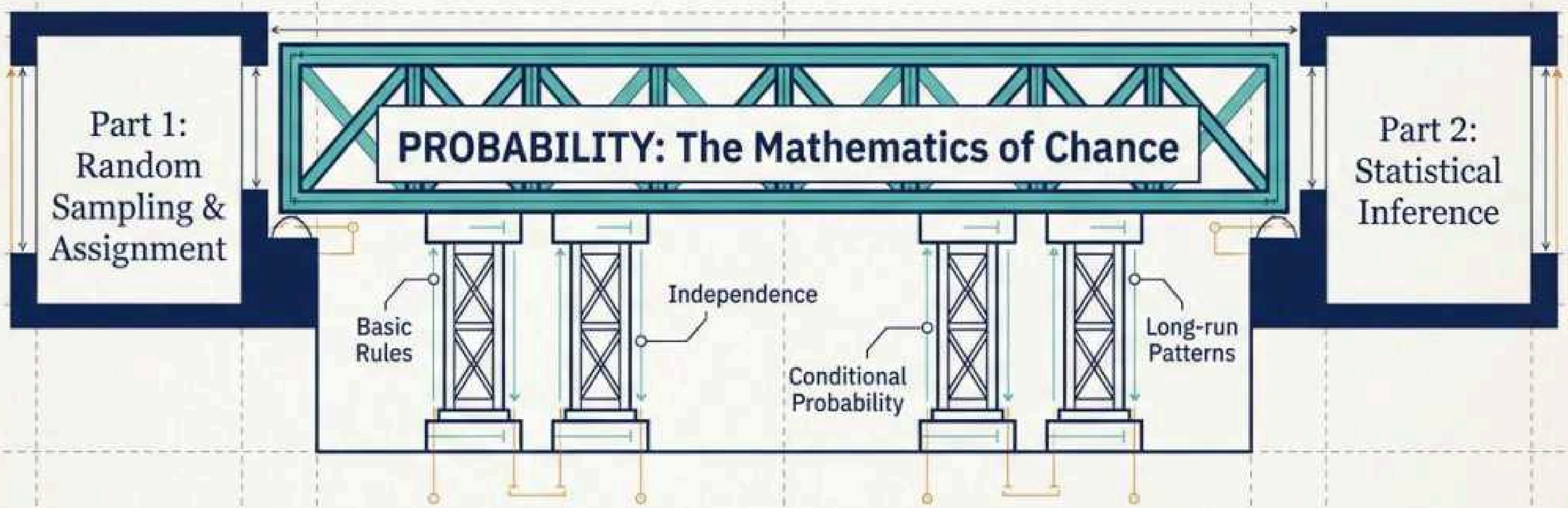
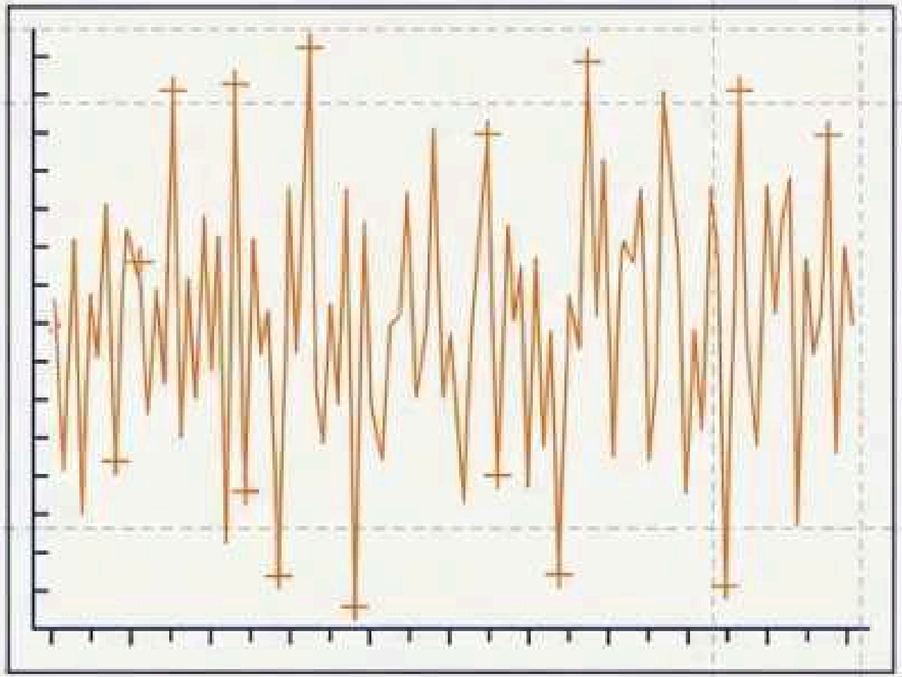
6.1



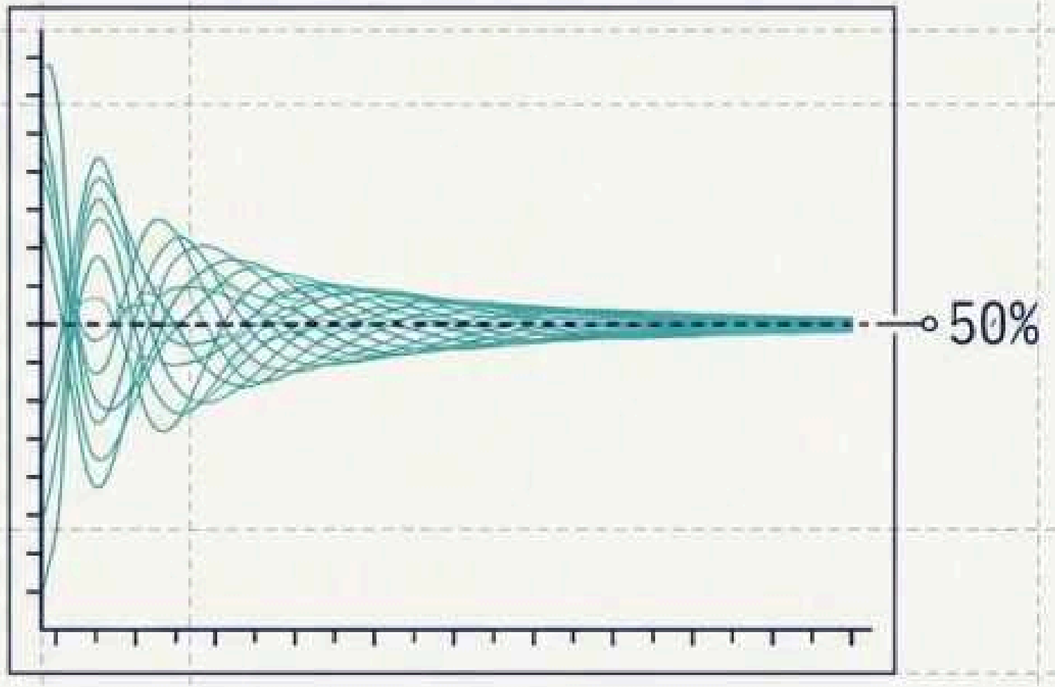
Probability explains why random sampling works—it is the indispensable bridge to inference.

