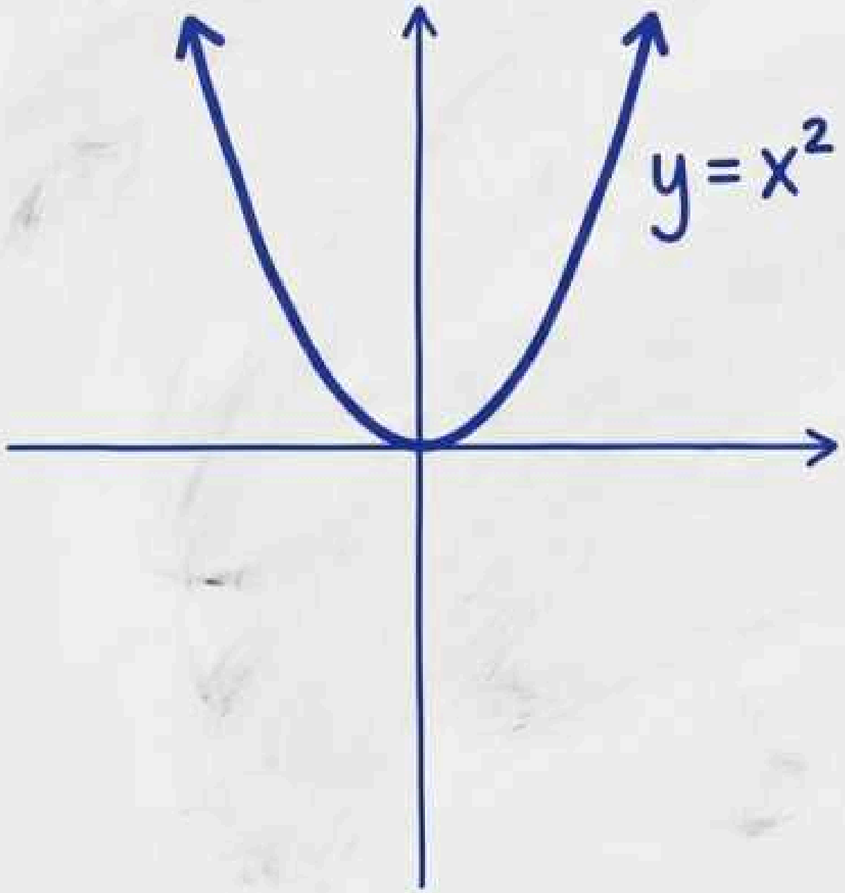
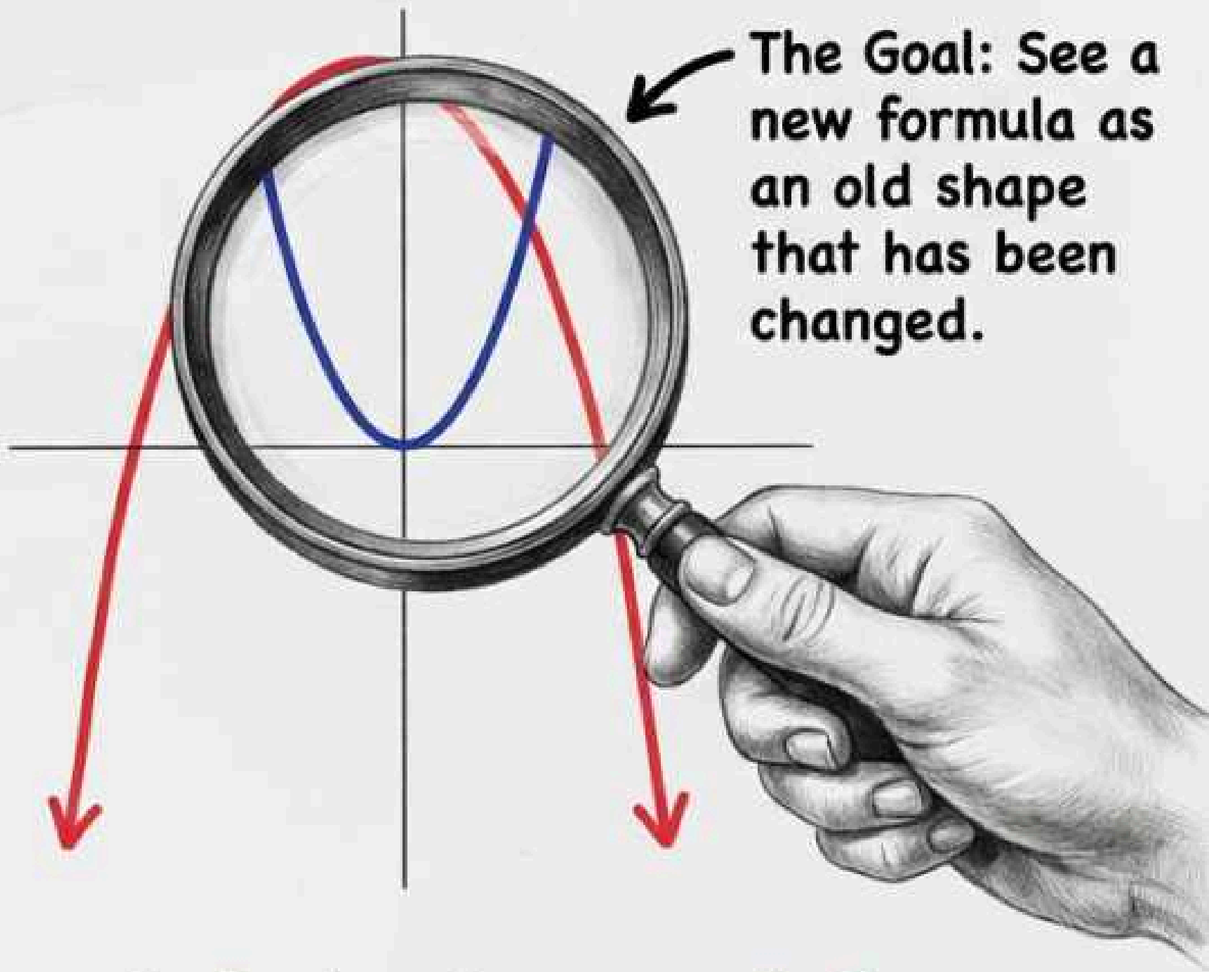
Transformations, Combinations, and Inverses

¶ 2.1





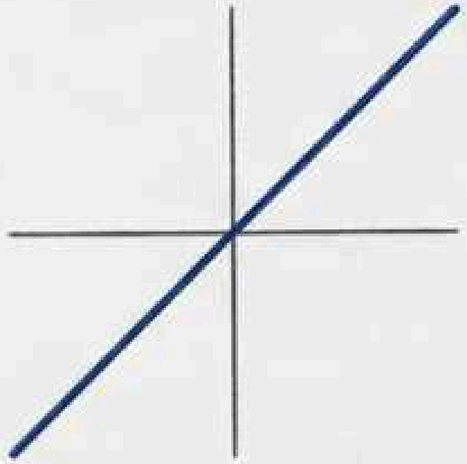
1. Basic Graphs → 2. Moves & Stretches → 3. Combinations → 4. Inverses

