$\sin(\theta) = \frac{y}{r}$

$\cos(\theta) = \frac{x}{r}$

$\sin^2(\theta) + \cos^2(\theta) = 1$

$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$

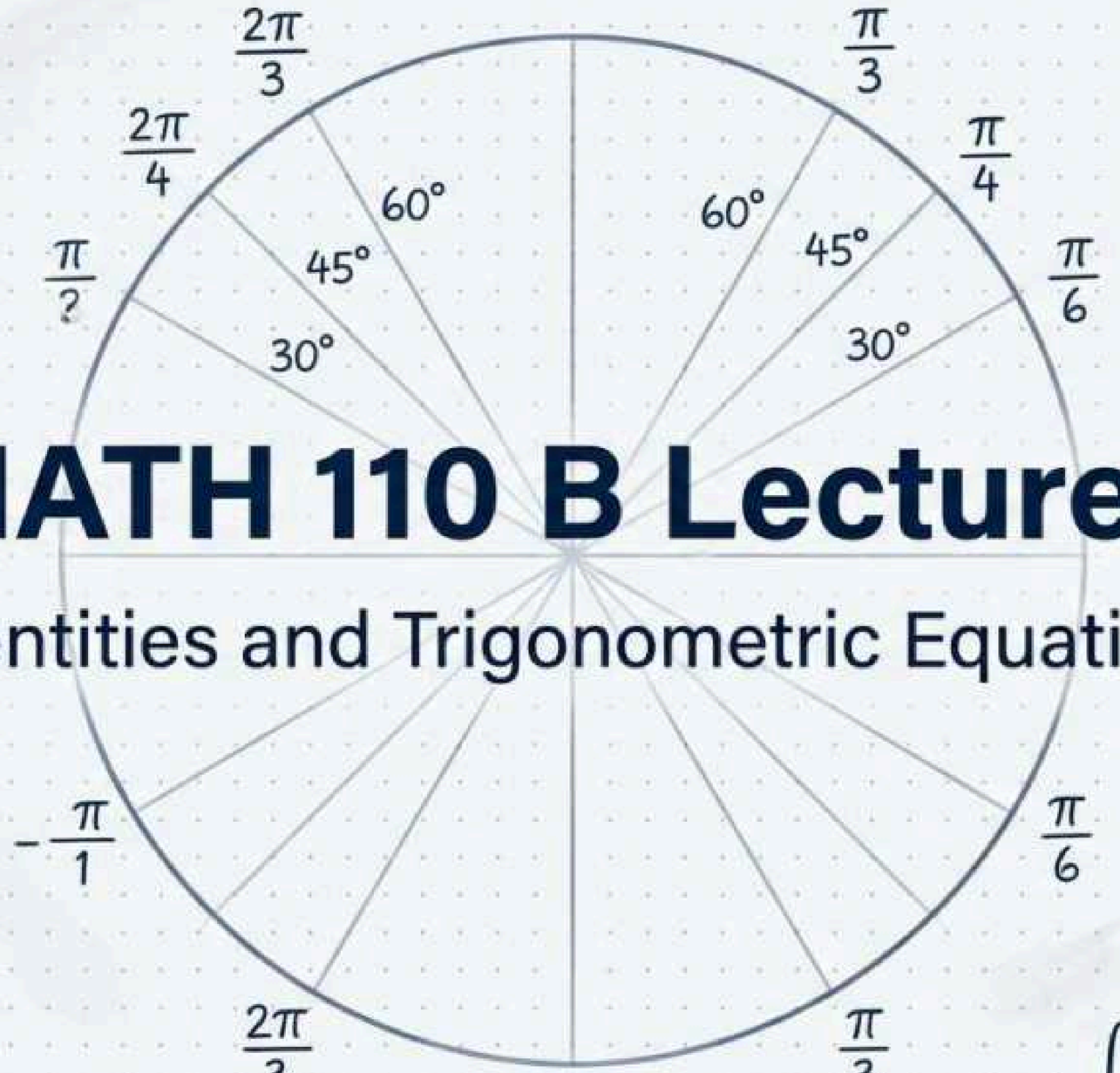
$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$