6.2

SHORT RUN



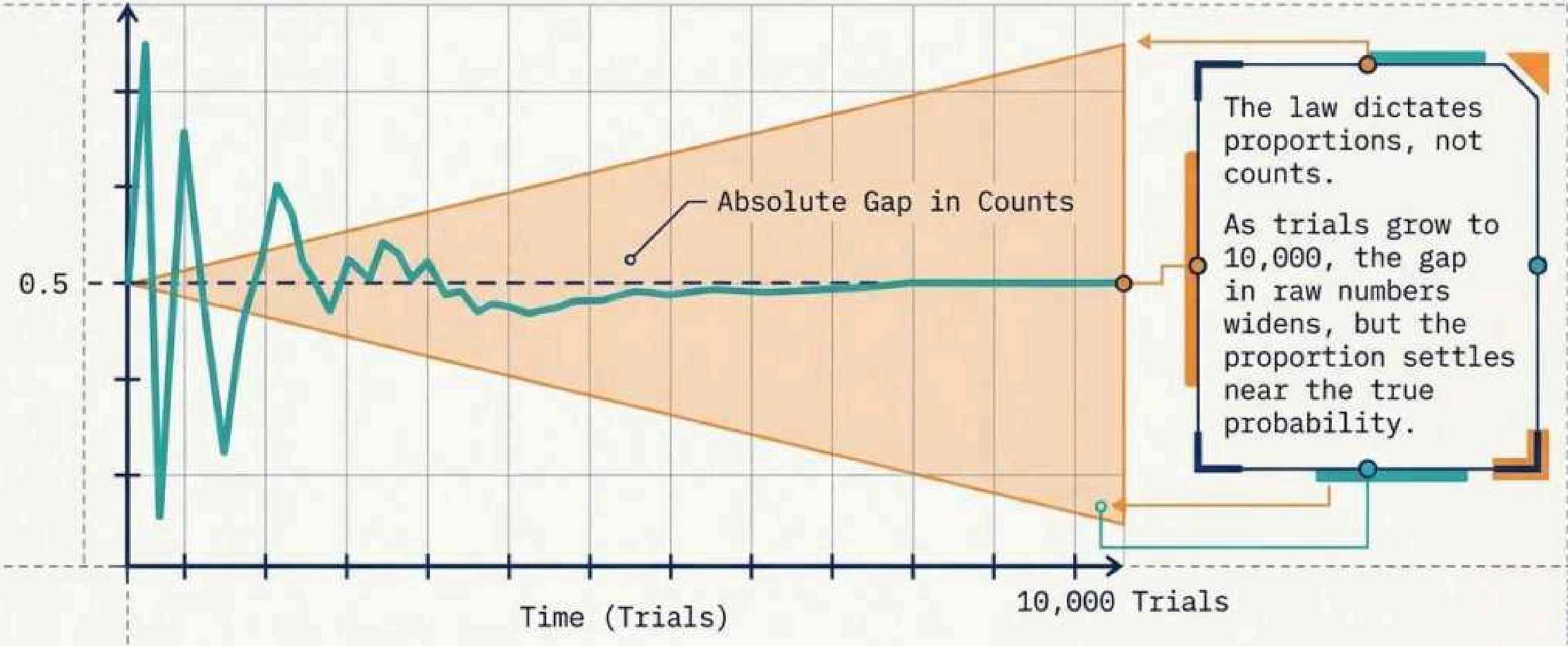
LONG RUN



A phenomenon is random if individual outcomes are uncertain, yet there is a regular pattern in the long run.

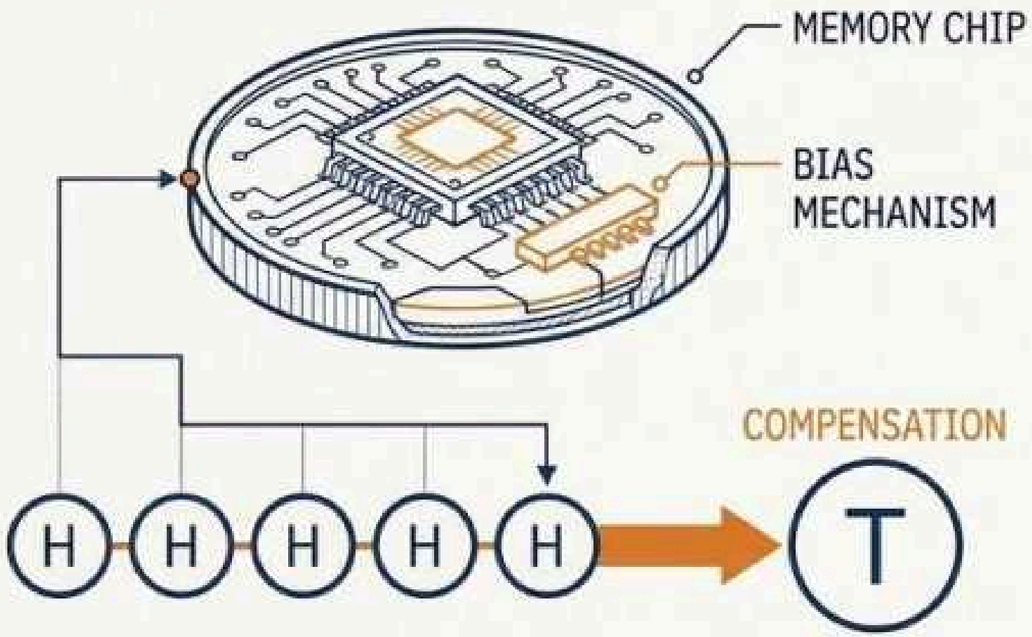
Probability = The proportion of times an outcome occurs in a very long series of repetitions.

6.3 The Law of Large Numbers

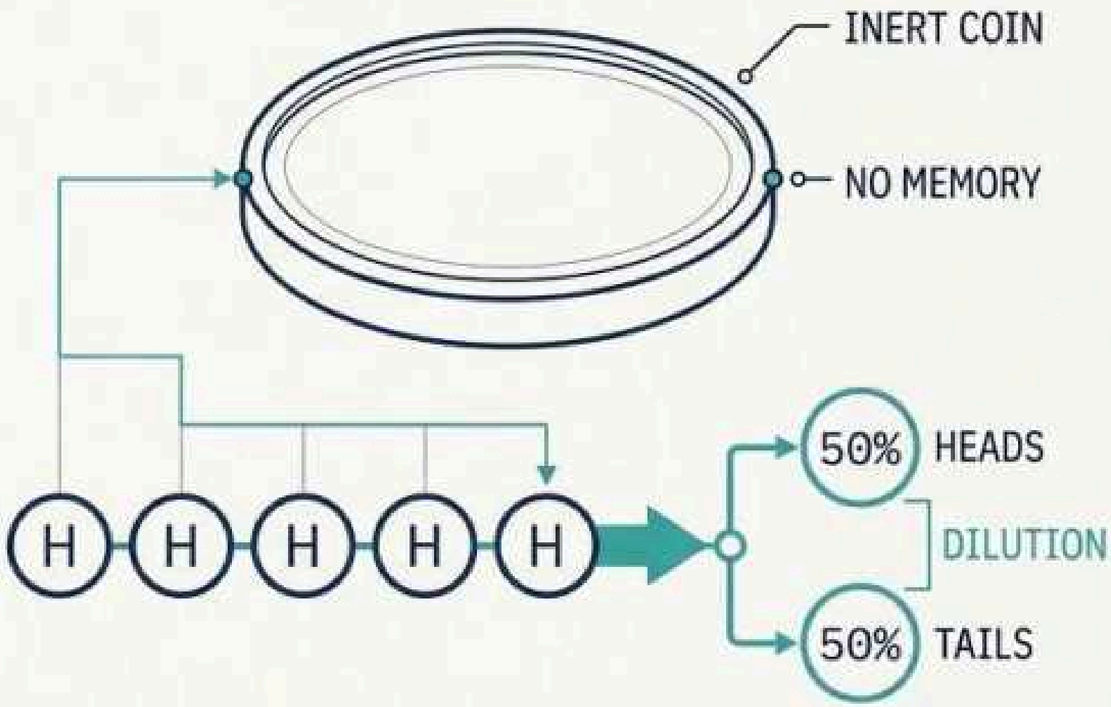


6.4

THE FALLACY



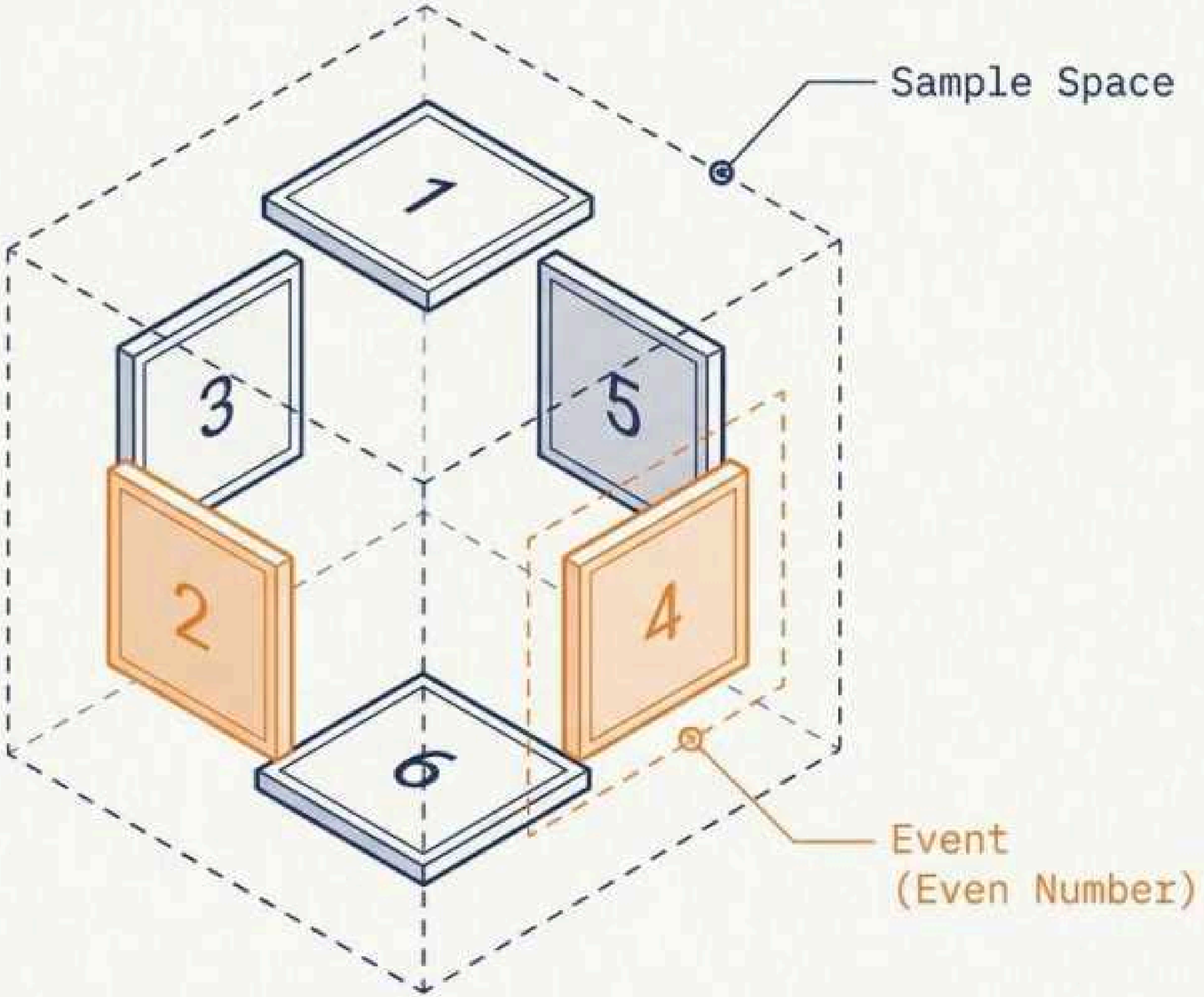
MATHEMATICAL REALITY



A FAIR COIN HAS NO MEMORY.