Gemini Notebook

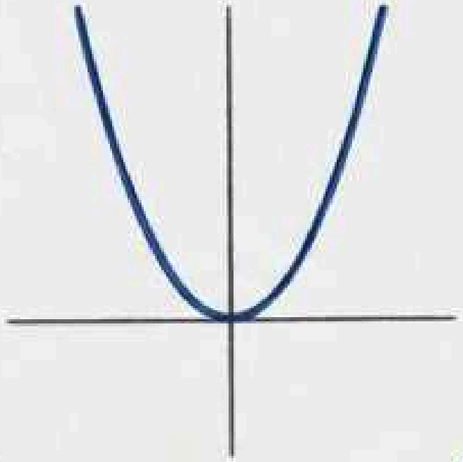
The Parent Functions Library

¶ 2.2

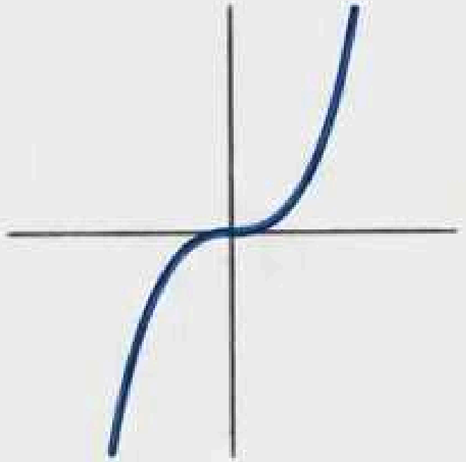
Learn these shapes to recognize most graphs.



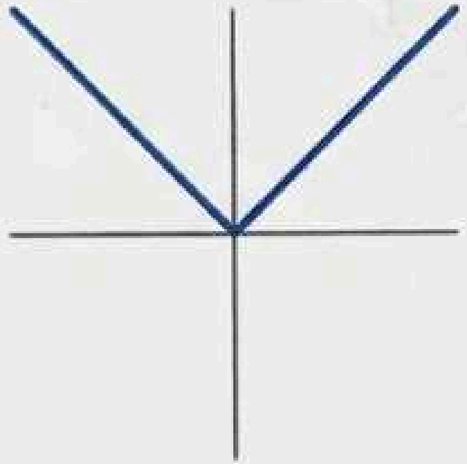
Line: $f(x) = x$



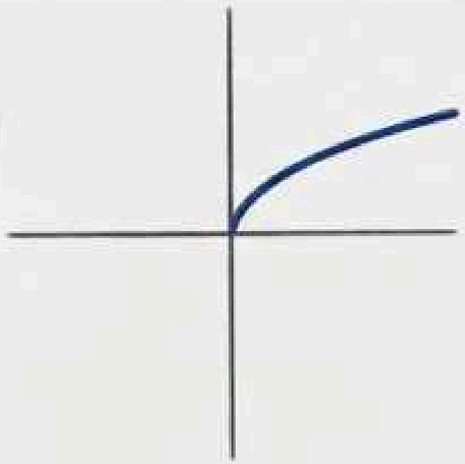
Parabola: $f(x) = x^2$



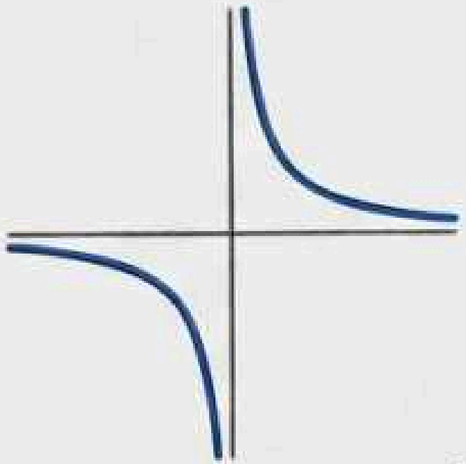
Cubic (Odd): $f(x) = x^3$



Absolute Value (V): $f(x) = |x|$



Square Root: $f(x) = \sqrt{x}$



Hyperbola: $f(x) = \frac{1}{x}$

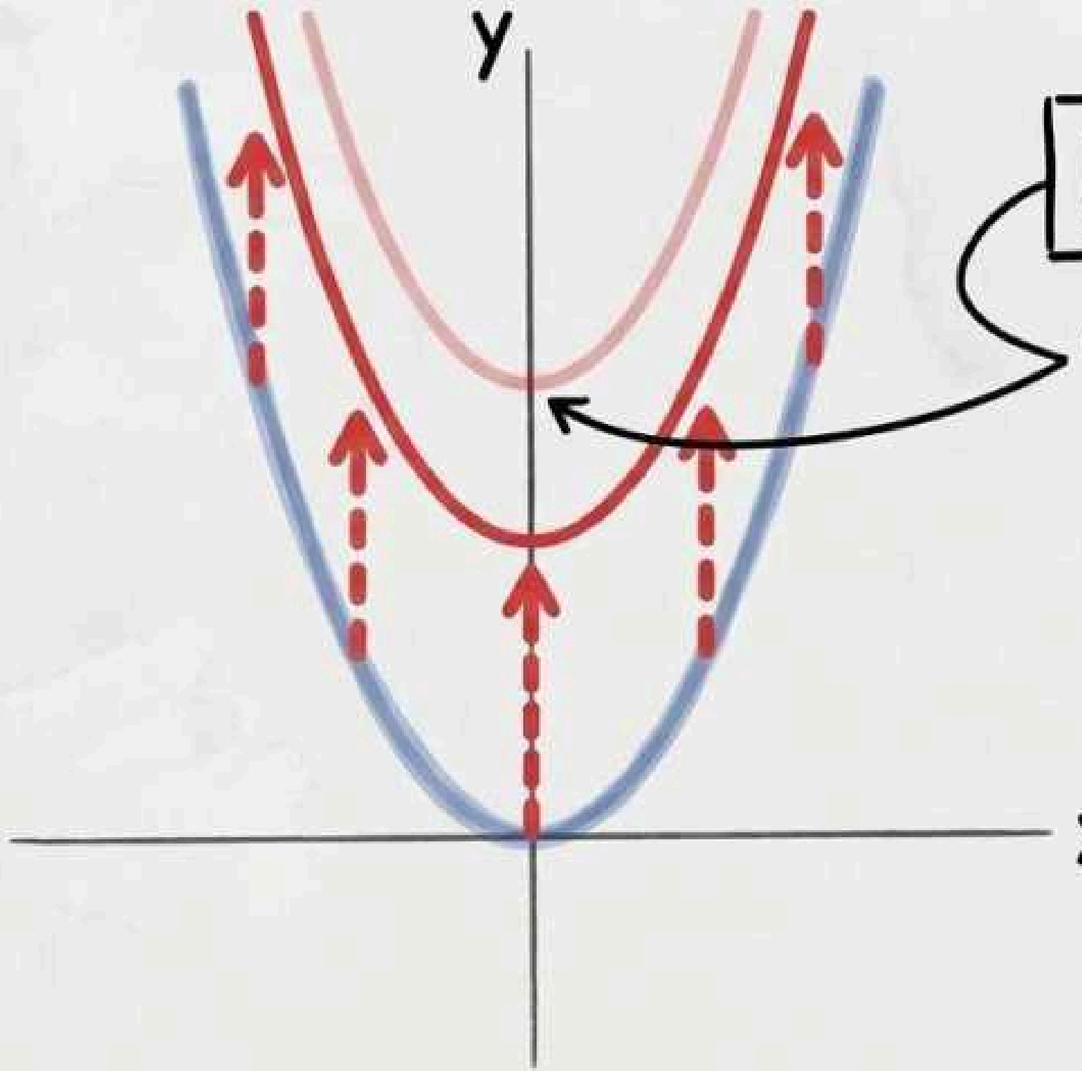
Gemini Notebook

Vertical Shifts: Output Changes

¶ 2.3

Rule: $f(x) + k$

- Moves UP by k (if positive)
- Moves DOWN (if negative)



Example: $x^2 + 2$

Vertex moves from (0,0) to (0,2)

Every point moves straight up by the exact same amount.