$x^2 + y^2 = 1$

$\sin(\theta) = \frac{y}{r}$

$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$

$x^2 + y^2 = 1$



MATH 110 B Lecture 5

Identities and Trigonometric Equations

$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$

$x^2 + y^2 = 1$

$A = \frac{1}{2}r^2\theta$

$x^2 + y^2 = 1$

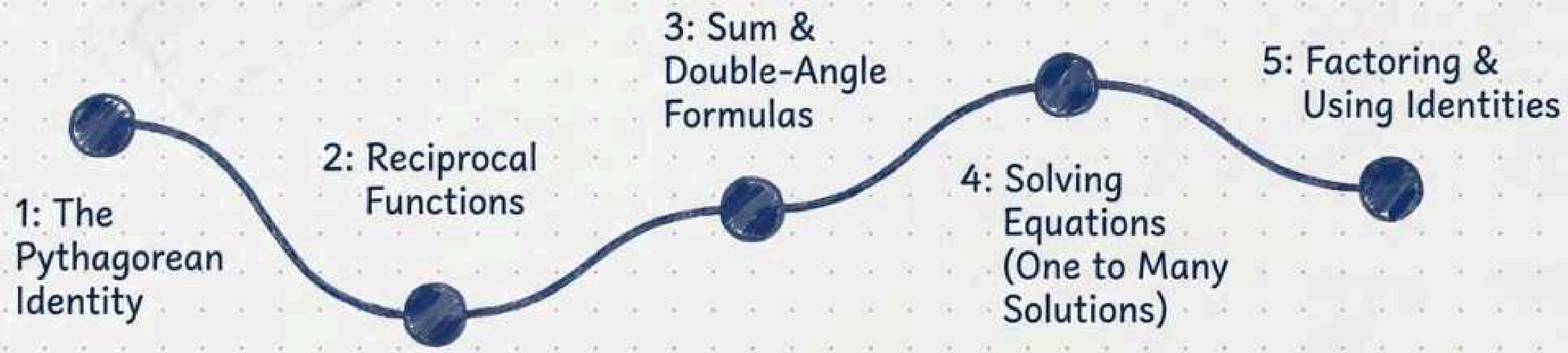
$A = \frac{1}{2}r^2\theta$

$\theta = \frac{s}{r}$

$\int \sin(x)\,dx = -\cos(x) + C$

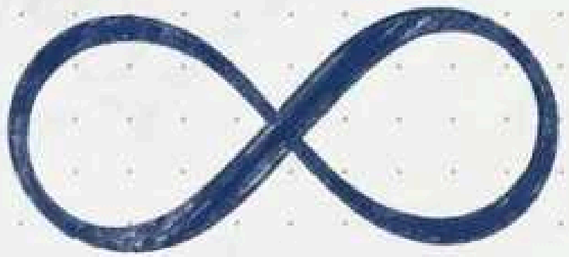
¶ 5.1

Welcome to Lecture 5.



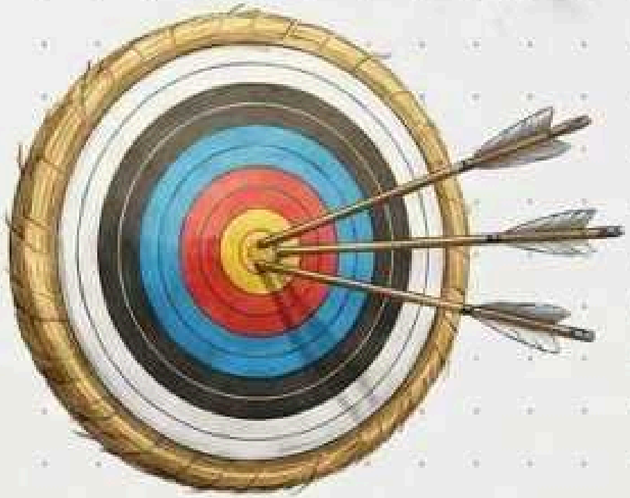
¶ 5.2

Identity



True for every allowed value.