The chance of heads after five heads is still exactly one half.
Proportions even out over time by dilution, not by compensation.

6.5

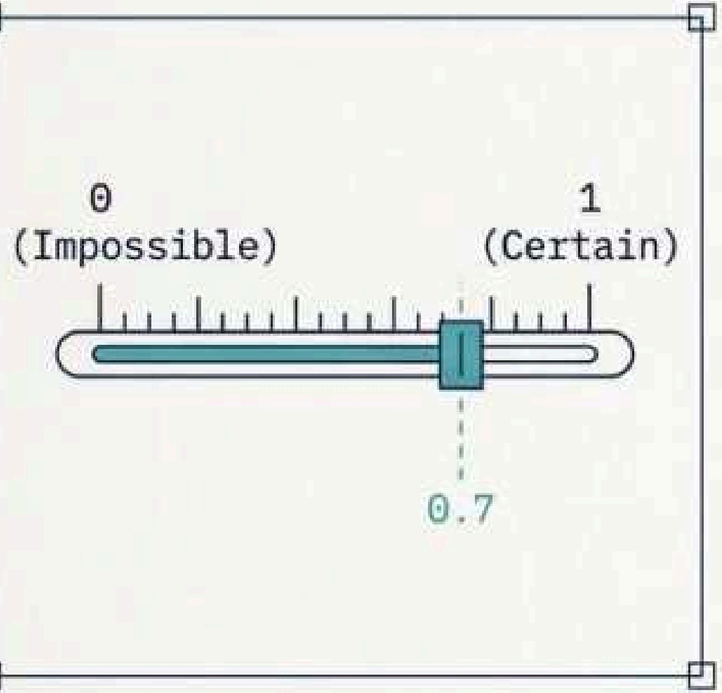


Sample Space:
The set of all possible outcomes.

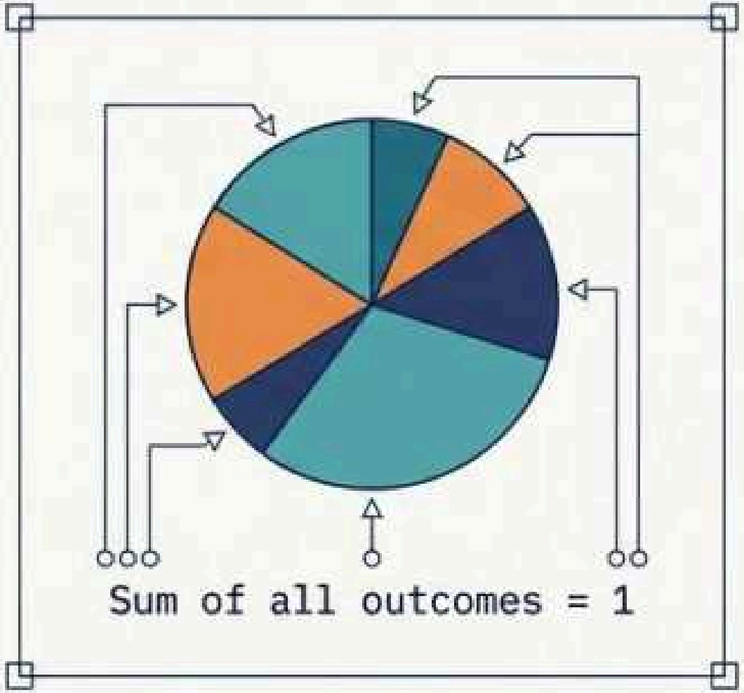
Event:
A collection of specific outcomes.

Probability =
Favorable Outcomes / Total Outcomes
(when equally likely)

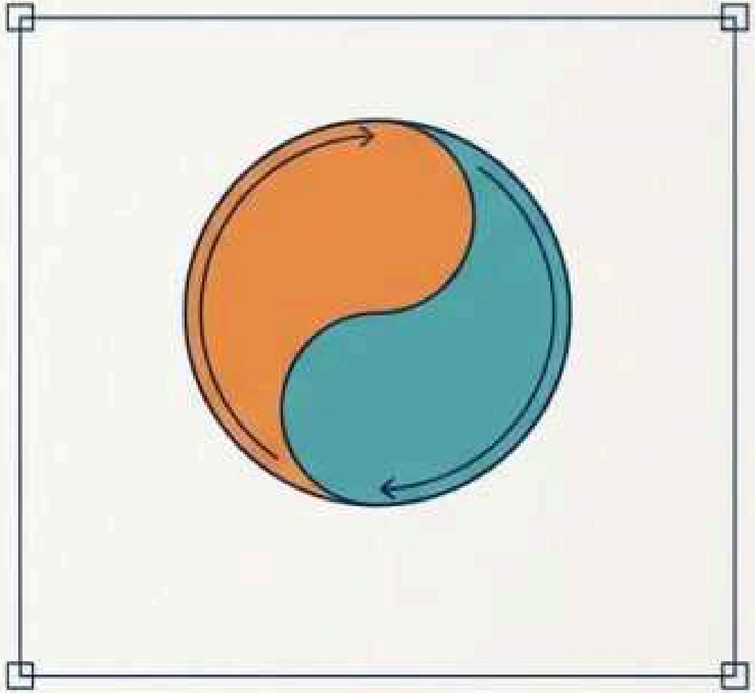
6.6



Rule 1: Every probability lies between 0 and 1.



Rule 2: All outcomes in the sample space sum to 1.

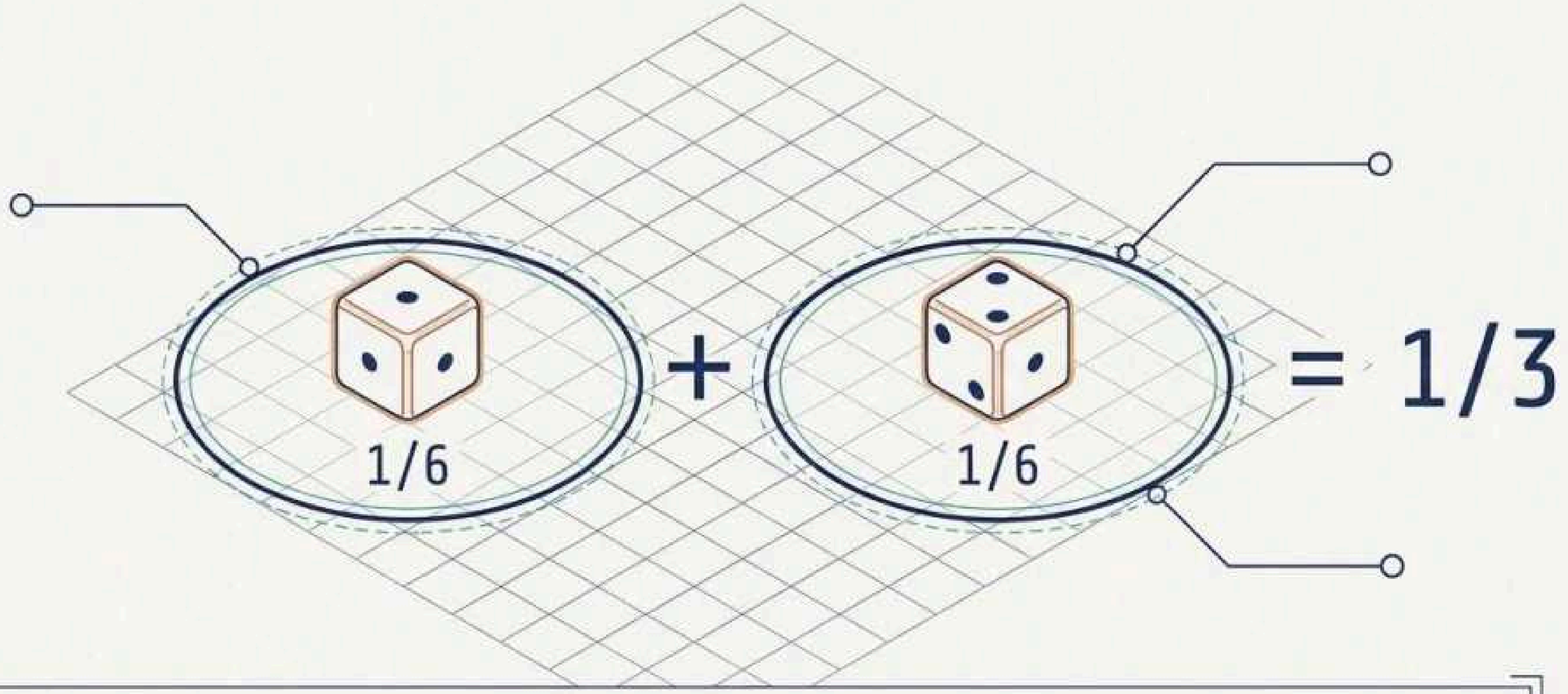


Rule 3: The Complement Rule

P(Not Event) = 1 - P(Event)

Example: Chance of not rolling a six is 1 - 1/6 = 5/6.

