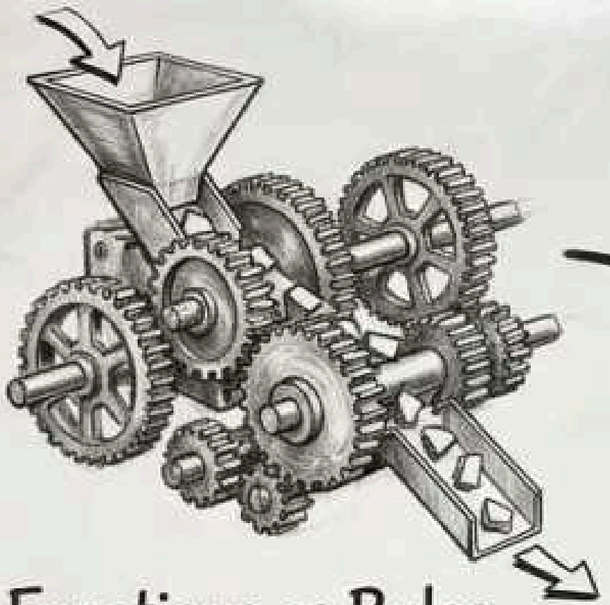
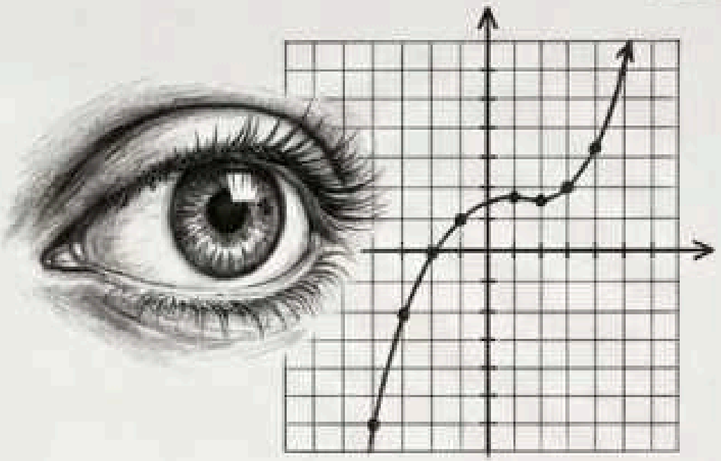
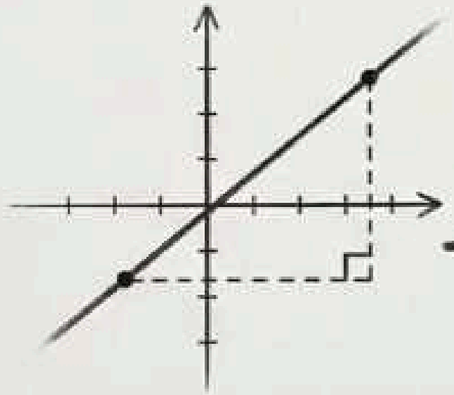
Functions & Their Graphs



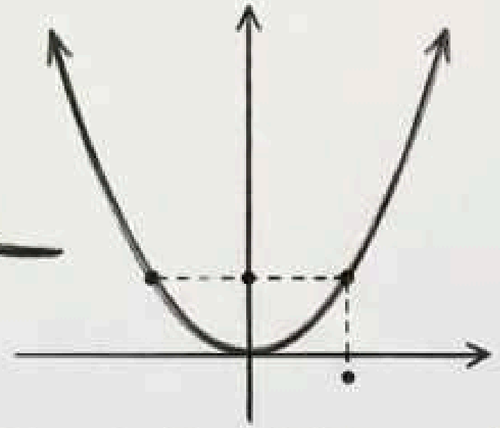
Functions as Rules



Reading Graphs



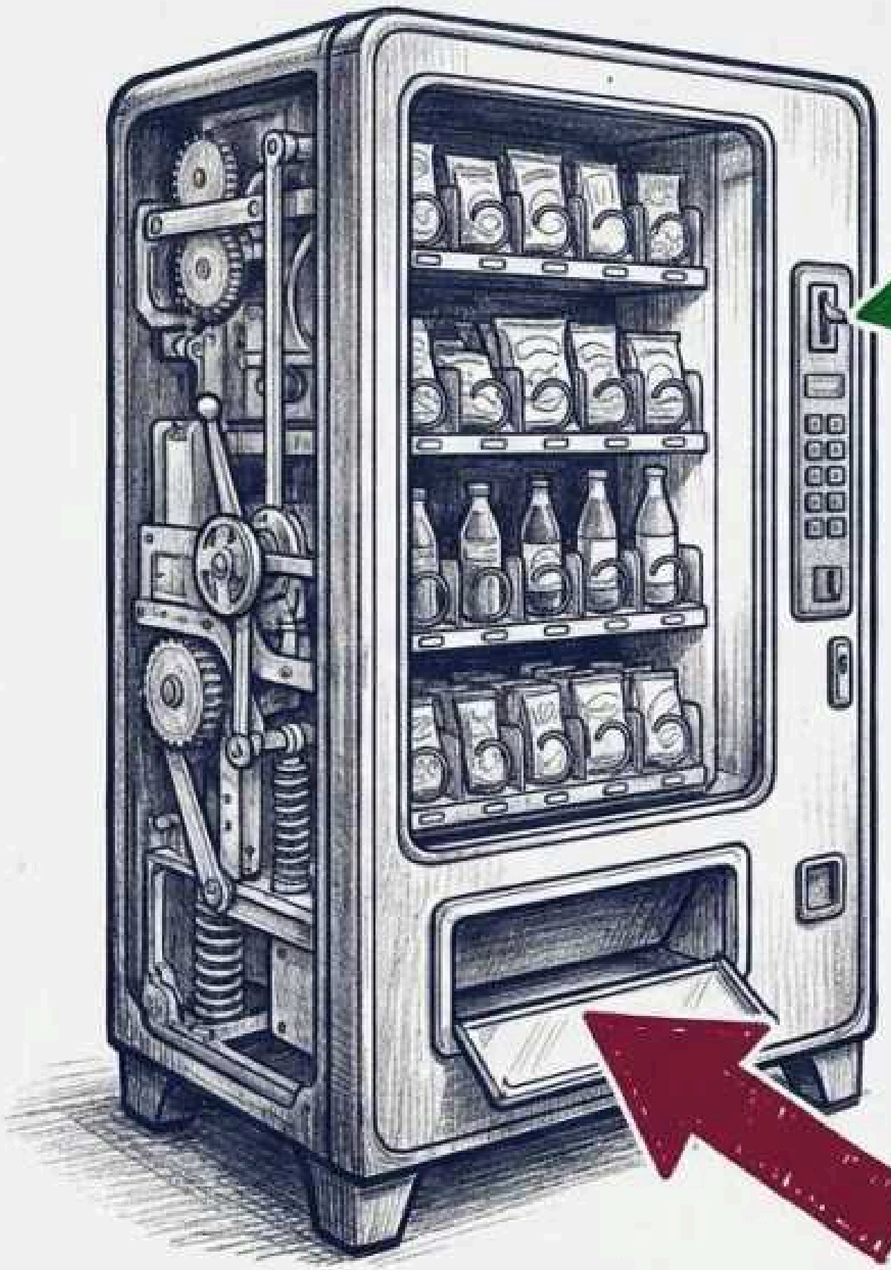
Linear Families



Parabolas & Beyond

Everything in the next five lectures builds on these first definitions.

Gemini Notebook



INPUT (Exactly One)

OUTPUT
(Exactly One Snack)

The Golden Rule:
A function is a rule that assigns to each input exactly one output.

The same input must NEVER yield two different outputs.

Gemini Notebook

$$f(x) = 2x + 3$$

Name of the function

The Input

The Rule

$$f(4) = 2(4) + 3 = 11$$

$$f(-1) = 2(-1) + 3 = 1$$

Note: f(x) DOES NOT mean f × x.