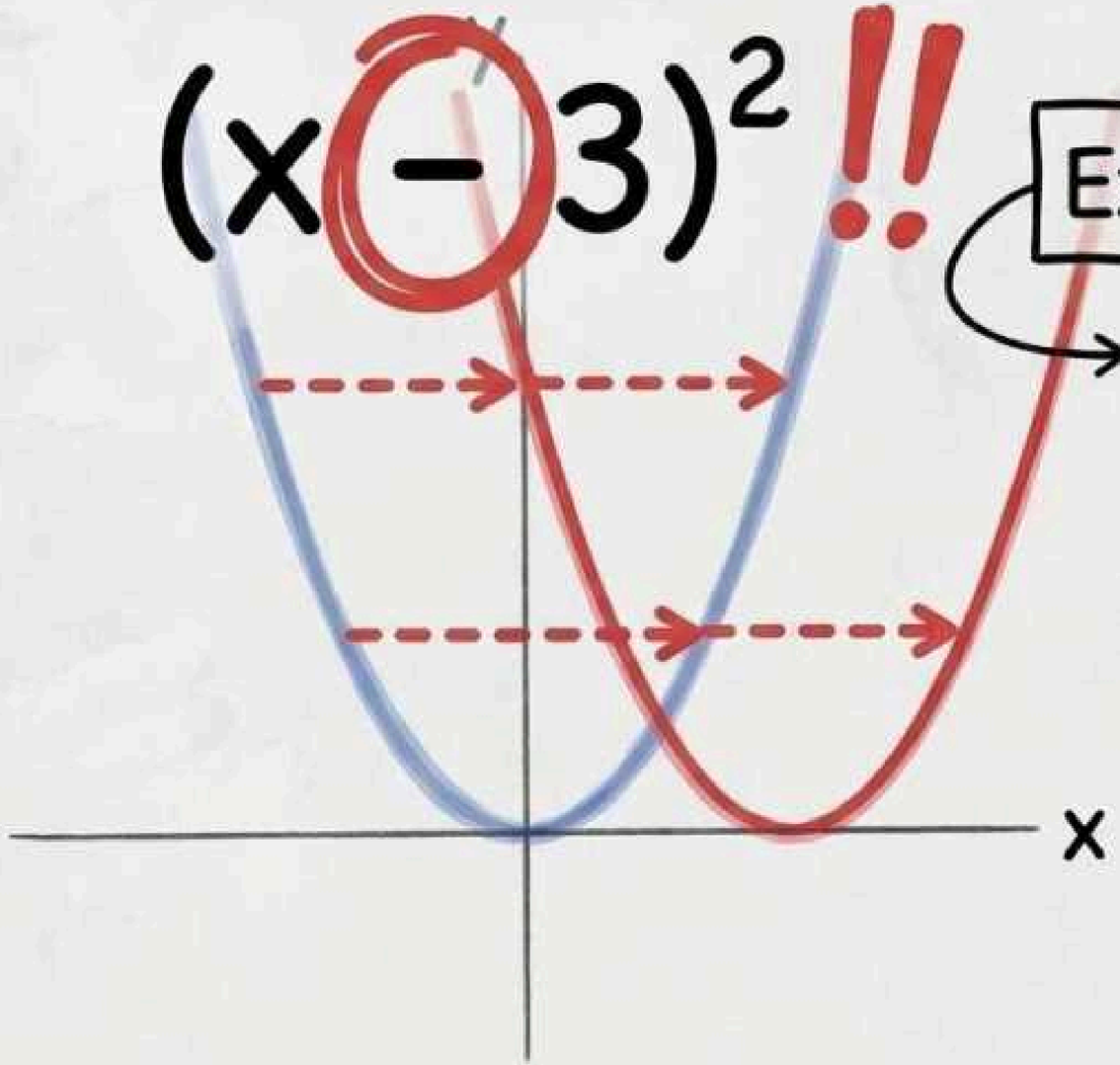
Gemini Notebook

Horizontal Shifts: Input Changes

¶ 2.4

Rule: $f(x - h)$
– Moves RIGHT by h units.

$(x - 3)^2$!!



Example: $(x - 3)^2$

→ Vertex moves to (3,0).

Note: A minus sign inside gives a move to the right!

Gemini Notebook

¶ 2.5

Why the Minus Sign Moves Right

Original Function: $f(x) = x^2$	Shifted Function: $f(x-3)$
At $x = 0 \rightarrow f(0) = 0^2 = 0$	At $x = 3 \rightarrow f(3-3) = 0^2 = 0$
At $x = 2 \rightarrow f(2) = 2^2 = 4$	At $x = 5 \rightarrow f(5-3) = 2^2 = 4$

The output at $x = 3$ is whatever the original function gave at 0.

Every original point is reached three units *later* in time (x). Therefore, the graph delays, moving right.