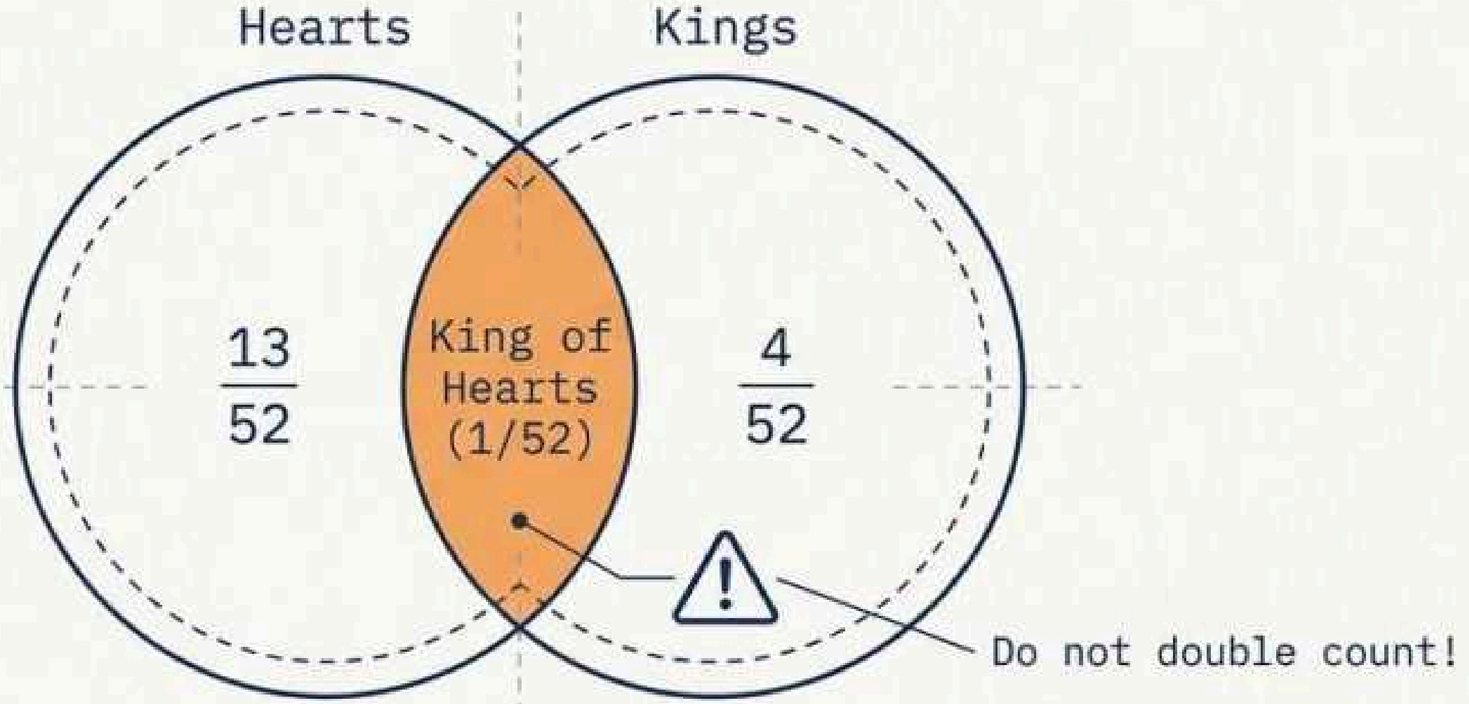
6.7



Disjoint / Mutually Exclusive: Events that cannot happen together.

P(1 or 2) = 1/6 + 1/6 = 1/3

6.8



General Addition Rule:

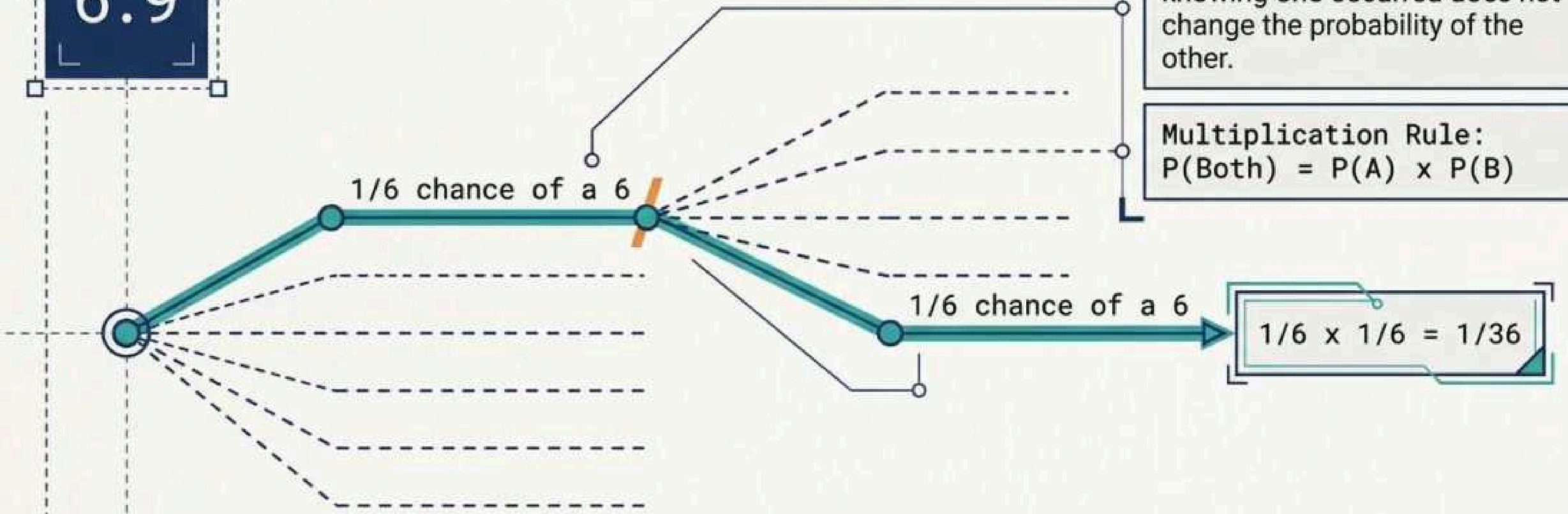
$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$13/52 + 4/52 - 1/52 = 16/52 \text{ (approx. } 0.31)$$