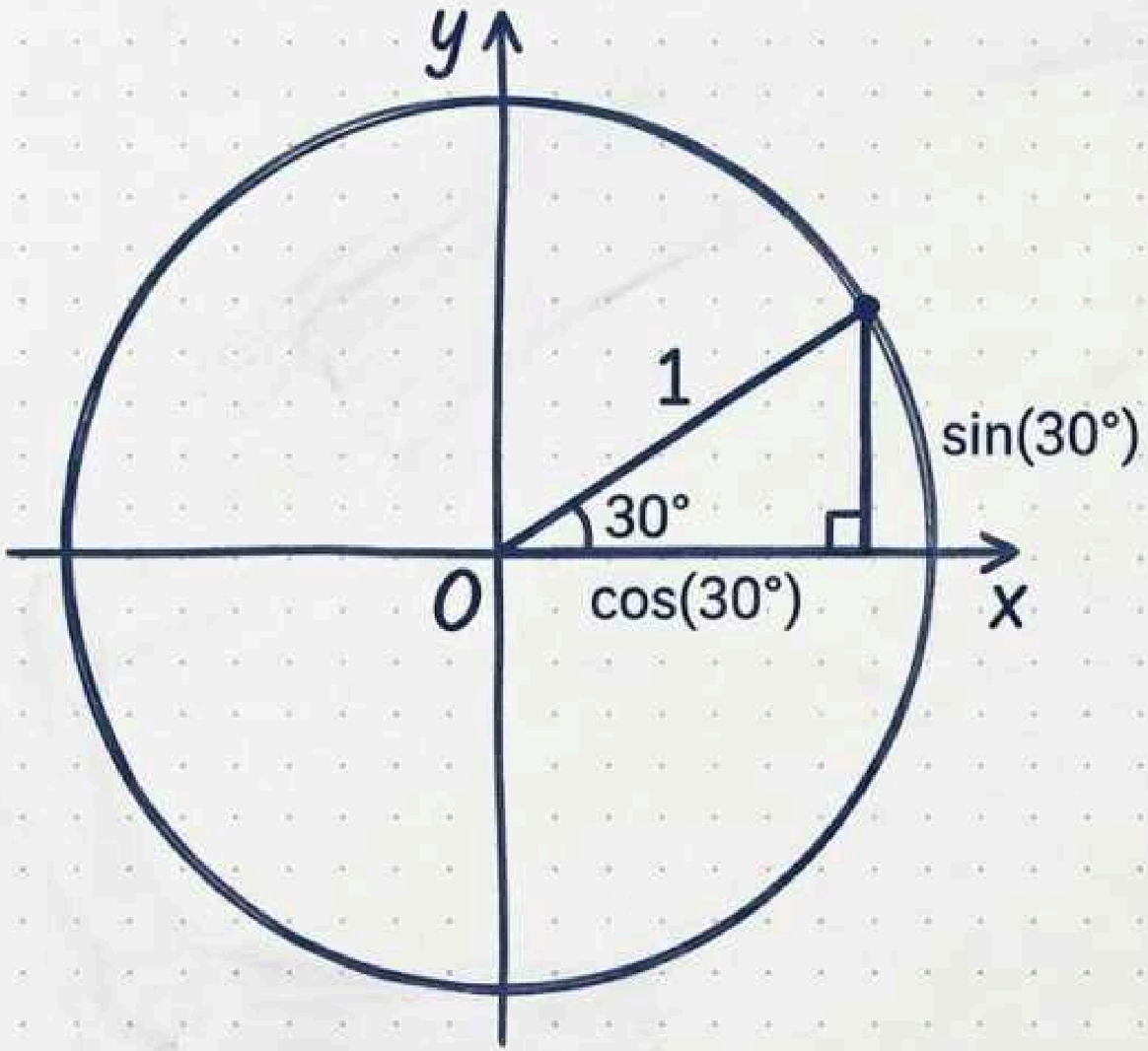
Ordinary Equation



True only for some values.

Goal: Learn the identities, then use them to find the angles that make statements true.

¶ 5.3



$$\sin^2\theta + \cos^2\theta = 1$$

The Test at 30°

$\sin(30°) = \frac{1}{2}$ → square is $\frac{1}{4}$

$\cos(30°) = \frac{\sqrt{3}}{2}$ → square is $\frac{3}{4}$

$$\frac{1}{4} + \frac{3}{4} = 1$$

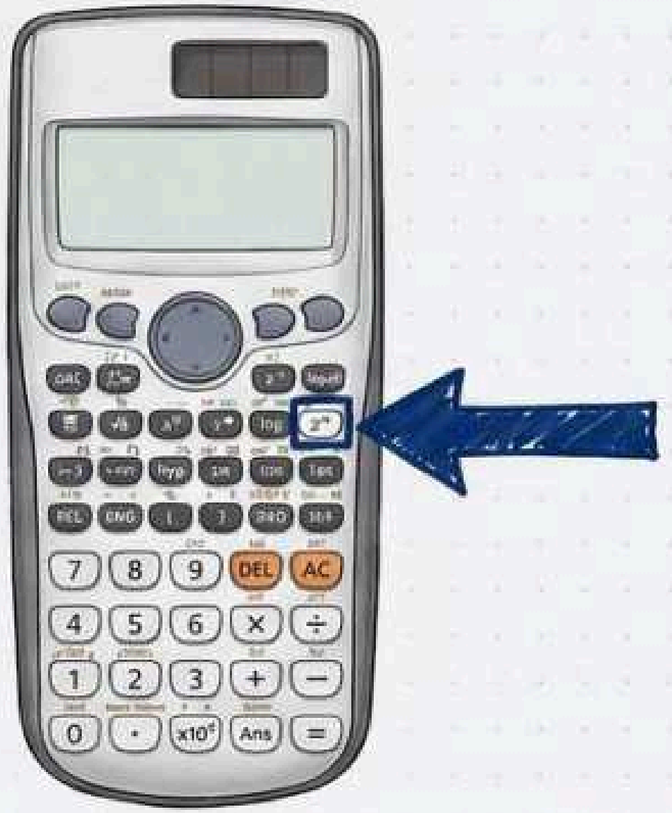
$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

Divide the first identity ($\sin^2 x + \cos^2 x = 1$) by $\cos^2(x)$...

$$1 + \tan^2(x) = \frac{1}{\cos^2(x)}$$

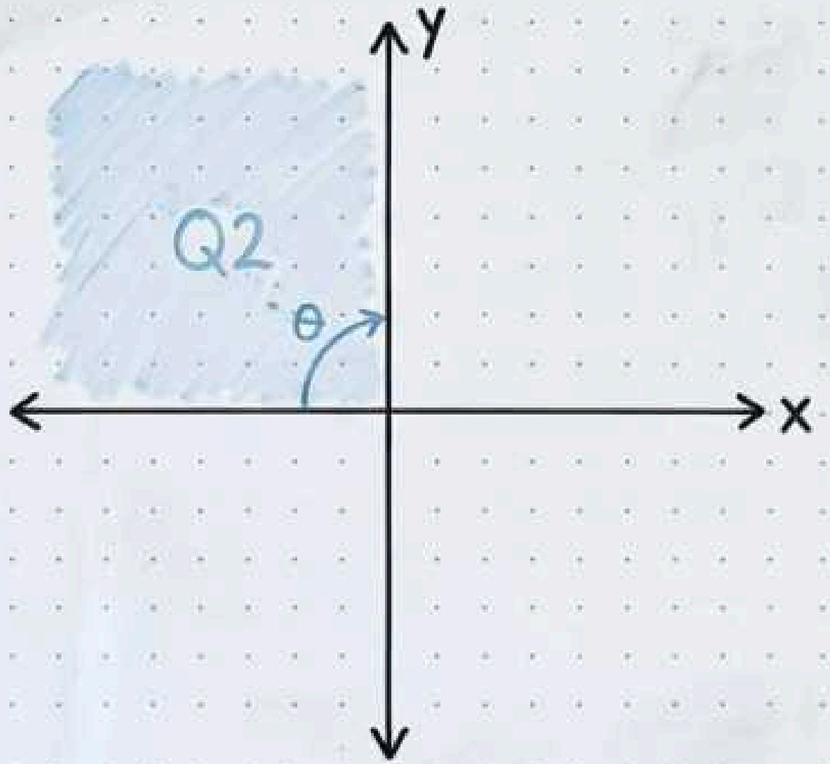
Useful when you know the tangent and need the others.