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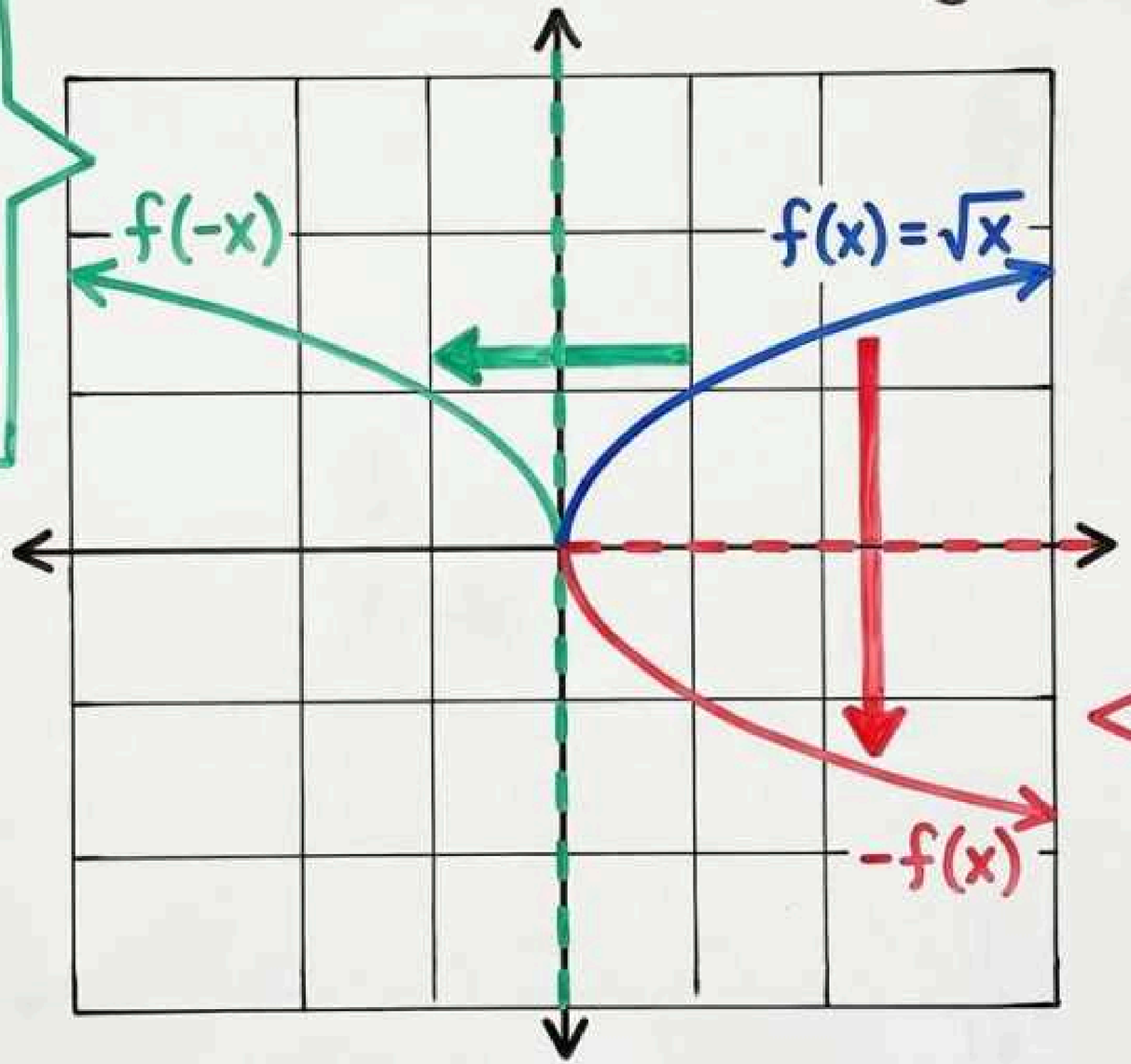
Reflections via Signs

¶ 2.6

Inside Minus:

$f(-x)$ → Flips over y-axis.

Example: $\sqrt{(-x}$ lives where $x \leq 0$.



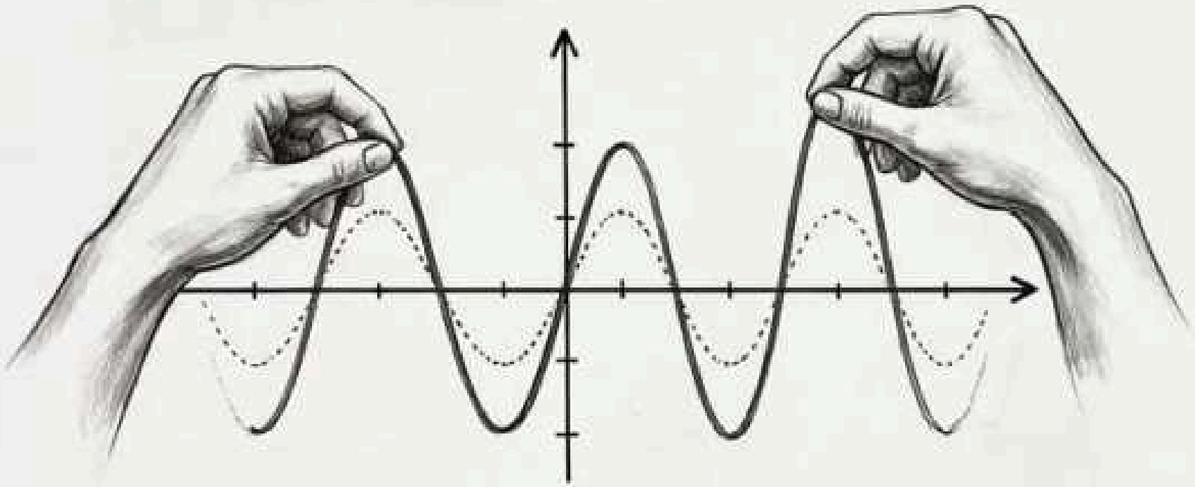
Outside Minus:

$-f(x)$ → Flips over x-axis.

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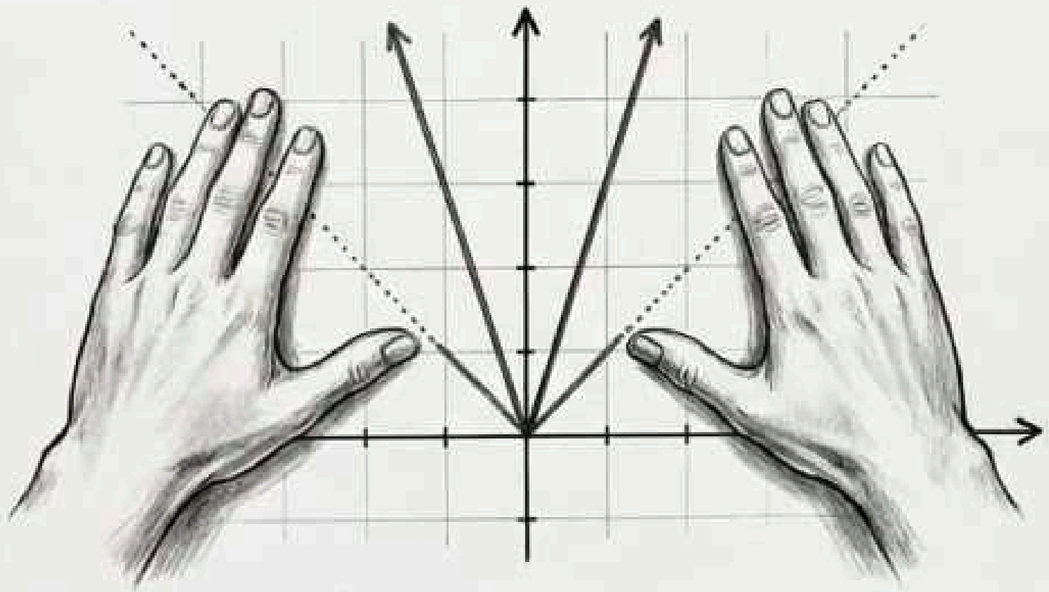
Stretches and Compressions

§ 2.7



Vertical: $A \cdot f(x)$

Stretched by factor A if > 1,
compressed if between 0 and 1.



Horizontal: $f(Bx)$

Squeezed horizontally by factor B.

Example: $|2x|$ is a V that is twice as narrow.