6.9

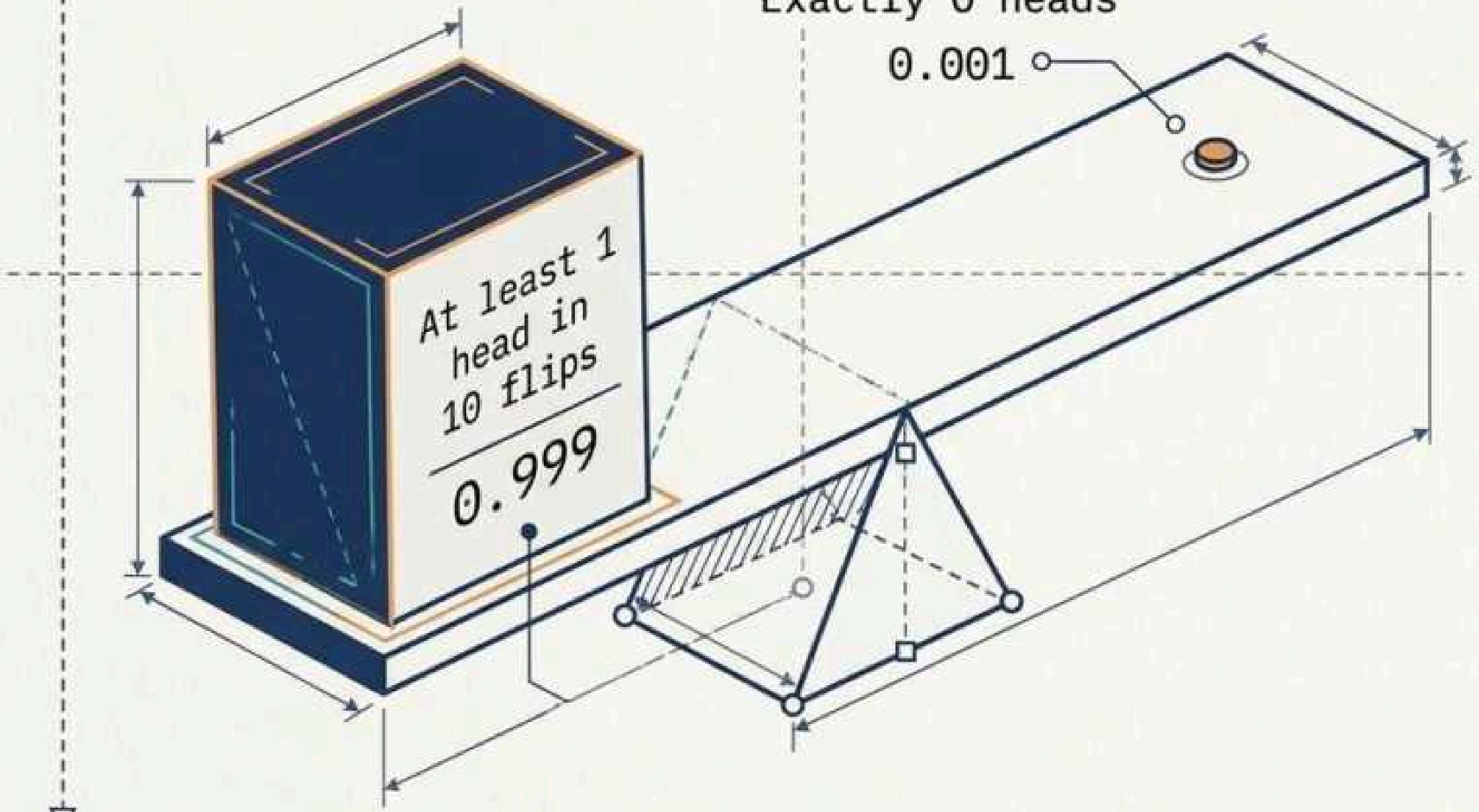
Two events are **independent** if knowing one occurred does not change the probability of the other.

Multiplication Rule:
P(Both) = P(A) x P(B)



⚠ CAUTION: Independence is an assumption to check, not a given.

6.10



The “At Least One” Trick: Calculate the exact opposite and subtract from 1.

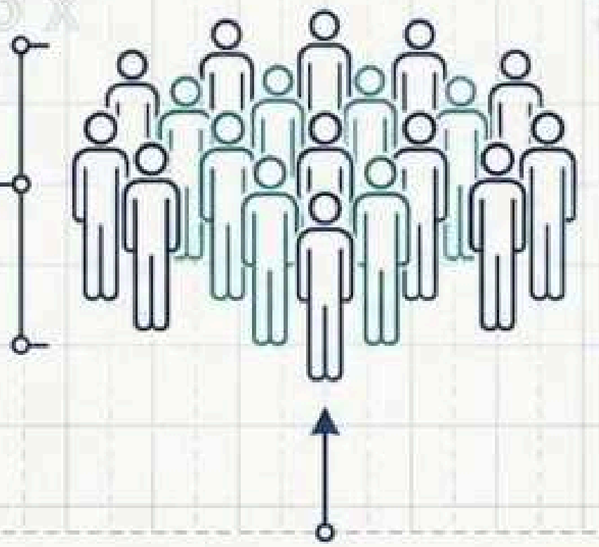
The chance of no heads is $(0.5)^{10} \approx 0.001$.

Therefore,
$1 - 0.001 = 0.999$.

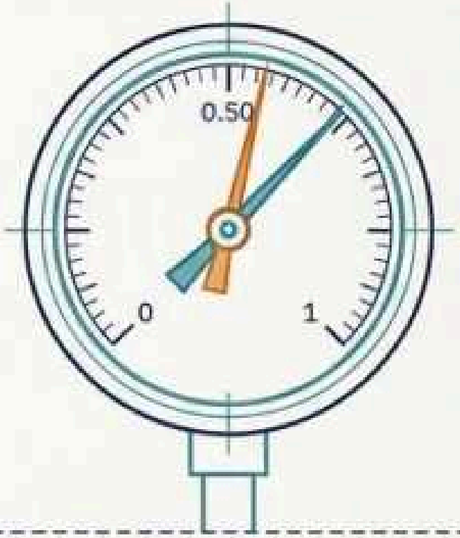
6.11 The Birthday Problem

364/365 x 363/365 x 362/365...
364/365 x 363/365 x 361/...
364/365 x 2/3...

Human Intuition:
Very unlikely!



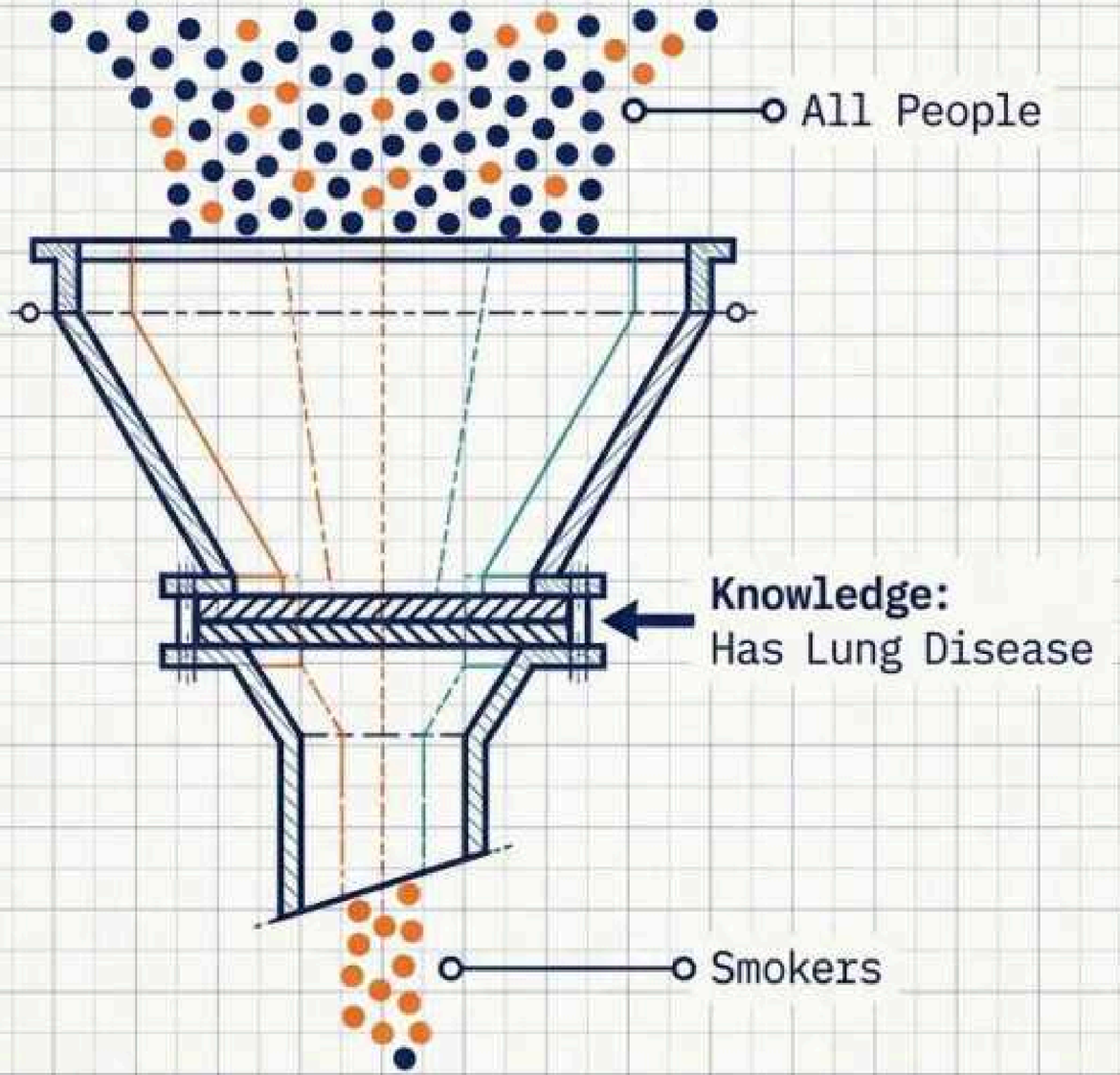
Math Reality: 0.507



In a room of 23 people, what is the chance at least two share a birthday?

Our intuition about coincidences is poor. By finding the chance that all 23 birthdays differ and subtracting from 1, the true chance is over one half (0.507).