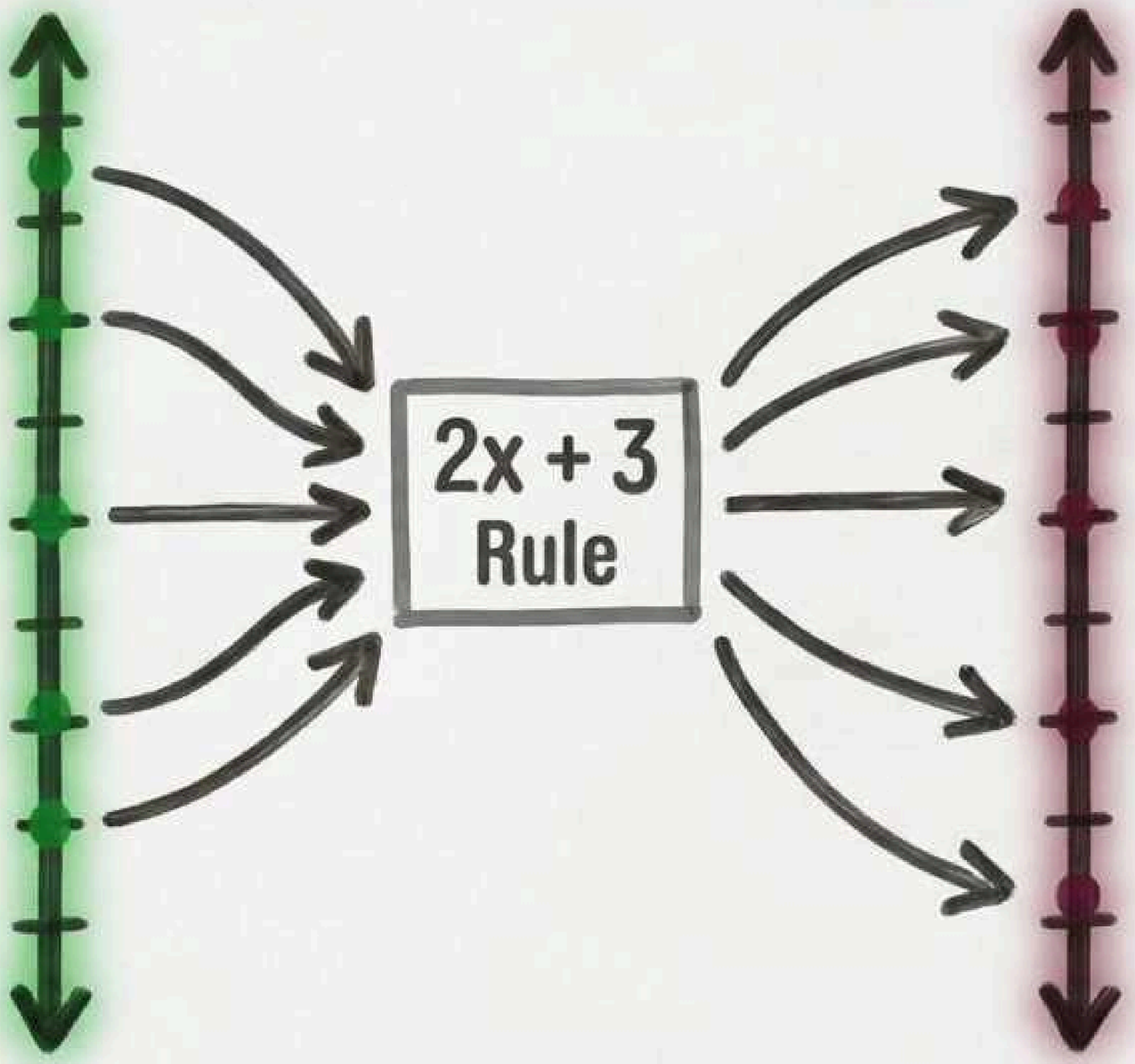
Gemini Notebook

DOMAIN: Allowed Inputs

RANGE: Actual Outputs

¶ 1.4

For $f(x) = 2x + 3$, any real number may go in.



Every real number can come out.

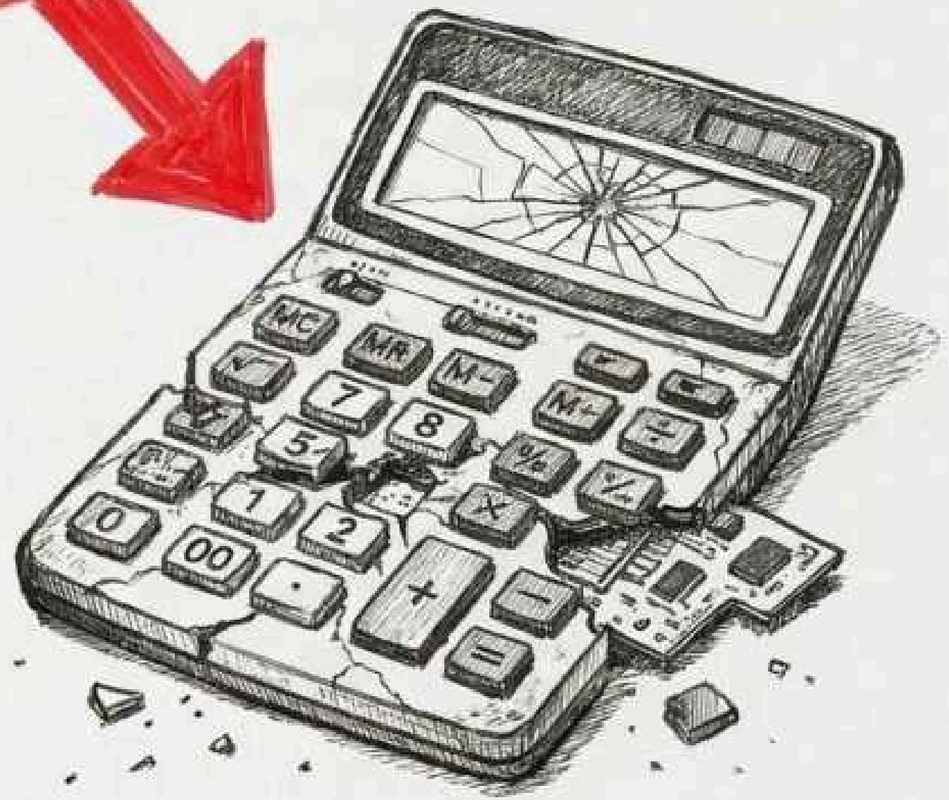
Gemini Notebook

$$f(x) = \frac{1}{(x-2)}$$

Restriction 1: You may not divide by zero.

$$x - 2 \neq 0$$

Domain: All real numbers EXCEPT $x = 2$.