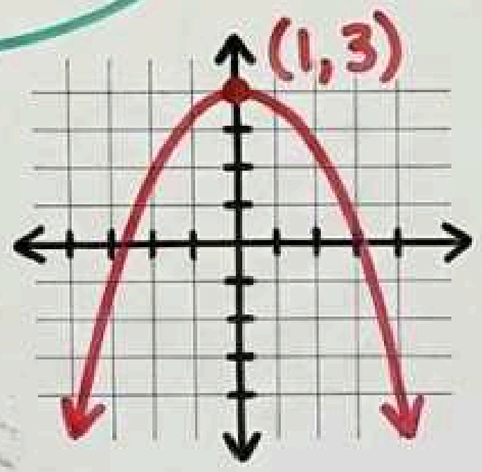
Gemini Notebook

Combining Transformations: Inside Out

¶ 2.8

$$g(x) = -2(x-1)^2 + 3$$

① Right 1 unit.
② Parent parabola.
③ Stretch by 2.
④ Flip upside down.
⑤ Lift 3 units.



Final Vertex: (1, 3)

✓ Check: $g(2) = -2(1)^2 + 3 = 1$

Gemini Notebook

The Vertex Form of a Quadratic

¶ 2.9

Vertex Form: $A(x-h)^2+k \longrightarrow$ Vertex is at (h,k)

Start: x^2-6x+5

↓

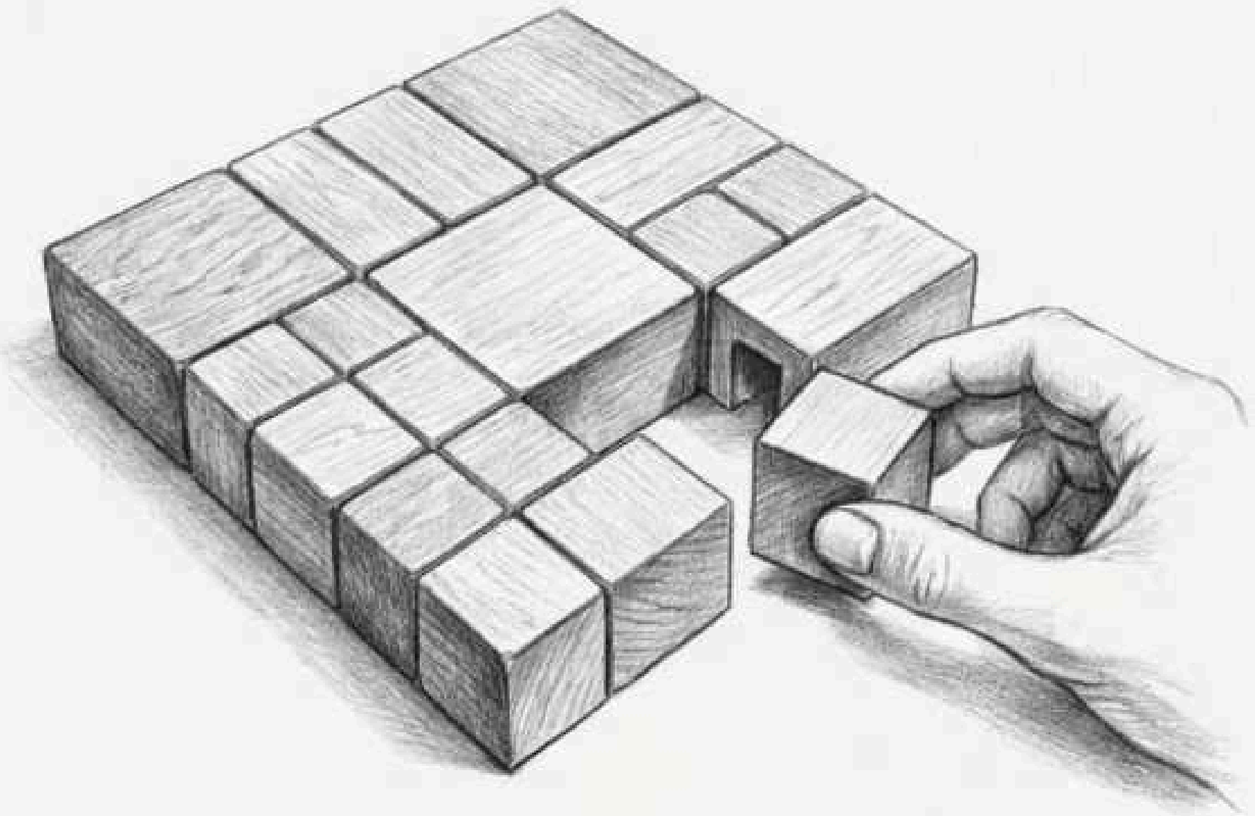
Half of -6 is -3.
Square it to get +9.

↓

$(x^2-6x+9)-9+5$

↓

Result: $(x-3)^2-4$



Gemini Notebook

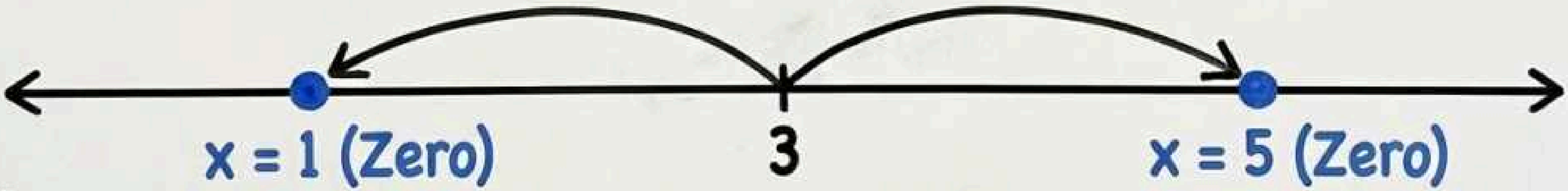
Steps for Completing the Square

¶2.10

The Algorithm:

1. Take half the coefficient of x. ✓
2. Square it. ✓
3. Add and subtract it. Group into a perfect square. ✓

Finding Zeros: $(x - 3)^2 - 4 = 0 \rightarrow (x - 3)^2 = 4 \rightarrow (x - 3) = \pm 2$



Zeros are at x = 1 and x = 5. Vertex is (3, -4).