6.12

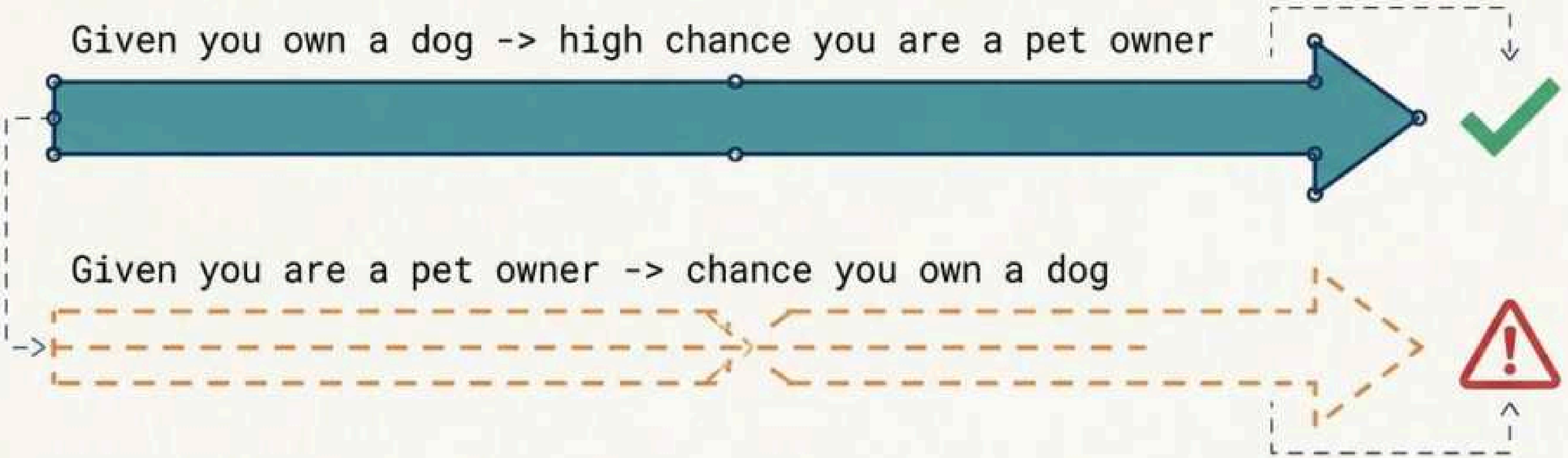


Conditional Probability: Probabilities change when we learn something new.

$$P(A|B) = \frac{P(\text{Both})}{P(B)}$$

The chance of being a smoker changes drastically if you know the person has lung disease.

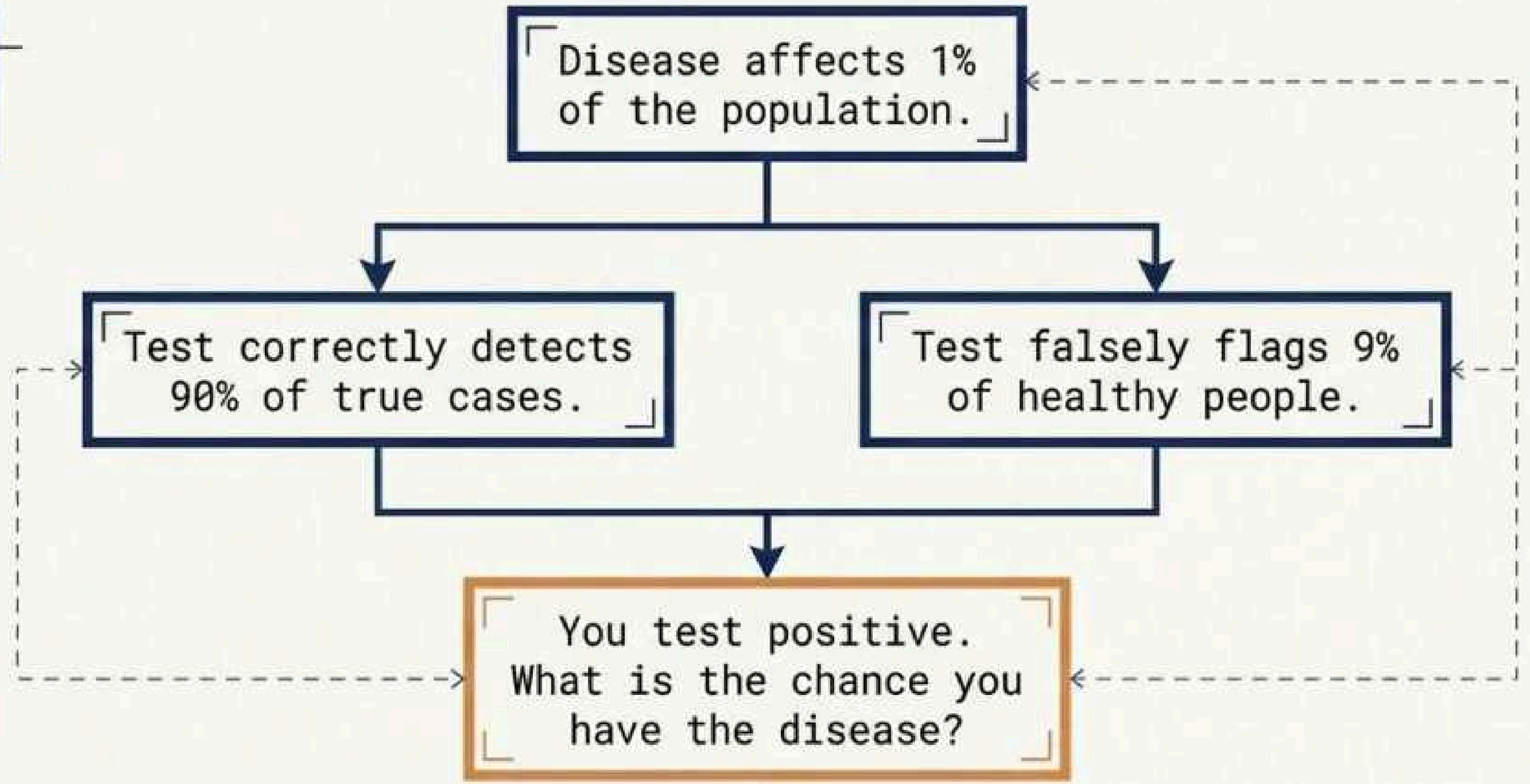
6.13



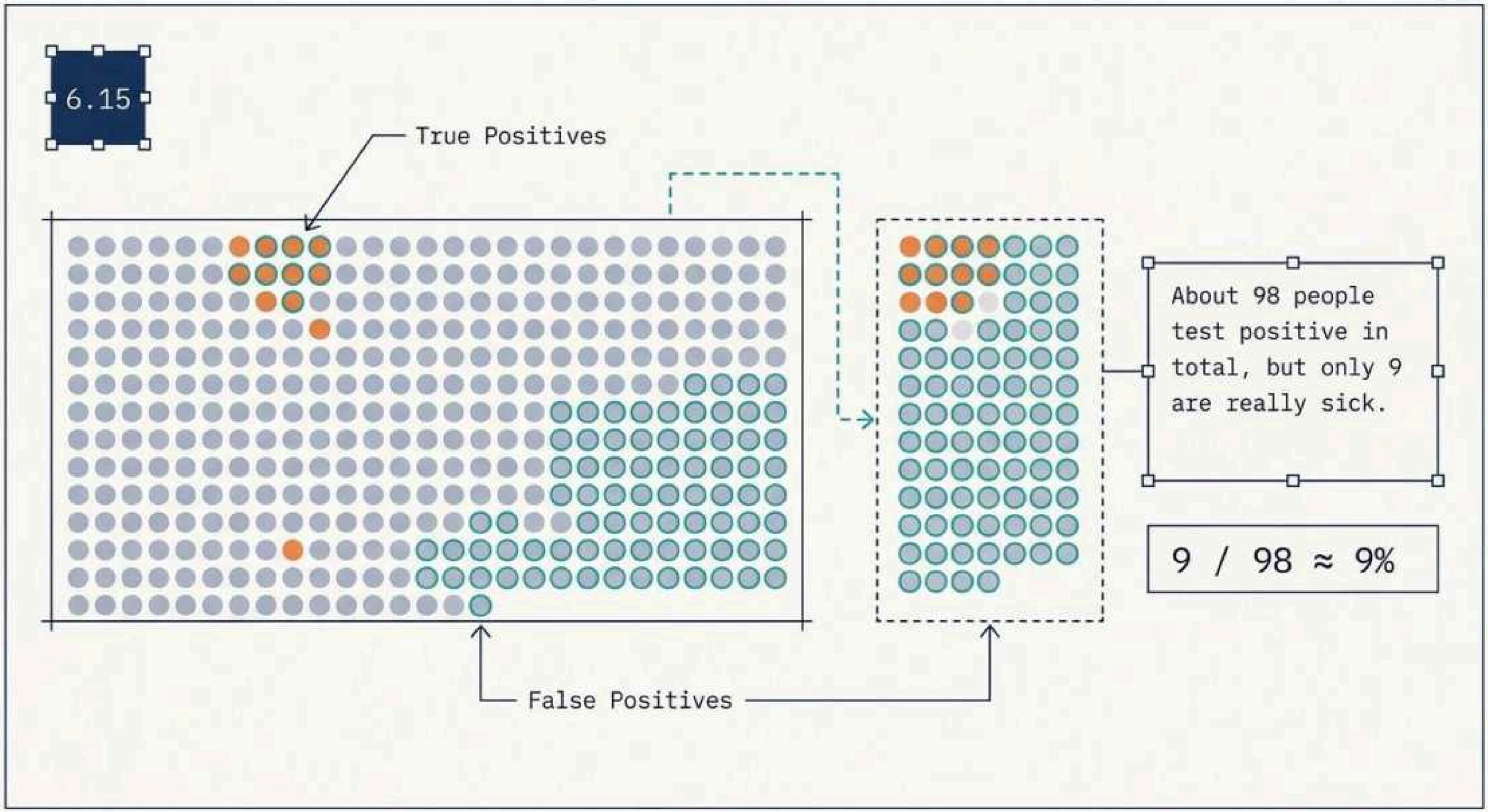
The Fallacy of the Transposed Conditional: Confusing P(A|B) with P(B|A)

This logical error frequently appears in courtrooms and clinics.