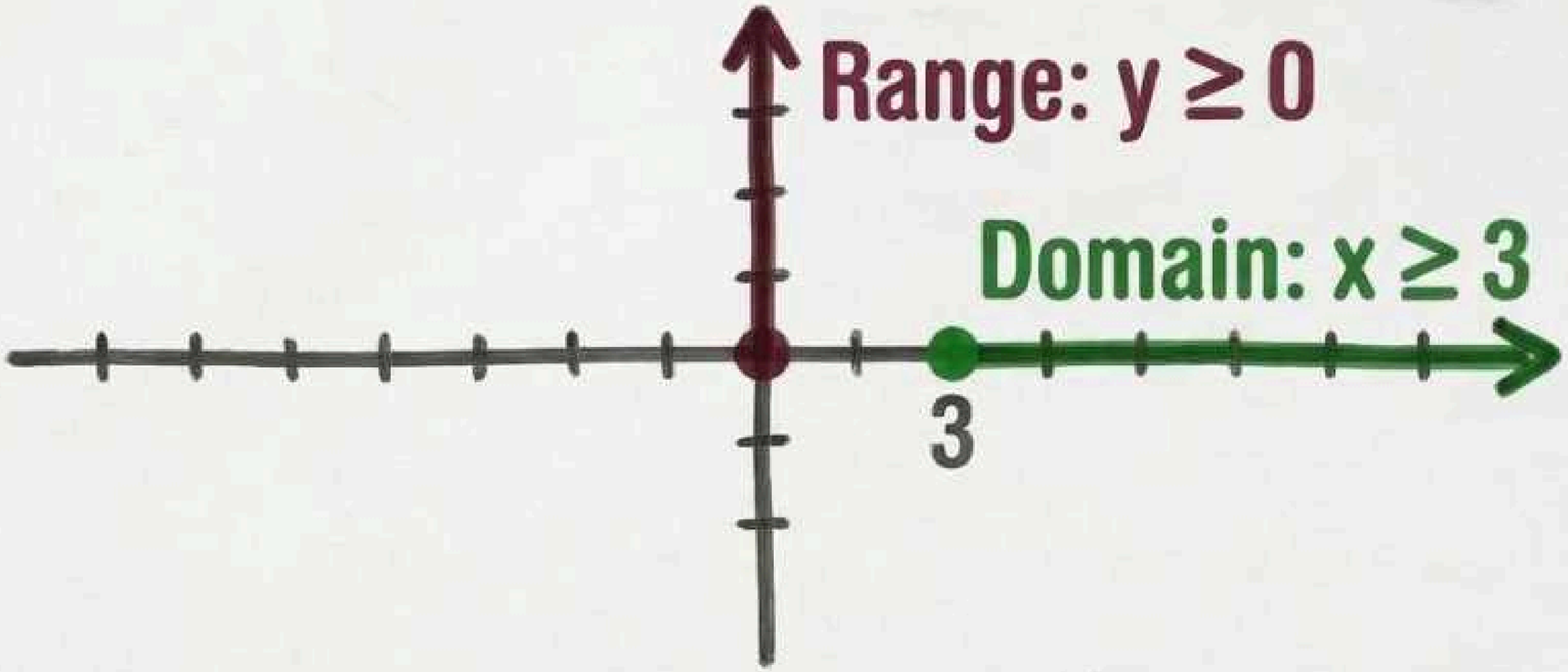


Gemini Notebook

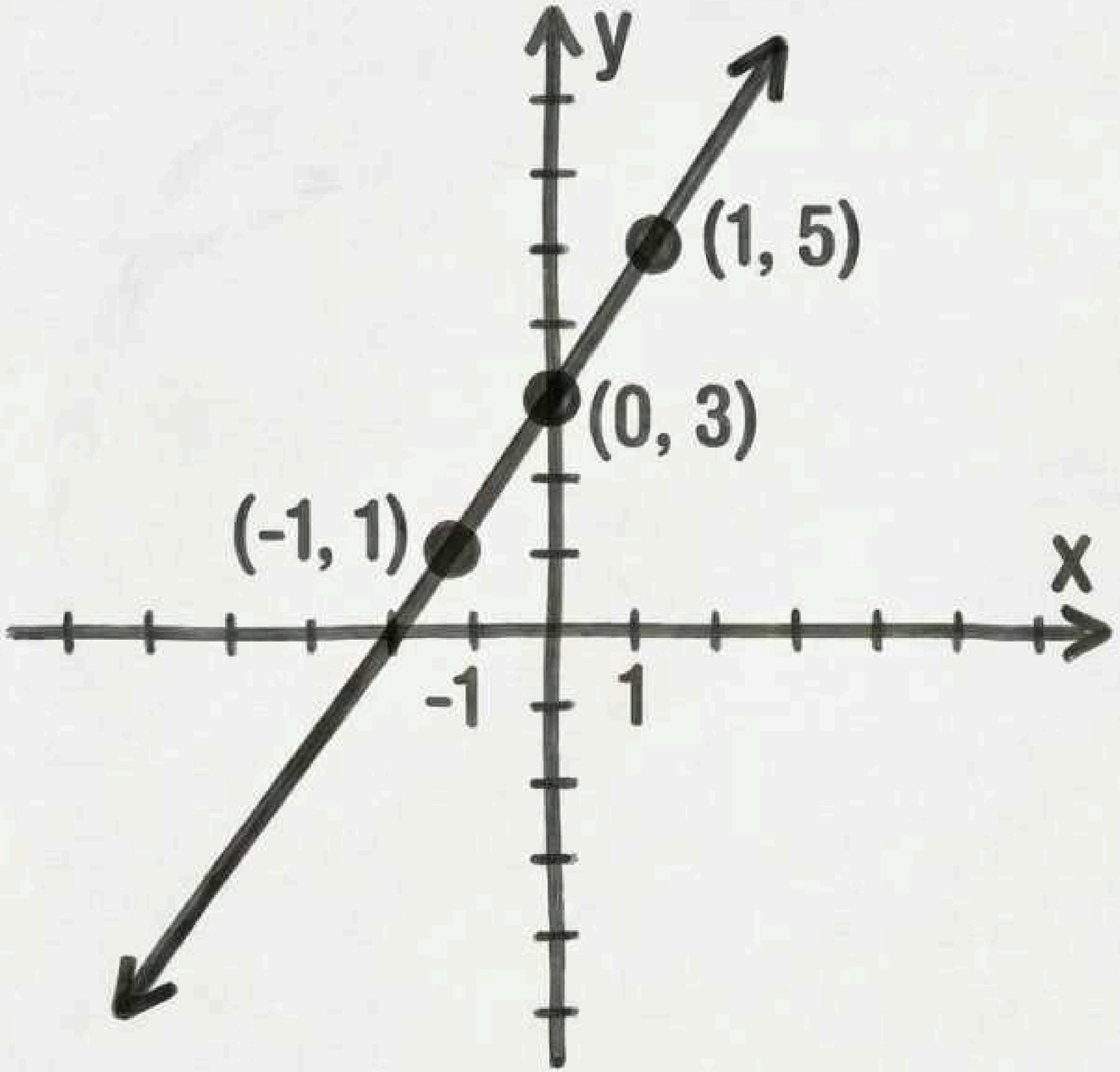
$$f(x) = \sqrt{(x-3)}$$

Restriction 2: No negative square roots in real numbers.

