $t = 2$ minutes.
$\cos\left(\frac{\pi}{2}\right) = 0$
Height = 22m (level with center).

Check 3: $t = 8$ minutes.
Rider is back at 2m.

Check 2: $t = 4$ minutes.
$\cos(\pi) = -1$
Height = 42m (the very top).

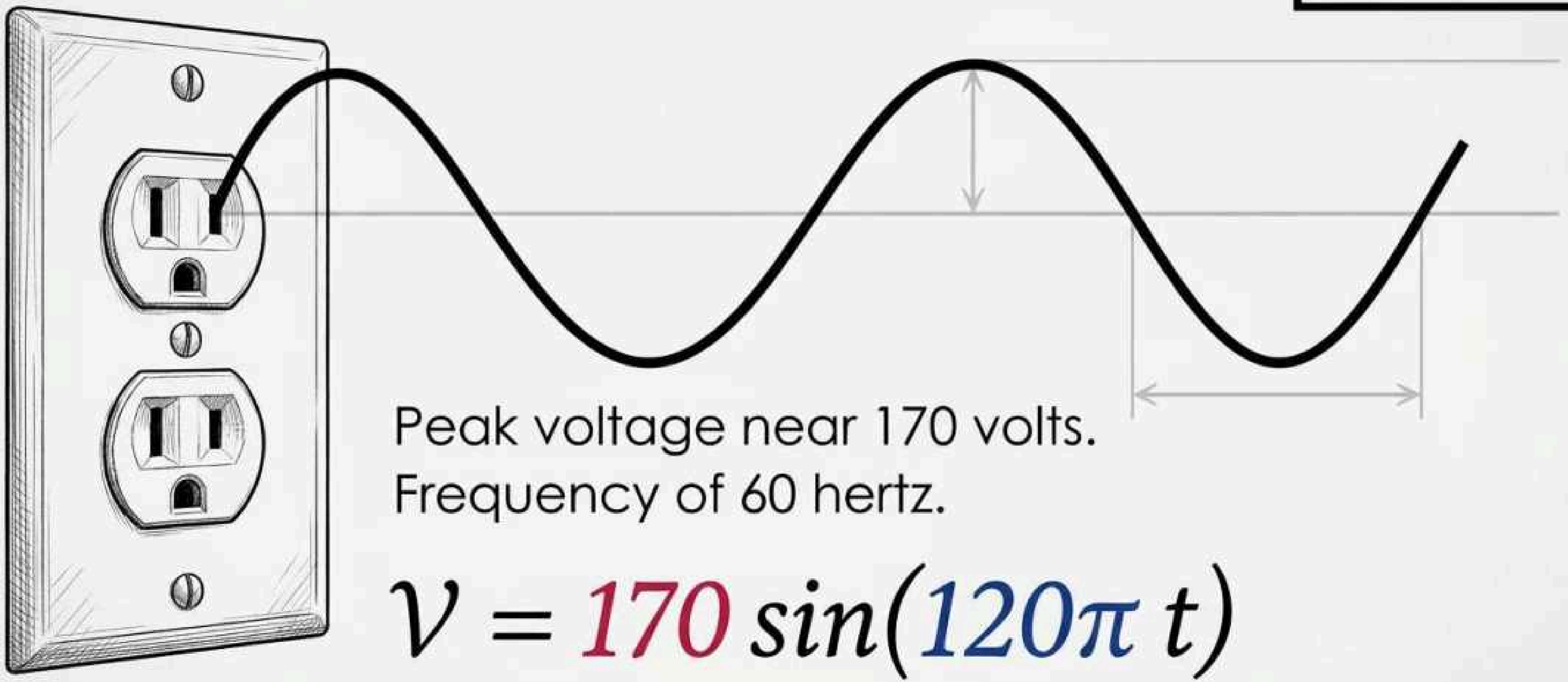




The cosine form, with a minus sign, matches a start at the bottom.

¶ 4.18

Alternating current is a sine wave.