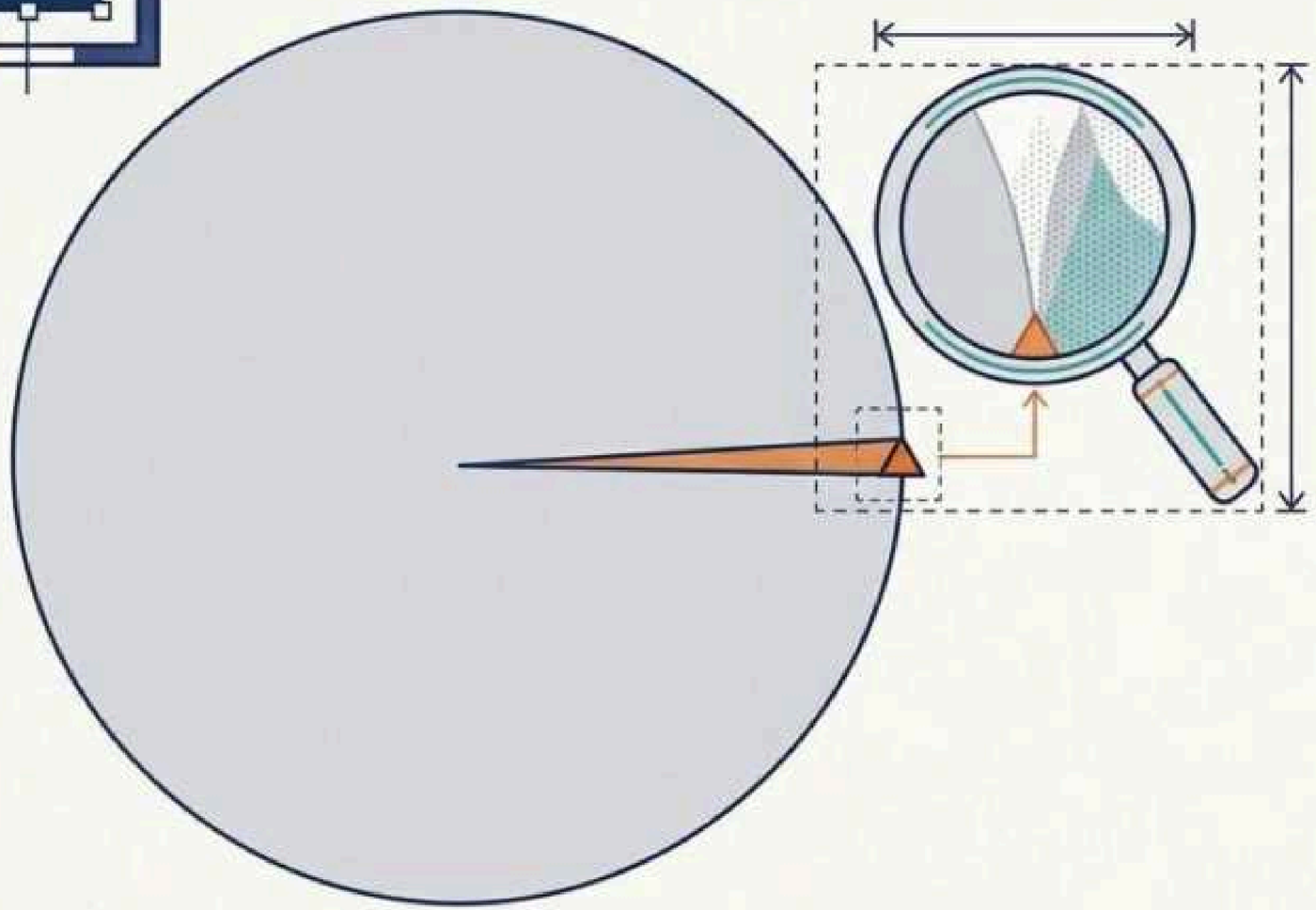
6.14



Most people guess about 90 percent.


6.15
True Positives
About 98 people test positive in total, but only 9 are really sick.
9 / 98 ≈ 9%
False Positives

6.16

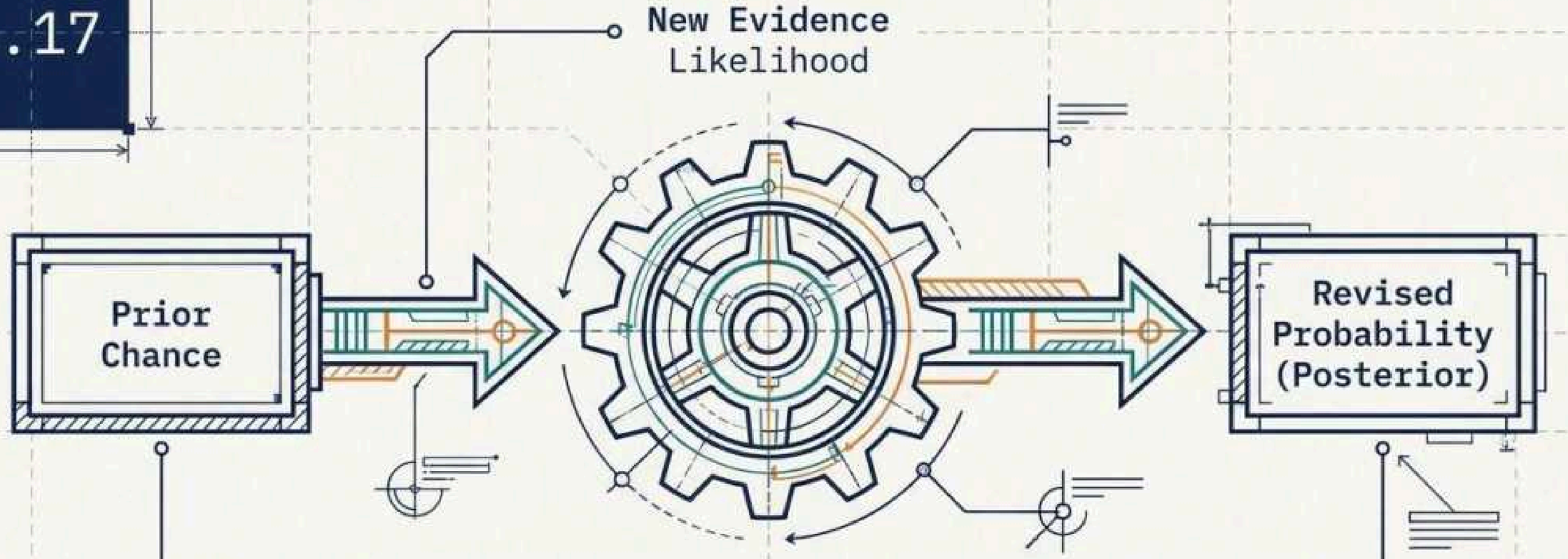


The Base Rate Effect

When a condition is rare, even a fairly accurate test produces far more false alarms than true cases.

- The same mathematics rule security screening and fraud alerts. A positive result raises the chance (1% to 9%), but it is far from certain.