$$x - 3 \geq 0 \longrightarrow x \geq 3$$



Gemini Notebook

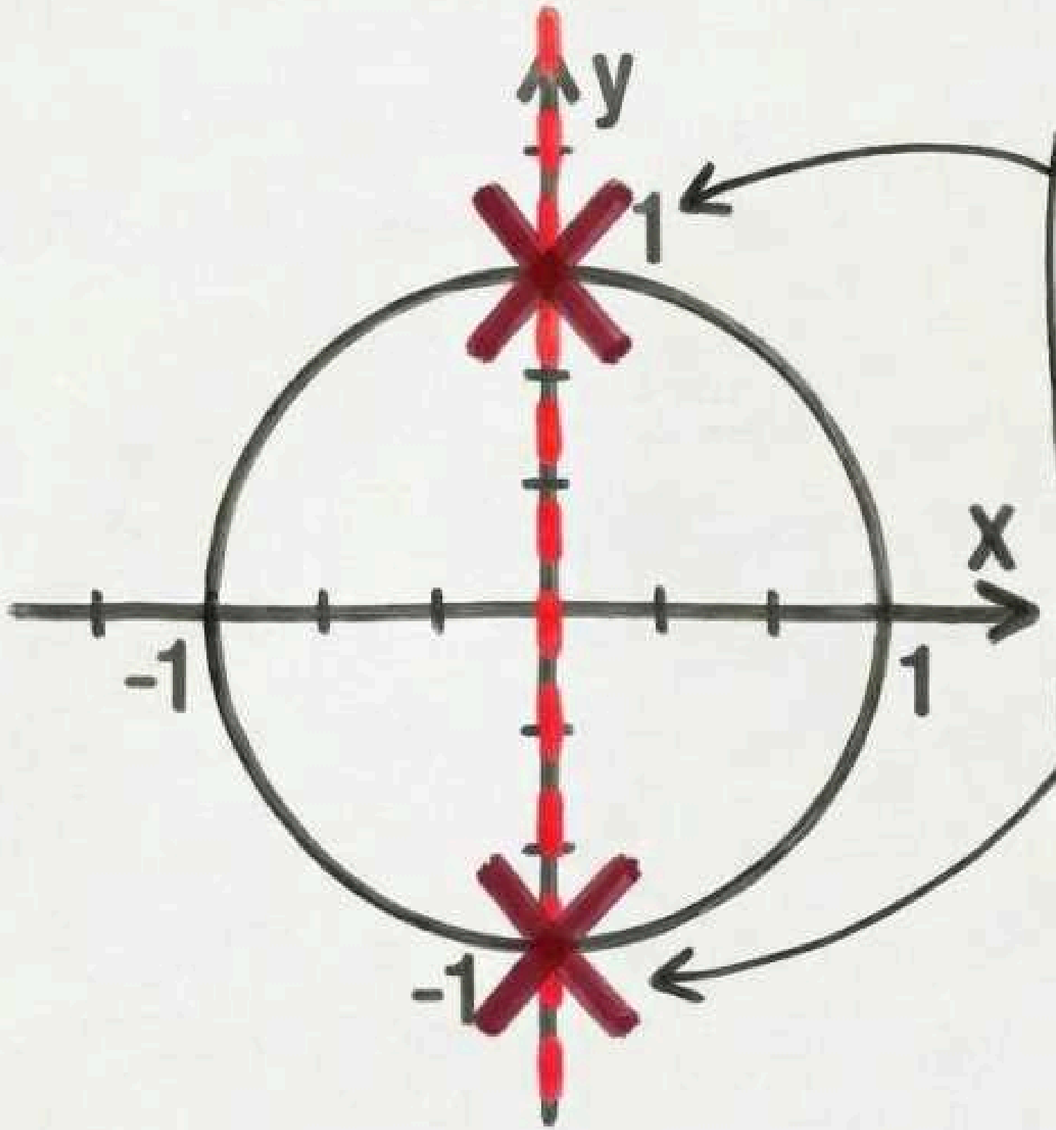


x	f(x)
-1	1
0	3
1	5

The graph is the set of all points (x, f(x)). Input runs horizontal; output runs vertical. Join the points to reveal the curve.

Gemini Notebook

The Vertical Line Test



Fails the Test:

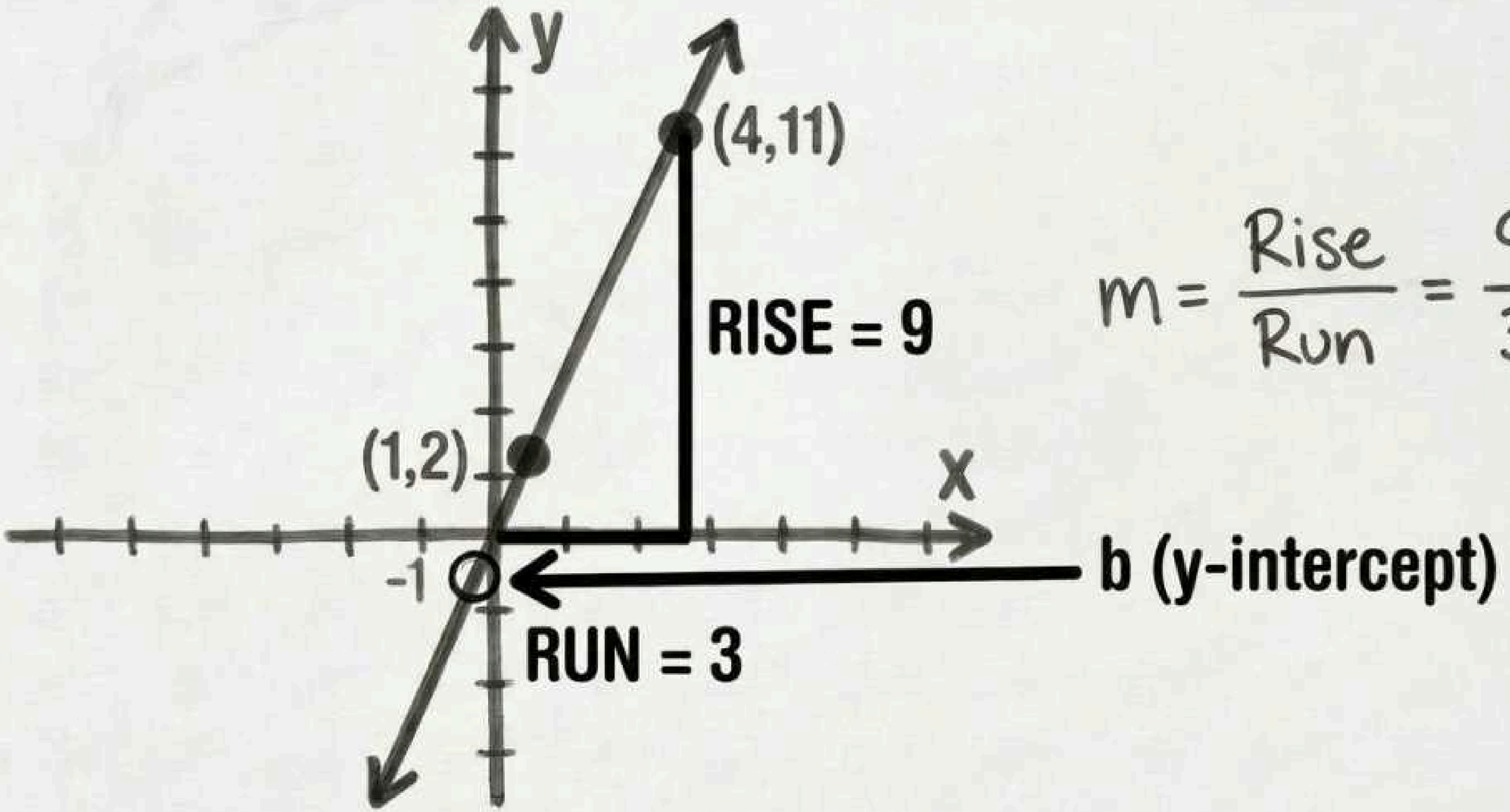
The line x=0 meets the curve twice.

One input yields TWO outputs.

A circle is NOT a function.