$\cos(x)$	$\leftrightarrow$	$\sec(x) = \dfrac{1}{\cos(x)}$ (Secant)
$\sin(x)$	$\leftrightarrow$	$\csc(x) = \dfrac{1}{\sin(x)}$ (Cosecant)
$\tan(x)$	$\leftrightarrow$	$\cot(x) = \dfrac{1}{\tan(x)}$ (Cotangent)



Rarely needed in this course, but older books use them. Calculators lack these buttons—just press the reciprocal key.

¶ 5.6



Step 1:

Given: $\sin = \frac{3}{5}$, Angle in Q2.

Step 2:

Compute: $\cos^2 = 1 - \left(\frac{3}{5}\right)^2 = 1 - \frac{9}{25} = \frac{16}{25}$

Step 3:

Resolve: $\cos = \pm\frac{4}{5}$.

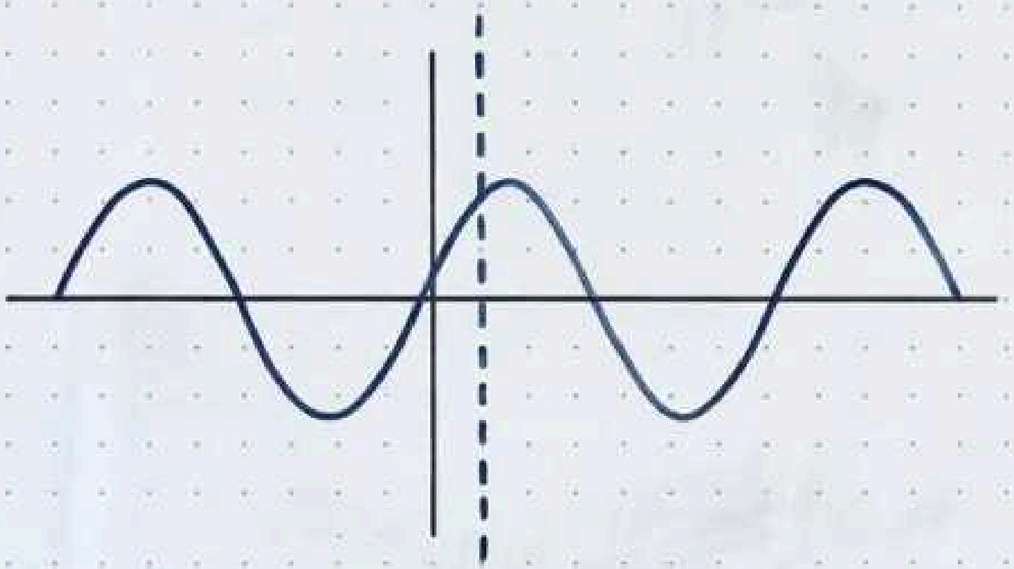
In Q2, cosine is negative!

$\rightarrow \cos = -\frac{4}{5}$

Step 4:

Tangent Calculation: $\frac{\frac{3}{5}}{-\frac{4}{5}} = -\frac{3}{4}$

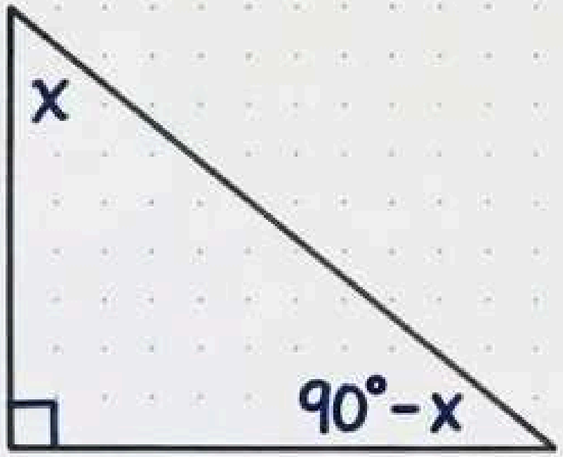
Negative Angles



$$\sin(-x) = -\sin(x)$$

$$\cos(-x) = \cos(x)$$

Complements



$$\sin(90° - x) = \cos(x)$$

$$\cos(90° - x) = \sin(x)$$

$$\sin(A + B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$

$$\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

Notice the minus sign in the cosine formula!

Find Exact Value: sin(75°)

¶ 5.9



$$\sin(45°)\cos(30°) + \cos(45°)\sin(30°)$$

$$(0.7071 \times 0.8660) + (0.7071 \times 0.5)$$

$$\approx 0.9659$$

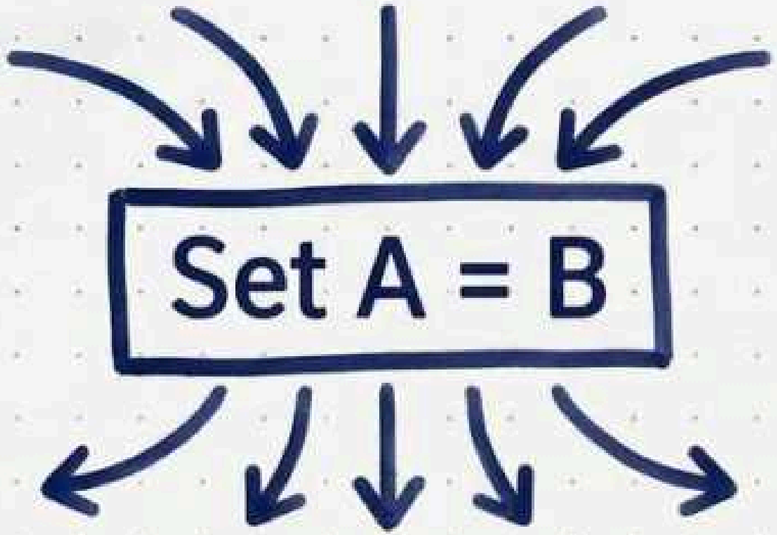
¶ 5.10

$$\sin(A+B) = \sin(A) + \sin(B)$$

The Proof (Why it fails)

$$\sin(30°) + \sin(45°)$$
$$= 0.5 + 0.7071$$
$$\approx 1.2071$$

No sine can exceed 1!
Check it once and remember it.