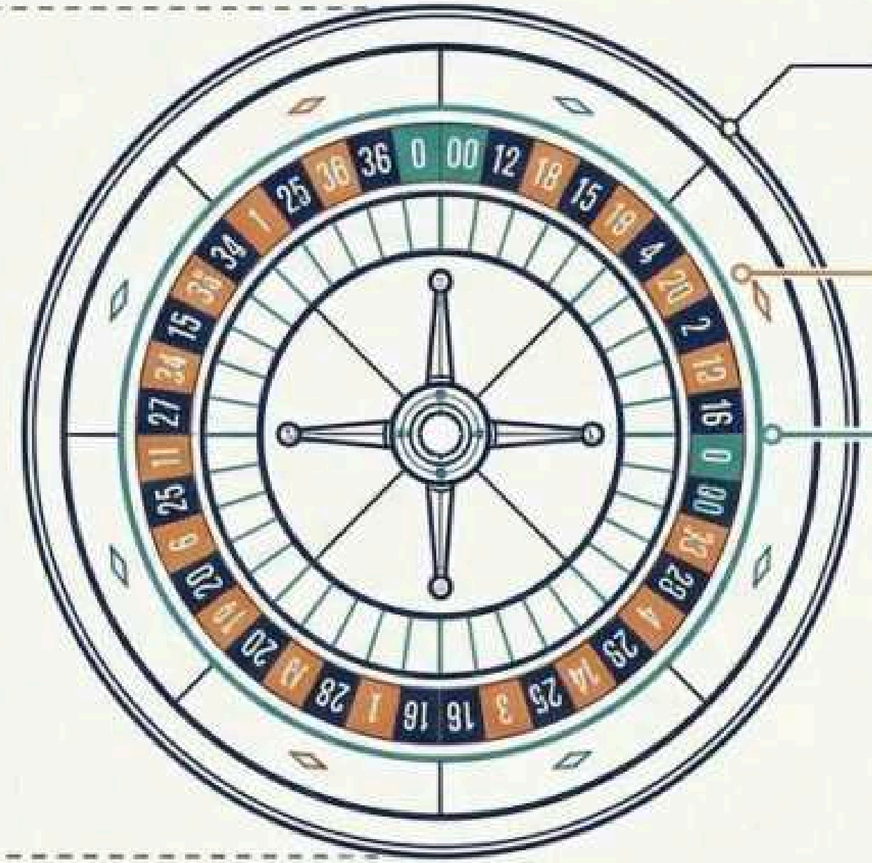
6.17



Bayes' Theorem

- The formal rule for revising probability when new evidence arrives.
- Combines prior chance with evidence likelihood. It serves as the foundation for an entire school of statistics.

6.18



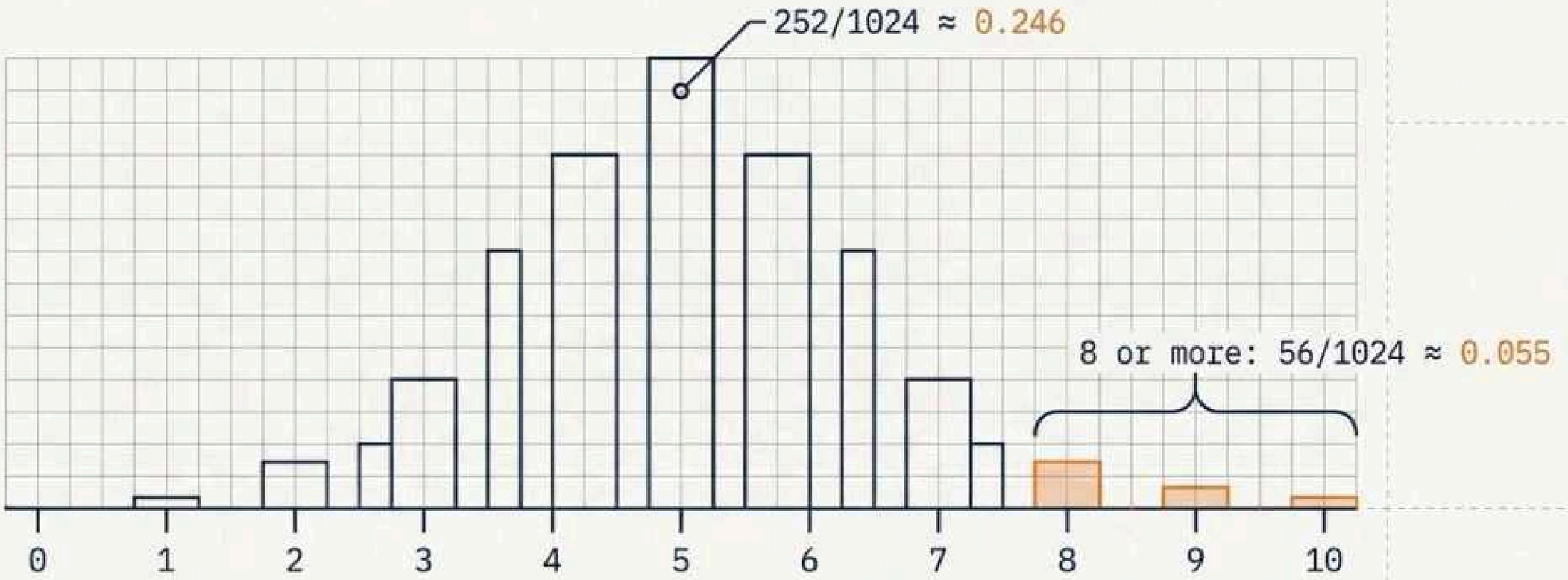
Random Variable: A number determined by a chance outcome.

Expected Value: The long-run average. Multiply each value by its probability and add.

The House Edge: The casino mathematically guarantees it keeps 5 cents on each dollar in the long run.

Win	+$1 x (18/38)
Lose	-$1 x (20/38)
Net Expected Value =	**-$0.05**

6.19

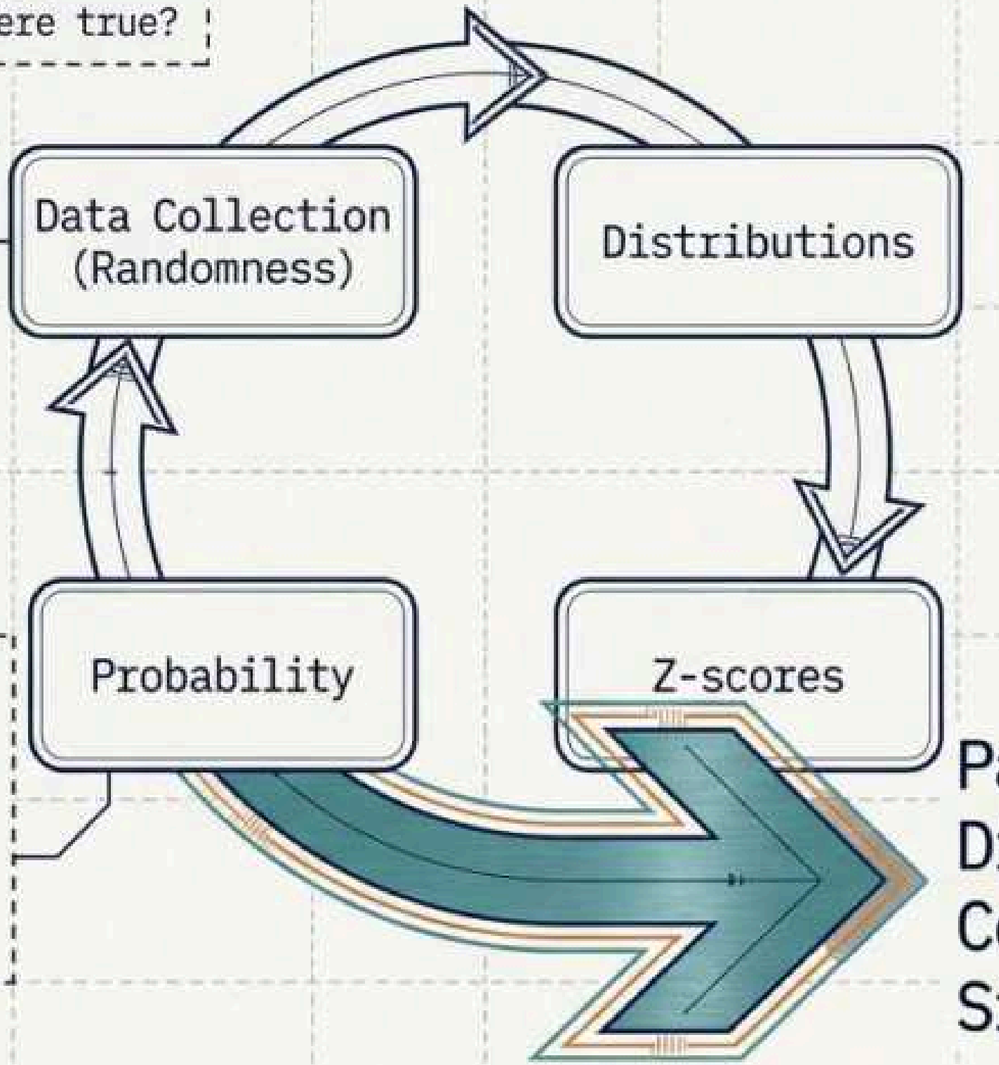


The Binomial Distribution

- Counts successes in a fixed number of independent trials.
- Such calculations tell us exactly how surprising a result would be if a baseline assumption (like a fair coin) were true.

6.20

How surprising would these data be if a certain claim were true?



Statistics learns from data despite variation. Probability is the ultimate connector between the sample and the population.

Part 2: Sampling Distributions, Confidence Intervals, Significance Tests.