Gemini Notebook

$f(x) = mx + b$



$$m = \frac{\text{Rise}}{\text{Run}} = \frac{9}{3} = 3$$

Gemini Notebook

Write the Equation (Using slope m=3 and point (1,2))

$$y-2=3(x-1)$$

↓

Expand to

$$y=3x-1$$

Always check against the second point (4, 11):

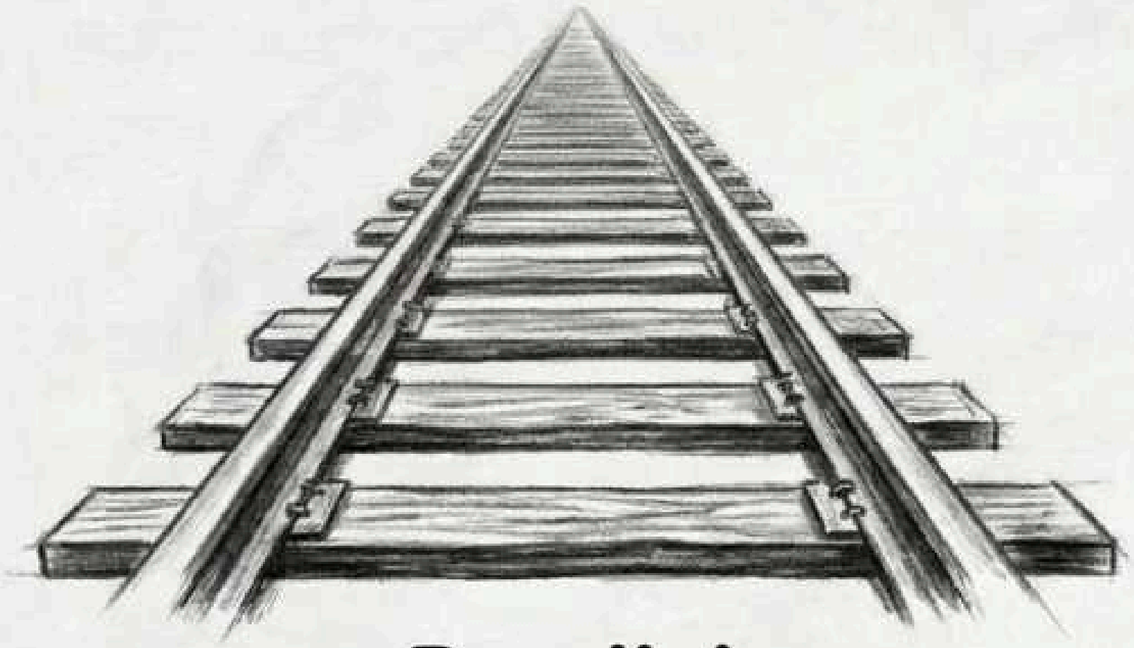
$$3(4)-1$$

$$=12-1$$

$$=11$$

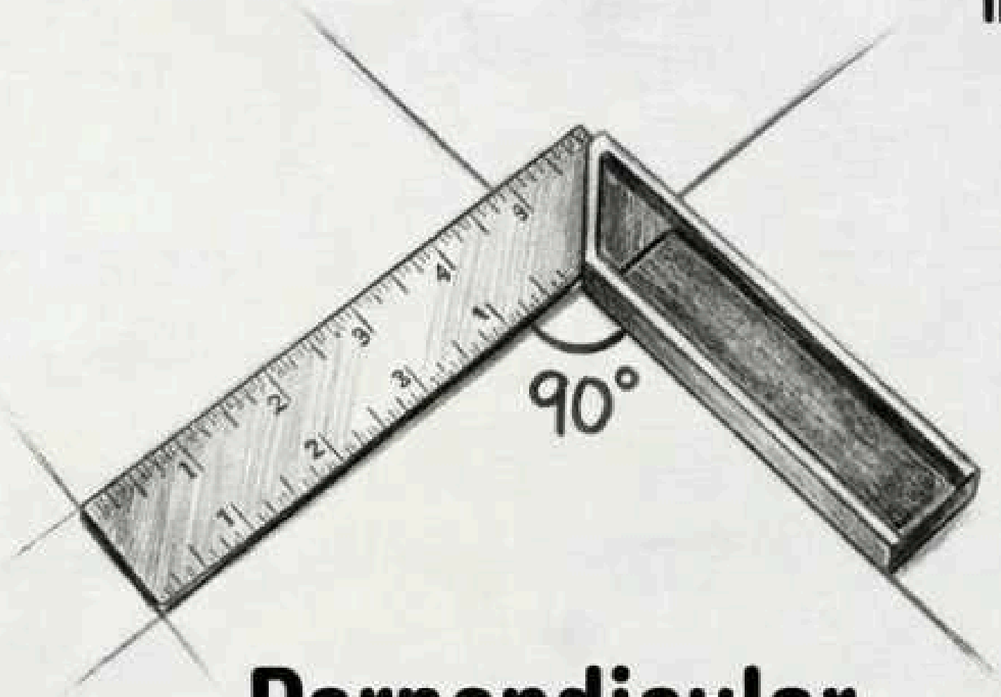


Gemini Notebook



Parallel

Slopes are equal ($m_1 = m_2$).



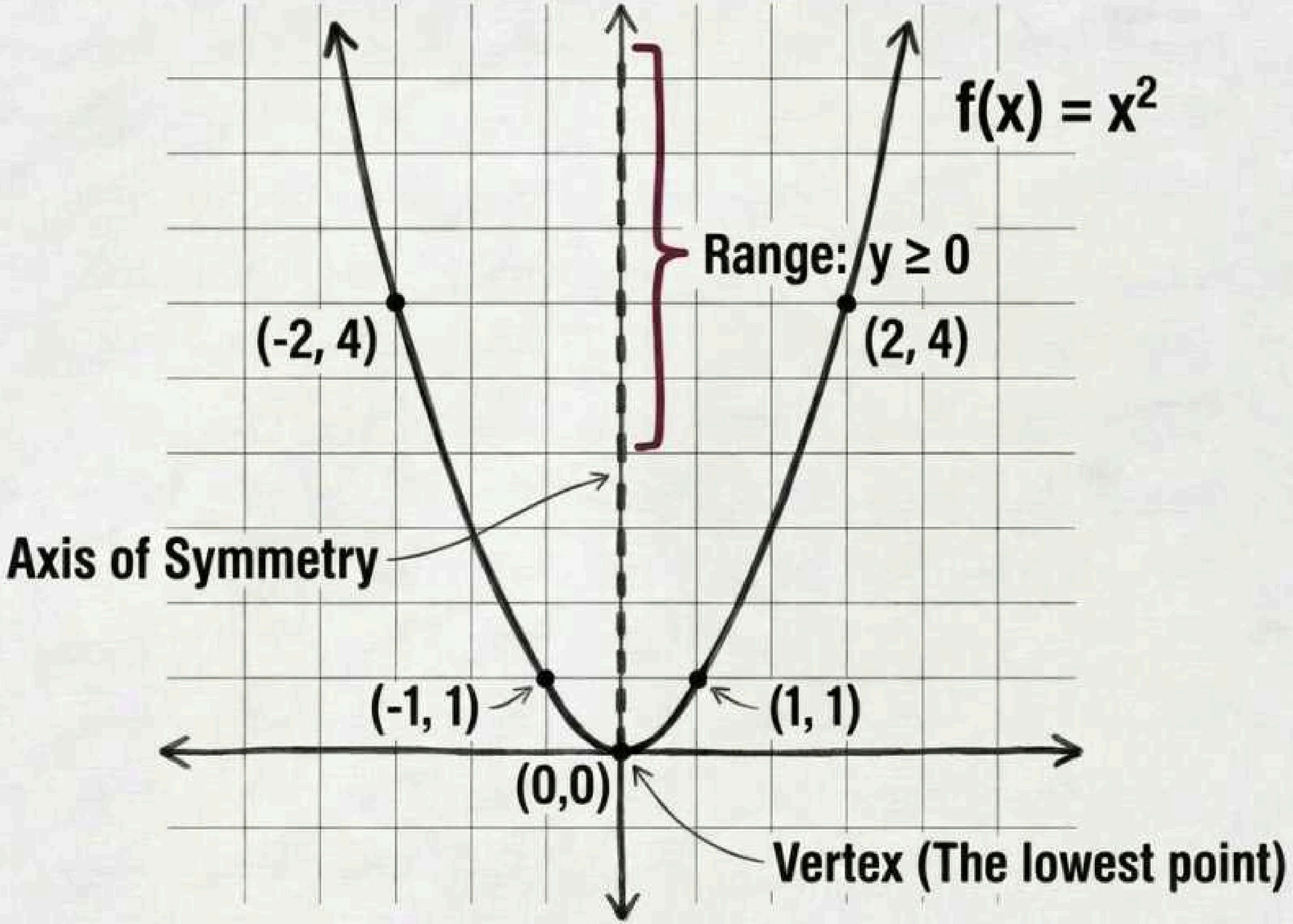
Perpendicular

Slopes are negative reciprocals ($m_1 \cdot m_2 = -1$).