Gemini Notebook

Arithmetic Combinations

¶ 2.11

Given $f(x) = x^2$ and $g(x) = x - 1$

Operation	Notation	Result
Sum	$f + g$	$x^2 + x - 1$
Product	$f \cdot g$	$x^3 - x^2$
Quotient	$\frac{f}{g}$	$\frac{x^2}{x-1}$

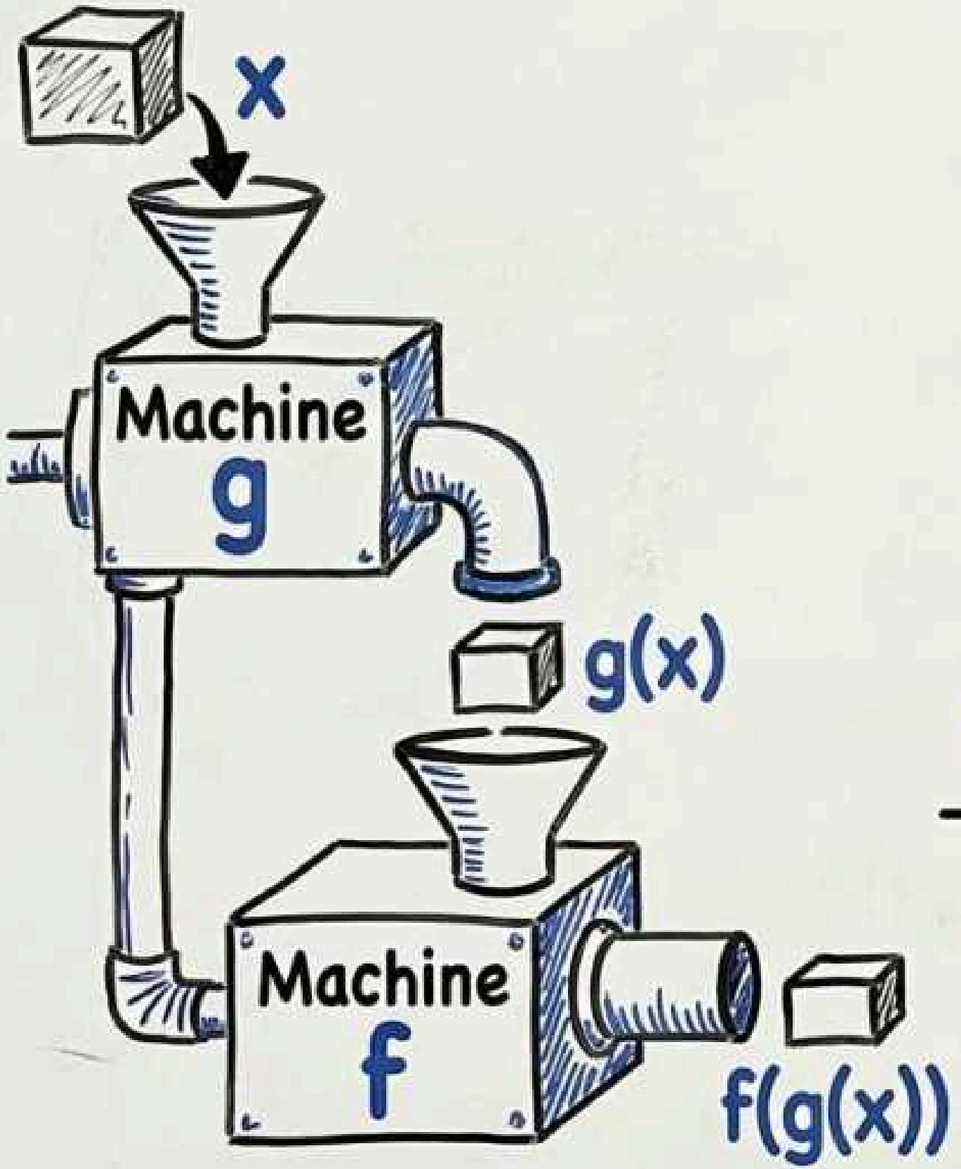


Domain Rule: $x \neq 1$, because $g(1) = 0$. No dividing by zero.

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Composition: Feeding Functions

¶ 2.12



Composite $f(g(x))$: Do g first, then f. Order matters!

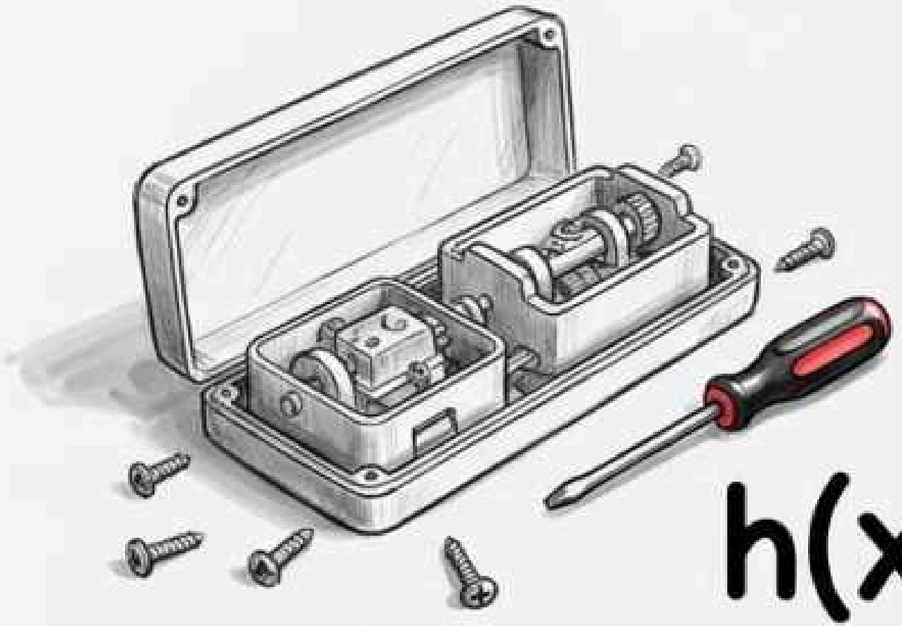
Let $f(x) = x^2$, $g(x) = x + 3$

$f(g(x)) = (x + 3)^2$	$g(f(x)) = x^2 + 3$
At $x = 2$, Output is 25.	$\neq$
	At $x = 2$, Output is 7.

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De-composition: Taking Formulas Apart

¶ 2.13



$$h(x) = \sqrt{2x - 1}$$

Outer function (f):
$\sqrt{(\text{input})}$

Inner function (g):
$2x - 1$

Domain Need: Inside of root must be ≥ 0.

So, $2x - 1 \geq 0 \rightarrow x \geq \frac{1}{2}$.

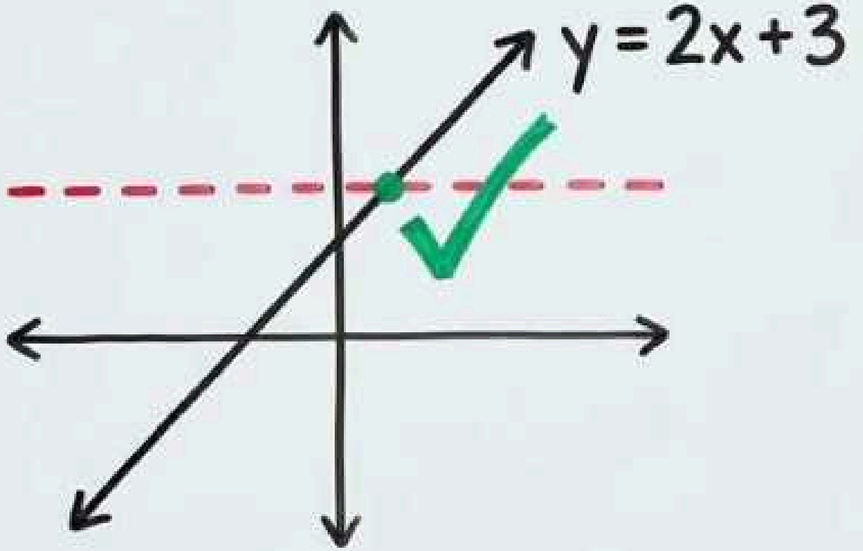
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One-to-One Functions & The Line Test