$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

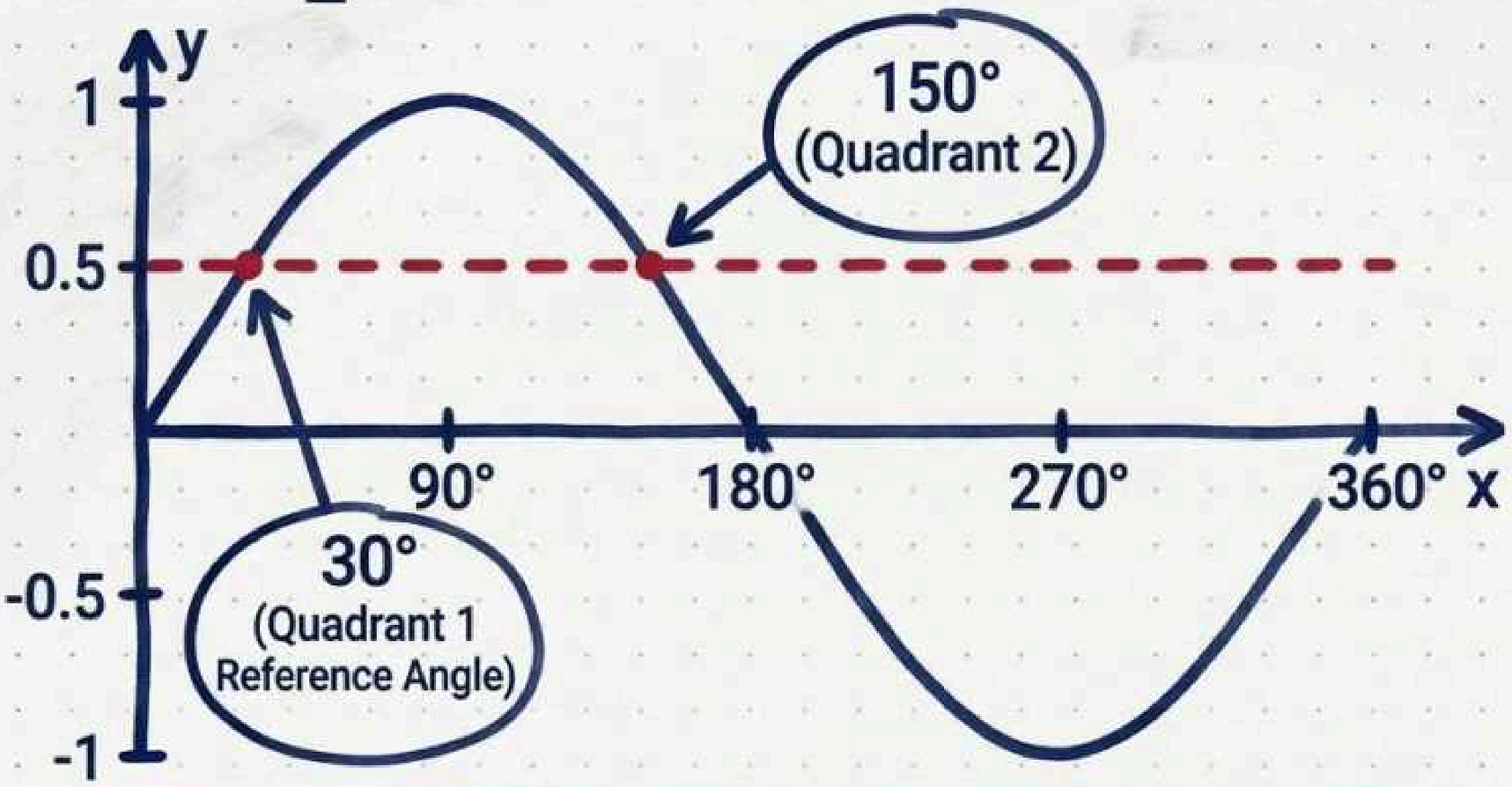
Verification Test at x = 30°

$\sin(60°)$: $2 \times \frac{1}{2} \times 0.8660 = 0.8660$

$\cos(60°)$: $\frac{3}{4} - \frac{1}{4} = \frac{1}{2}$

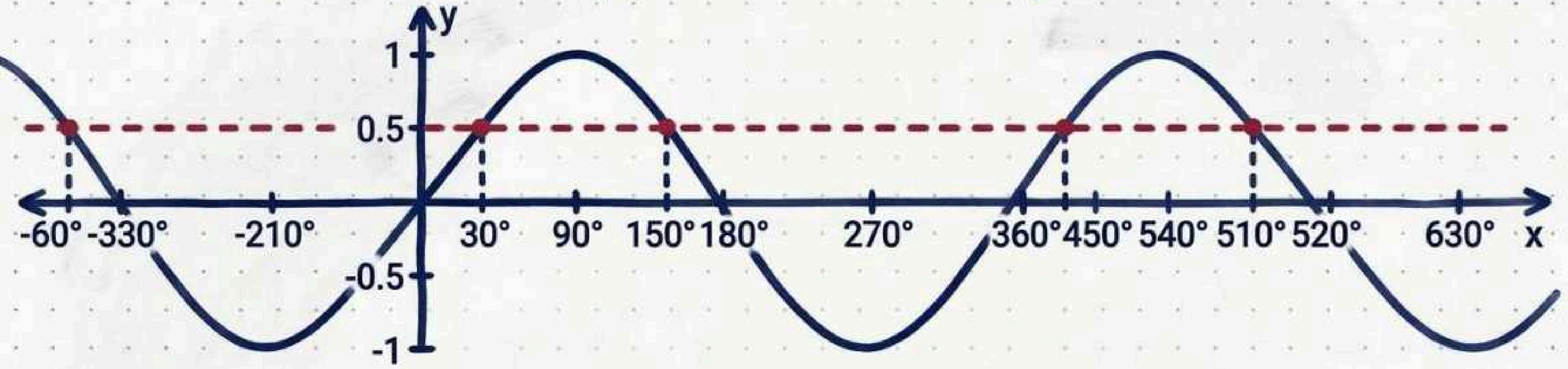
Solve $\sin(x) = \frac{1}{2}$ for $x \in [0, 360°]$

¶ 5.12