$m = 3$ pairs with $m = -\frac{1}{3}$

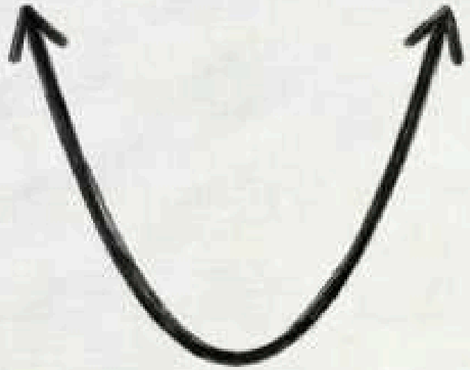
Horizontal lines = Slope 0. Vertical lines = Not a function.

Gemini Notebook



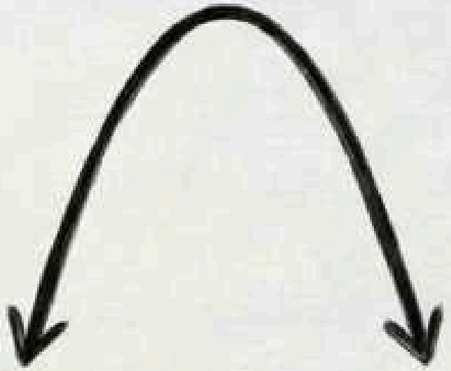
Gemini Notebook

Shape



A > 0: Opens Up

$$f(x) = Ax^2 + Bx + C$$



A < 0: Opens Down

Vertex

$$\text{Vertex } x = -\frac{-B}{2A}$$

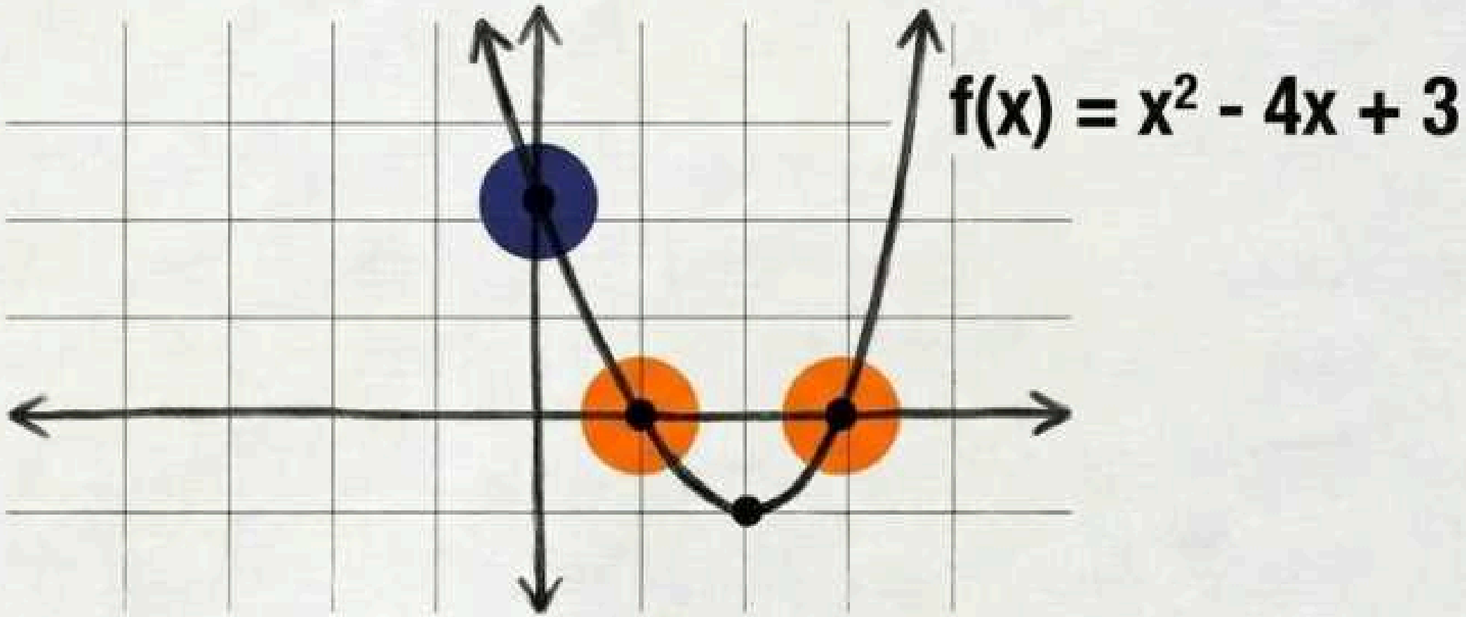
$$f(x) = x^2 - 4x + 3 \quad x = \frac{-(-4)}{2(1)} = 2$$

$$f(2) = 4 - 8 + 3 = -1$$

Vertex located at (2, -1).

Gemini Notebook

Zeros of a Function



Zeros are inputs that make the output zero.

Solve: $x^2 - 4x + 3 = 0$

Factor: $(x - 1)(x - 3) = 0$

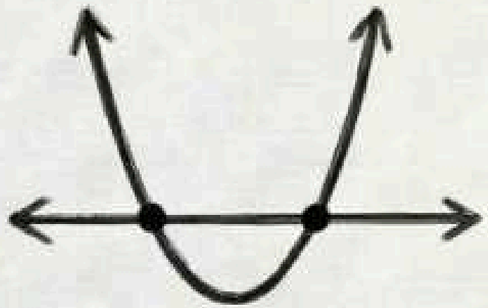
Zeros are 1 and 3.

y-intercept is $f(0) = 3$

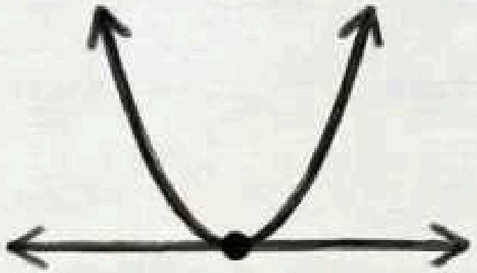
Gemini Notebook

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

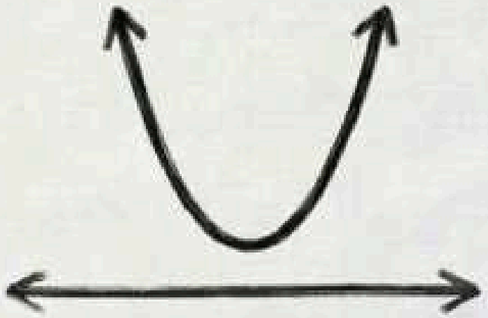
Positive (+)
Two real zeros



Zero (0)
One real zero



Negative (-)
No real zeros



Gemini Notebook

Solve: $2x^2 + 3x - 2 = 0$

Step 1 (Discriminant): $B^2 - 4AC = 9 - 4(2)(-2) = 9 + 16 = 25$

(Note: $\sqrt{25} = 5$**)**

Step 2 (Formula): $$x = \frac{-3 \pm 5}{4}$$

Step 3 (Results): $$x = \frac{2}{4} = \frac{1}{2}$$

$$x = \frac{-8}{4} = -2$$

Check $x = \frac{1}{2}$

$$2\left(\frac{1}{4}\right) + 3\left(\frac{1}{2}\right) - 2 = \frac{1}{2} + \frac{3}{2} - 2 = 2 - 2 = 0$$