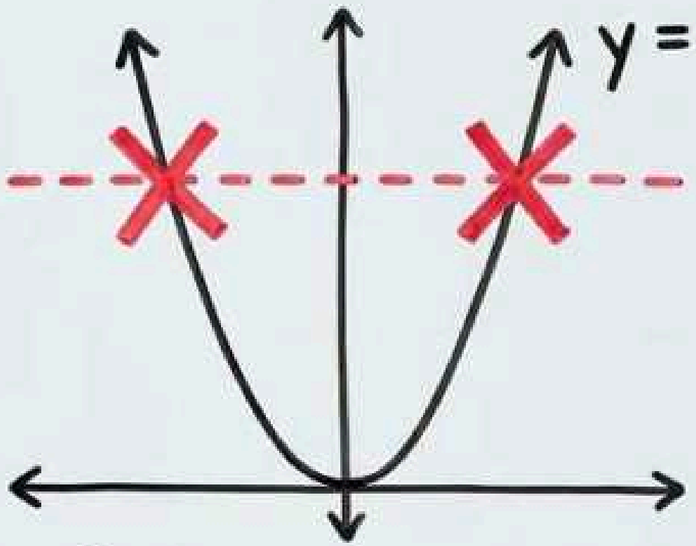
¶ 2.14

Rule: Different inputs must give different outputs.

Horizontal Line Test: No horizontal line may meet the graph twice.



$2x + 3$ passes.



x^2 fails. (e.g., inputs 2 and -2 both give 4).

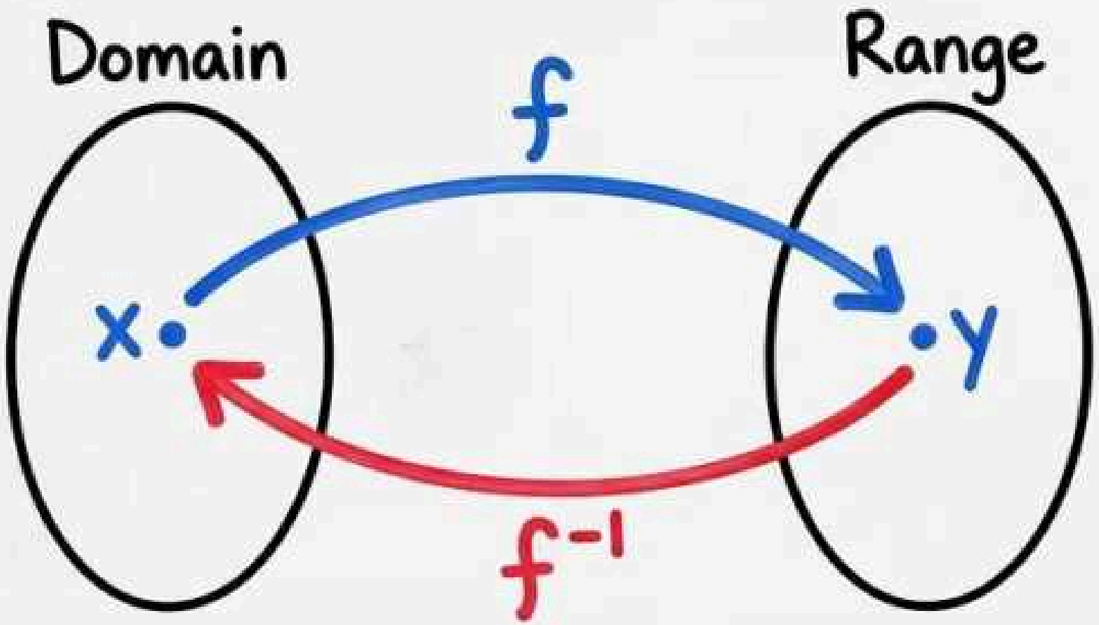
Only one-to-one functions have inverses.

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The Inverse Undoes the Function

¶ 2.15

Domain of Inverse
=
Range of f



Range of Inverse
=
Domain of f

CRITICAL WARNING

The inverse of $f(x)$ is **NOT** $\frac{1}{f(x)}$.

It is a completely different function that travels in reverse.

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How to Find an Inverse

¶ 2.16

Find the inverse for $f(x) = 2x + 3$

Write $y = 2x + 3$

Swap x and y

$x = 2y + 3$

Solve for y

$x - 3 = 2y$

$y = \frac{x-3}{2}$

Inverse is $\frac{x-3}{2}$

Check: $f(4) = 2(4) + 3 = 11$.

Inverse of $11 = \frac{11-3}{2} = \frac{8}{2} = 4$. It works!

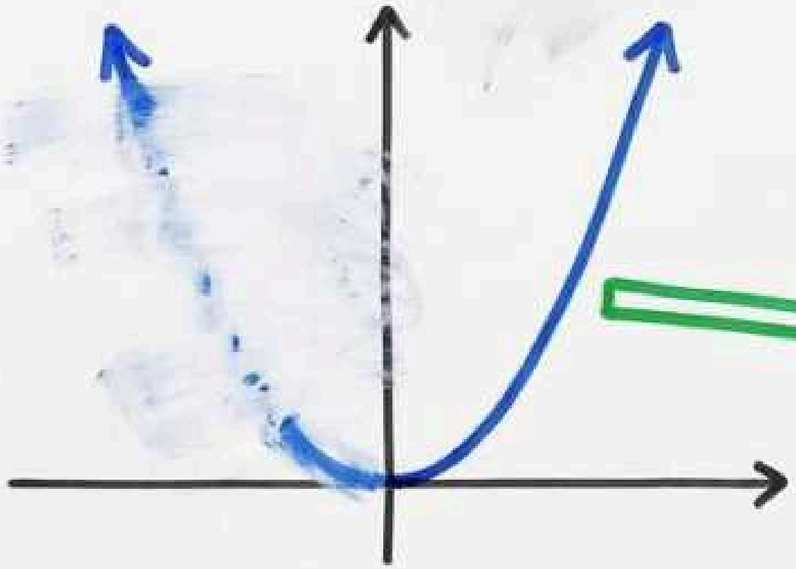
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Inverses and Domain Restriction

¶ 2.17

$f(x) = x^3 + 1$ → Swap & Solve → Inverse is $\sqrt[3]{(x-1)}$

The Standard Repair



x^2 is not one-to-one.