The horizontal line meets the wave twice in one period.

Infinite Solutions (General Form)

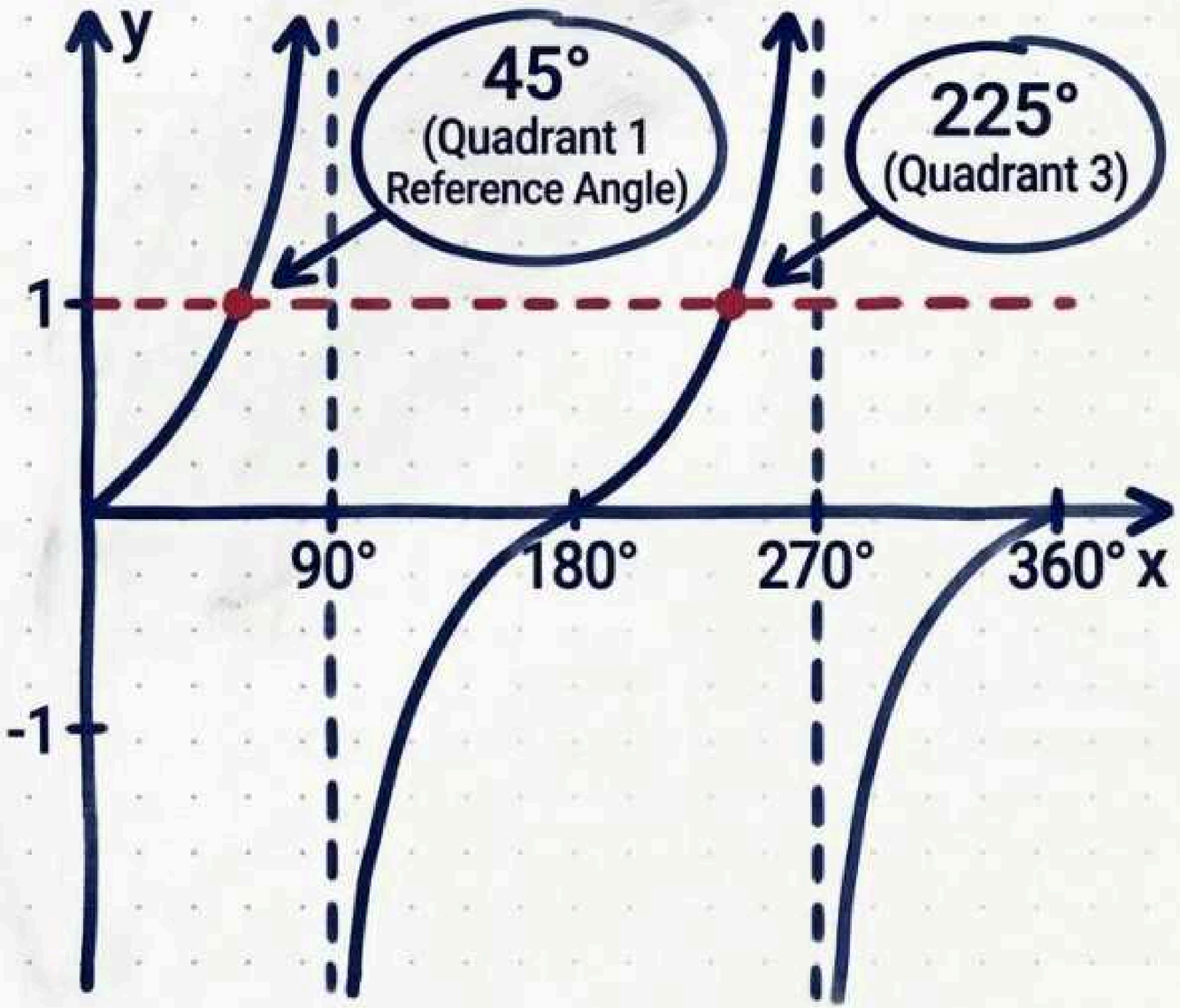


Degrees Output	Radians Output
$30° + 360°n$	$\frac{\pi}{6} + 2\pi n$
$150° + 360°n$	$\frac{5\pi}{6} + 2\pi n$
(where n is any whole number)	

Always state which interval the question asks for.

Solve tan(x) = 1, interval [0, 360°].