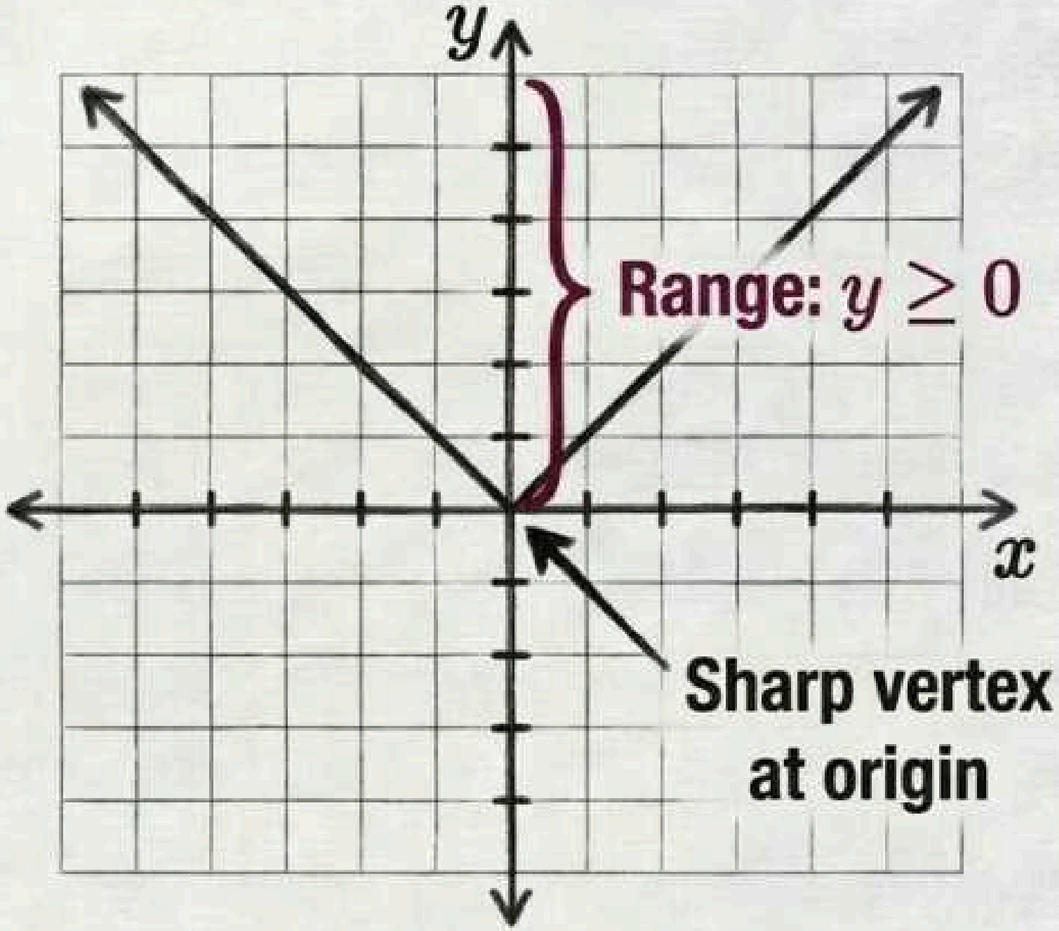


Gemini Notebook

Absolute Value

$$f(x) = |x| = \begin{cases} x & \text{when } x \geq 0 \\ -x & \text{when } x < 0 \end{cases}$$

$$|-3| = 3$$

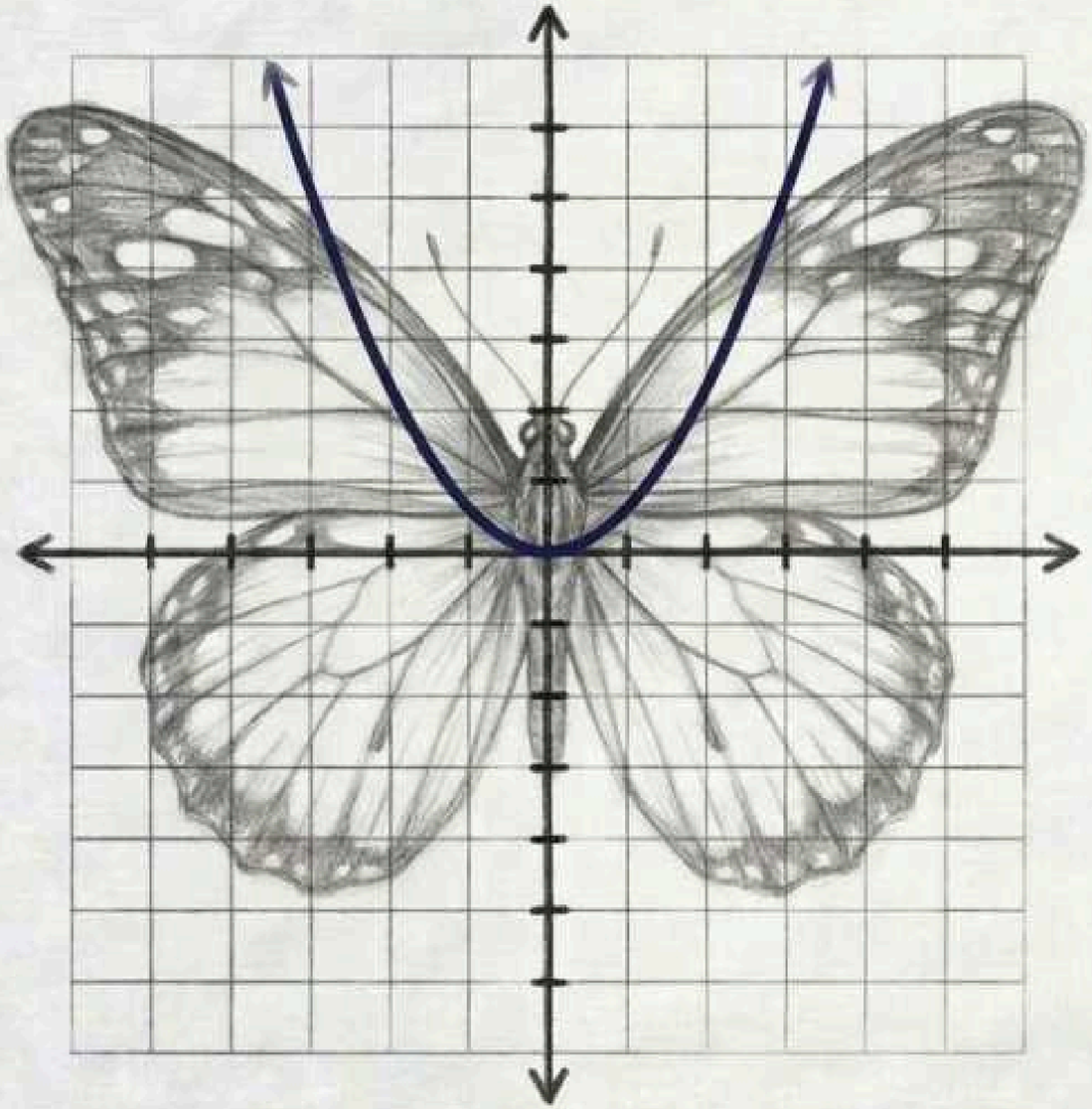


Defined in pieces. Note the sharp corner, distinct from a smooth parabola.