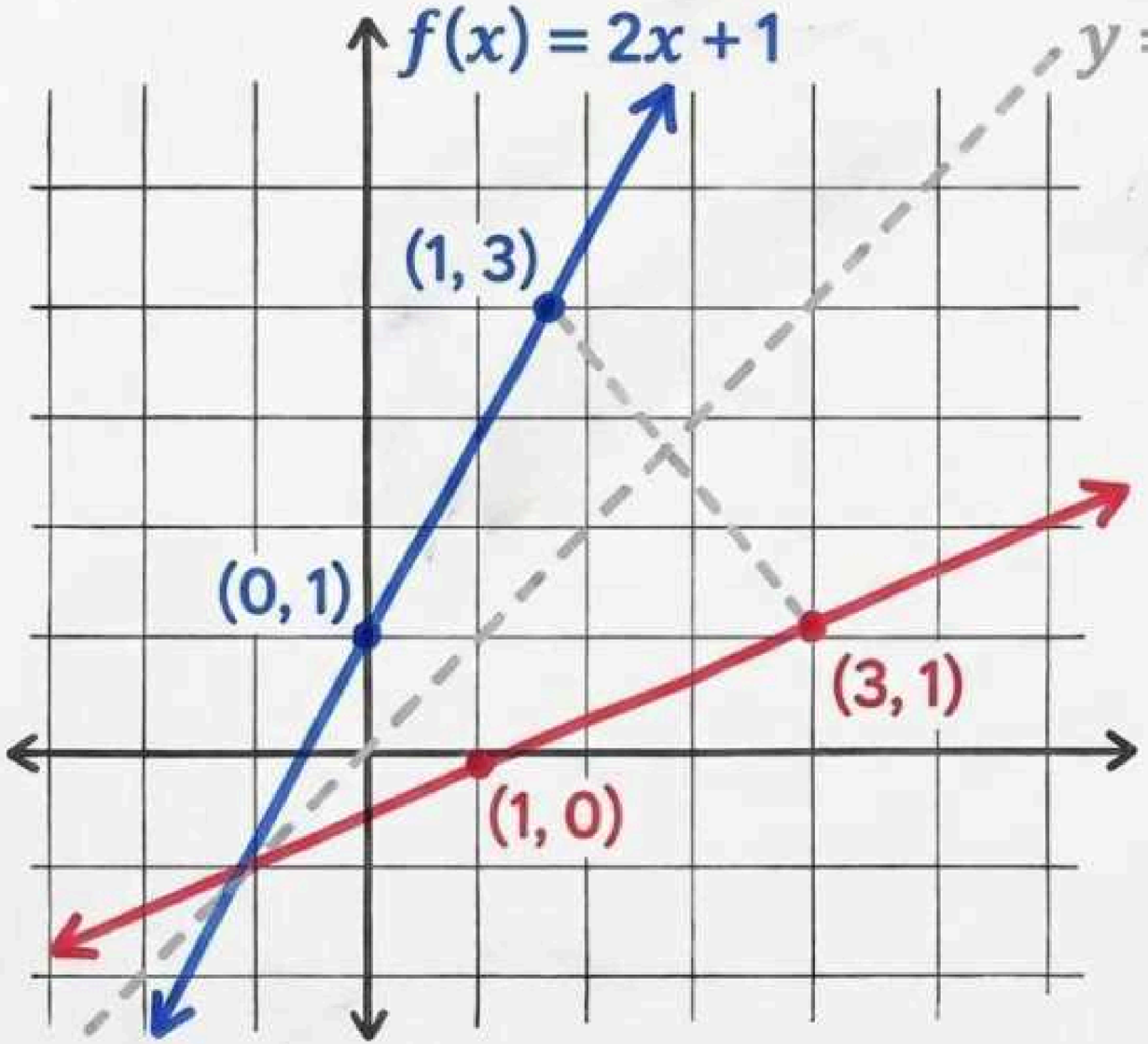
Restrict domain to $x \geq 0$.

Then, the inverse is safely $\sqrt{x}$.

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Graphical Symmetry of Inverses

¶ 2.18



Point (x, y) on f becomes point (y, x) on the inverse.

The two graphs are perfect mirror images across the line $y = x$.

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A Harder Inverse: Rational Functions

¶ 2.19

$$f(x) = \frac{x+1}{x-2}$$

Swap: $x = \frac{y+1}{y-2}$

Multiply out: $x(y-2) = y+1 \rightarrow xy - 2x = y+1$

Gather y terms: $xy - y = 2x + 1$

Factor out y: $y(x-1) = 2x+1$

Inverse: $y = \frac{2x+1}{x-1}$

Check:

$f(3) = 4$

$\text{Inverse}(4) = \frac{2(4)+1}{4-1} = \frac{9}{3} = 3.$

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Lecture Question & Looking Ahead

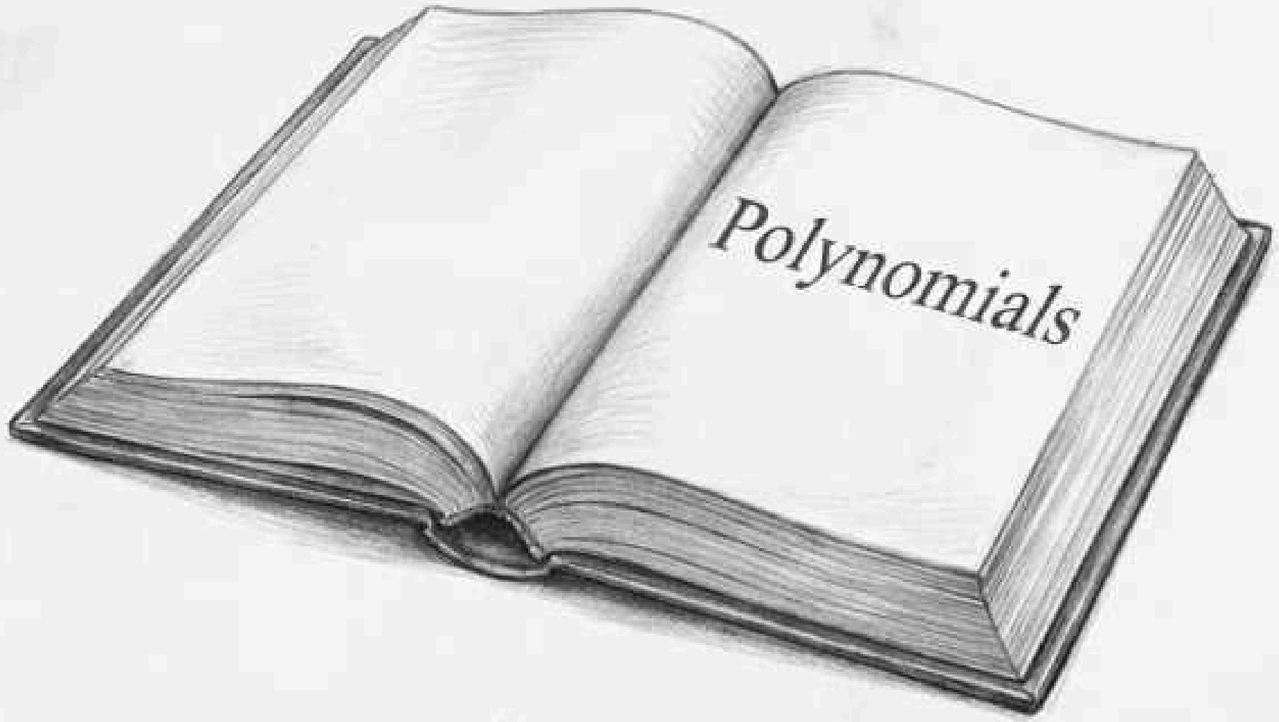
¶ 2.20

Homework

Find the inverse of

$$f(x) = 3x - 6.$$

How does its graph compare with the graph of f?



Up Next (Lecture 3): Polynomials — functions built entirely from powers of x.

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