¶ 5.14

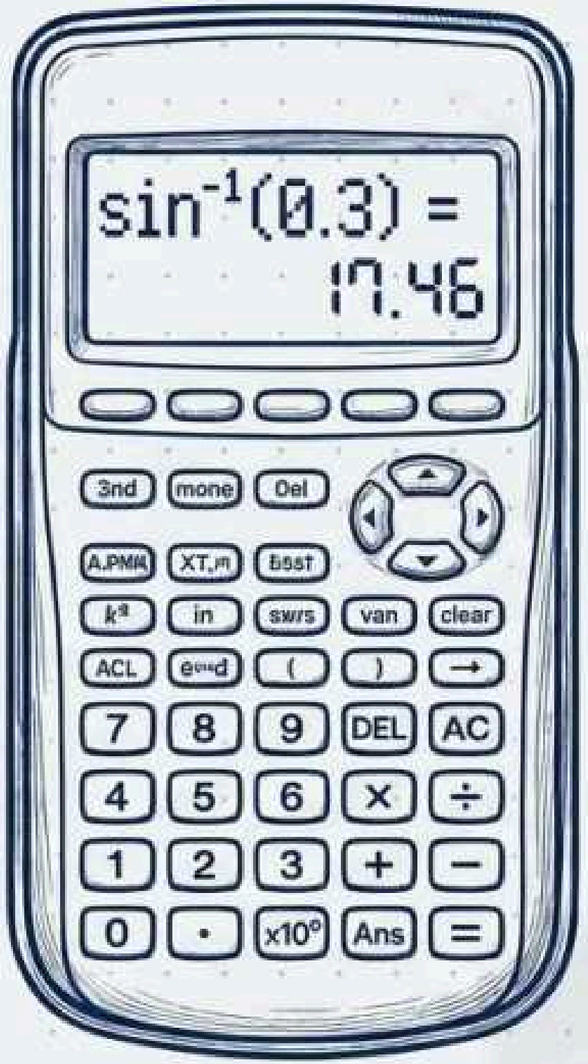


- Reference Angle: 45° (Positive in Q1, Q3).
- Solutions in interval: 45° and 225°.

General Form
45° + 180°n

Tangent's period is only 180°, not 360°.

Solve $\sin(x) = 0.3$



Step 1 (Calculator output):
Yields **17.46°** (Quadrant 1).

Step 2 (Human output):
Find the second solution in Quadrant 2:

$$180° - 17.46° = 162.54°$$

Radians Note: Solutions in radians are ≈ 0.3047 and ≈ 2.8369.

¶ 5.16

Solve $\cos(x) = -0.25$

Solution 1 (Calculator):

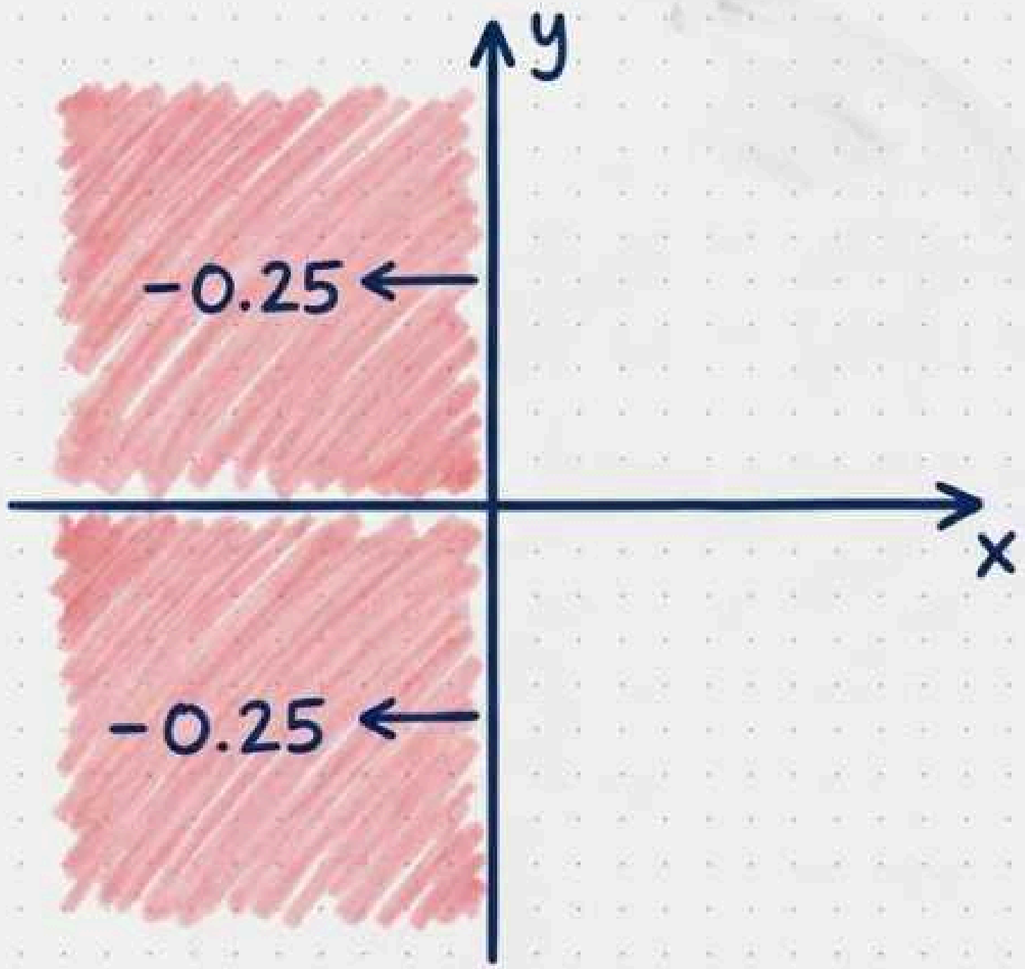
$\approx 104.48°$ (Q2)

↓

Solution 2 (The Hidden Answer):

The cosine is also negative in Q3.