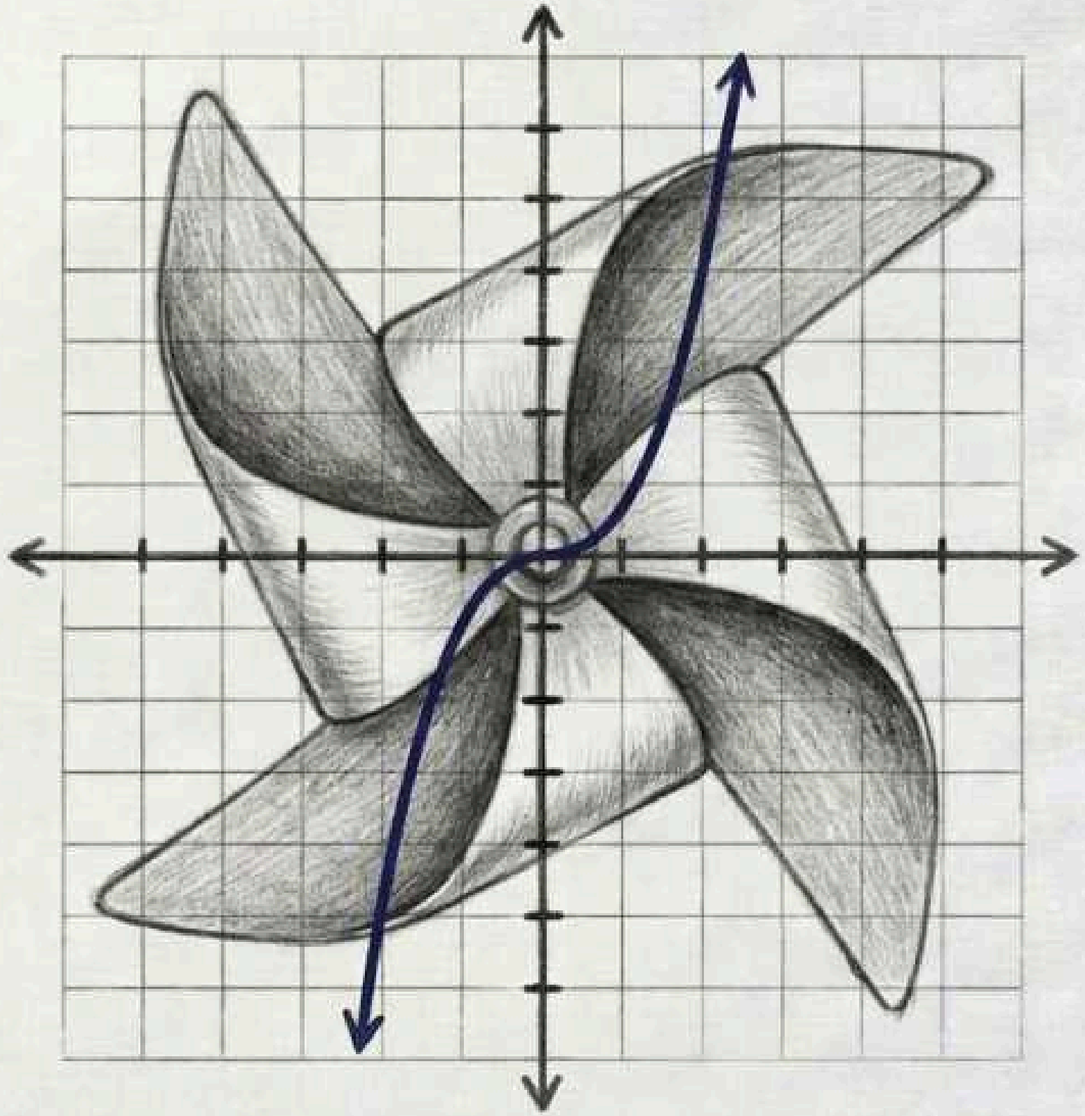
Gemini Notebook

¶ 1.18

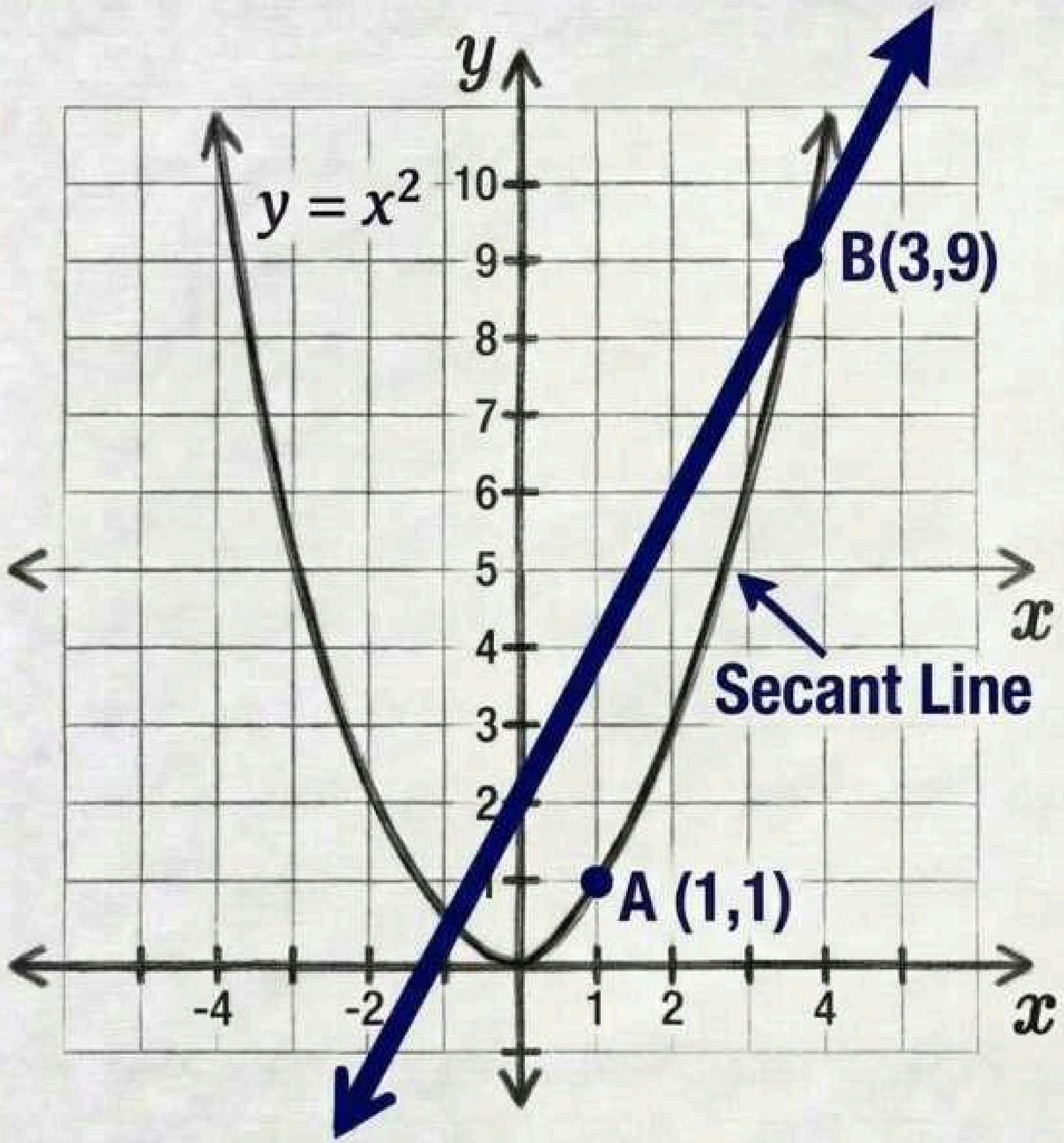
$f(-x) = f(x)$



$f(-x) = -f(x)$



Gemini Notebook



Average Rate of Change = Slope of the Secant Line.

$$\frac{\text{Change in Output}}{\text{Change in Input}} = \frac{9-1}{3-1}$$

$$= \frac{8}{2}$$

$$= 4$$

Gemini Notebook

Watch for Mistakes

¶ 1.20



Notation:
$f(x)$ is NOT multiplication $(f \times x)$.



Domains:
Do not forget to exclude values that make a denominator zero.



Formula:
Mix up signs in the quadratic formula at your own peril.

Challenge for Lecture 2

$$f(x) = \frac{\sqrt{x+5}}{x-1}$$

What is the domain?

Next lecture: Moving graphs around.

Gemini Notebook