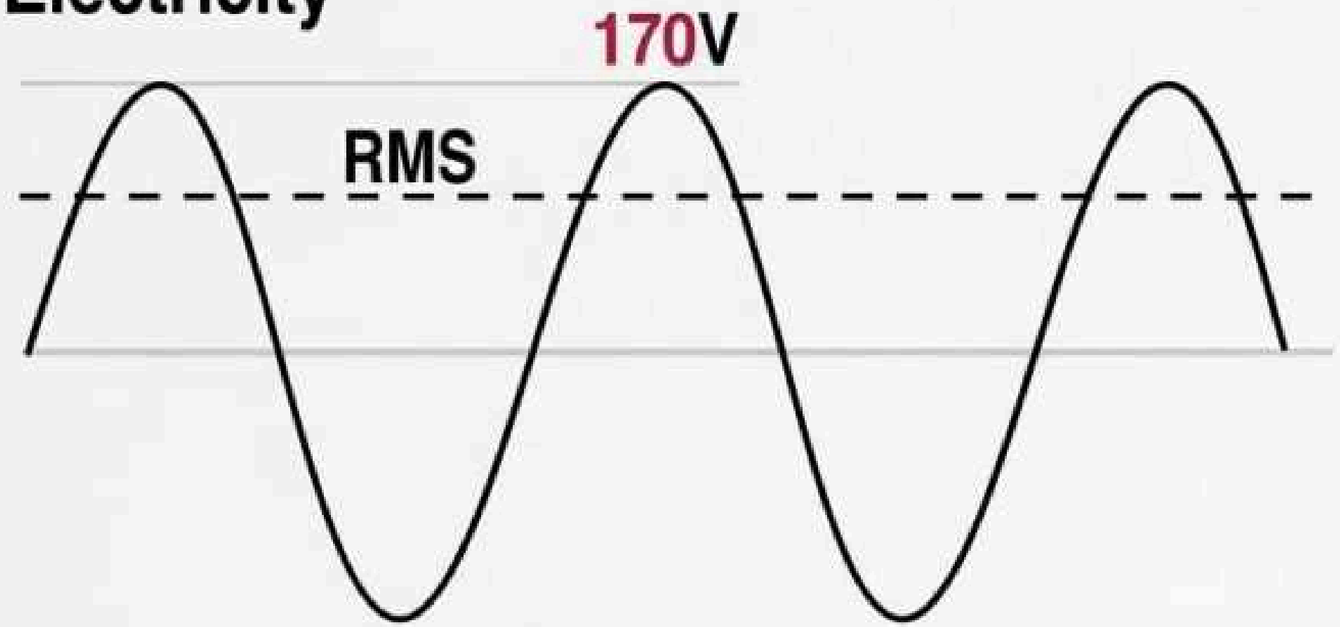
Peak voltage near 170 volts.
Frequency of 60 hertz.

$$\mathcal{V} = 170 \sin(120\pi\, t)$$

Period = 1/60th of a second (≈ 0.0167 seconds).

¶ 4.19

Electricity

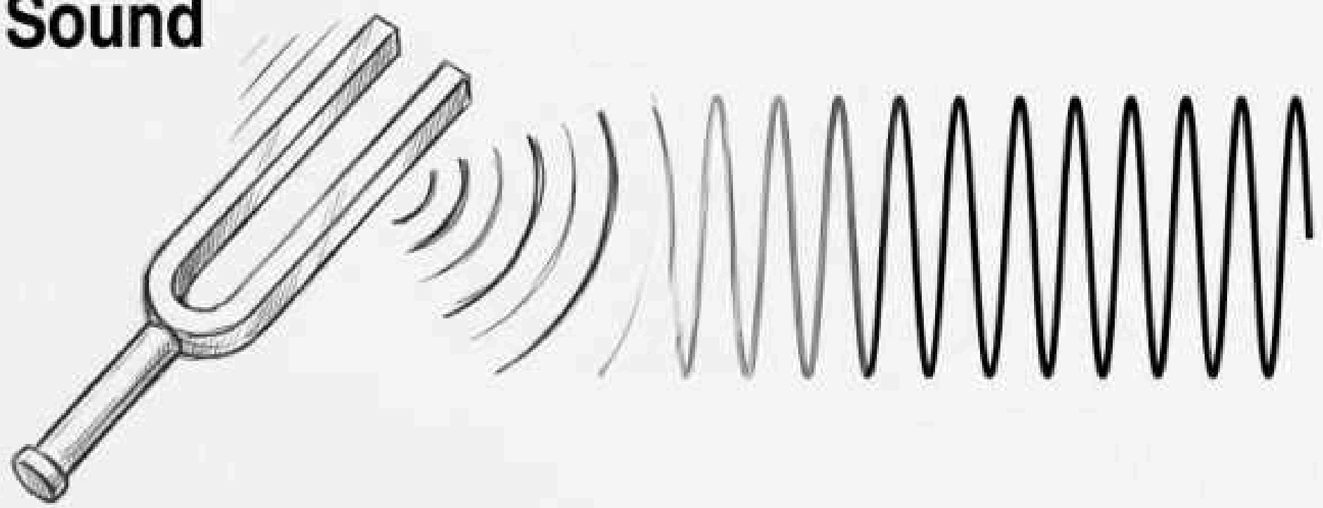


At t=0.002s, voltage is ≈ 116V.

Effective voltage $_{(RMS)} = \frac{\text{Peak}}{\sqrt{2}} \approx 120.2$ volts.

This is why household power is rated near 120V.

Sound



Sound works the same way:

Note A above middle C vibrates at 440 Hz.

Period ≈ 2.27 milliseconds.

How would you write a wave with amplitude 4, period 6, and a middle line at 10? Work out B, then write the equation.

$$y = [__]\sin([__]x) + [__]$$

Lecture 5: We study the identities that connect sines, cosines, and tangents, and we use them to solve equations.