$360° - 104.48° =$ **255.52°**



The calculator only gives **one** answer. **You must** find the others yourself.

Original Equation:
$2\sin^2(x) - \sin(x) - 1 = 0$

Translation Step:
Treat sine as an unknown → let $u = \sin(x)$ → $2u^2 - u - 1 = 0$

Factoring Step:
$(2\sin(x) + 1)(\sin(x) - 1) = 0$

Branching Solutions (Interval 0 to 360°)

$\sin(x) = -\frac{1}{2}$

210°, 330°

$\sin(x) = 1$

90°

¶ 5.18

$$2\sin^2(x) + 3\cos(x) = 3$$

↓

Substitution
Replace $\sin^2(x)$ with $(1 - \cos^2(x))$

↓

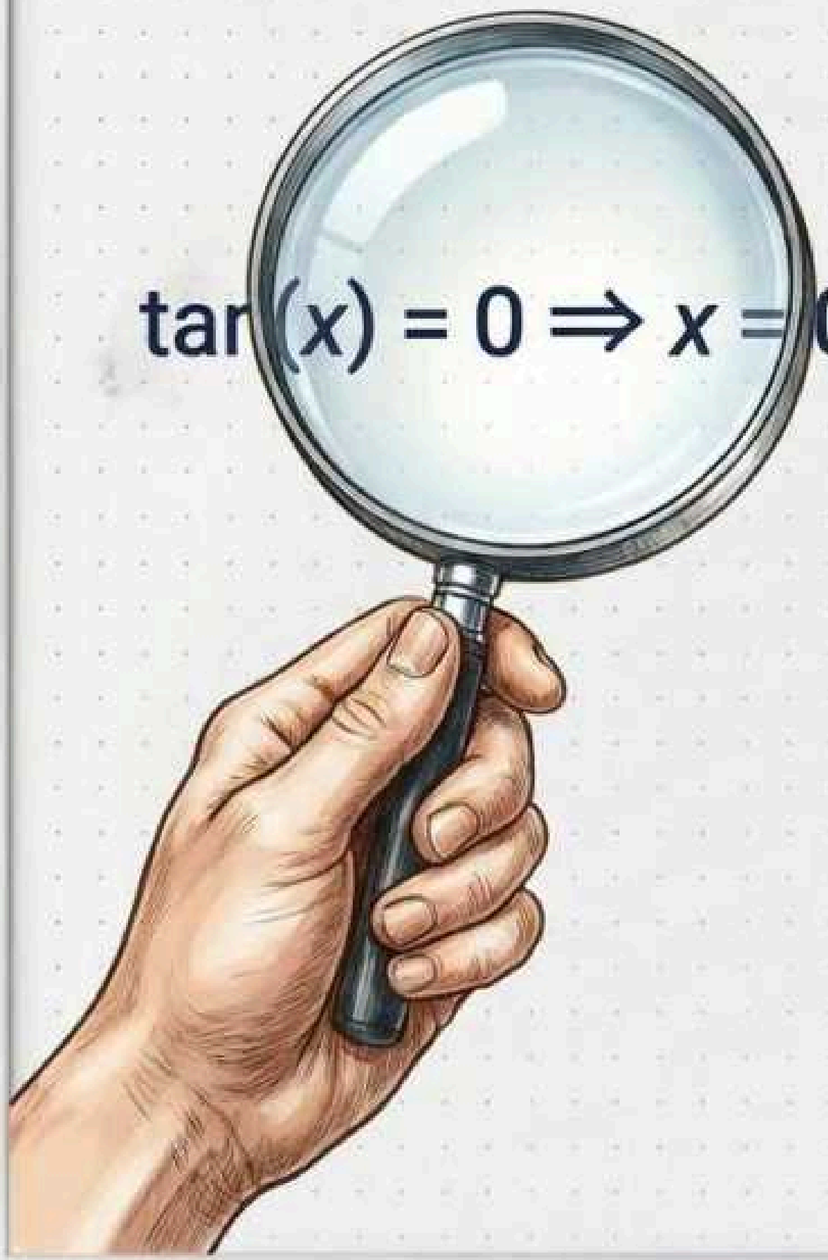
Uniform Equation
$2\cos^2(x) - 3\cos(x) + 1 = 0$

↓

Factoring
$(2\cos(x) - 1)(\cos(x) - 1) = 0$

↓

Final Angles
$\cos(x) = ½$ or $1 \rightarrow 0°, 60°, 300°$



- ☐ **Squaring?** Check if you introduced extra, invalid solutions. ✗
  (e.g., $\tan^2(x) = 1 \Rightarrow \tan(x) = \pm 1$)
- ☐ **Dividing?** Never divide by a function that could be zero (you will lose solutions). !
  (e.g., ÷ cos(x))
- ☐ **Quadrants?** Verify your positive/negative signs match the quadrant.
  (e.g., cos(x) < 0 in Q2 & Q3)
- ☐ **Calculator Mode?** Are you in Degrees or Radians? 🖩 (Check settings!)

A quick substitution catches most errors.

Solve $\cos(x) = \frac{\sqrt{2}}{2}$ from 0 to 2π.

Give answers in radians.

Next: Leaving the right triangle behind. Solving triangles of any shape using the Laws of Sines and Cosines.