MED 514 B / Lecture 2: Probability and Distributions

Key Concepts:
Statistical Foundations



- [x] Probability rules
- [x] Conditional probability
- [x] Sensitivity and specificity
- [x] Predictive values
- [x] Binomial and normal distributions

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¶ 2.1

The Scale of Probability

Long-run frequencies



0
(Impossible)

0.5
(As likely as not)

1
(Certain)



Careful judgement

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¶ 2.2

Two Basic Rules of Probability

The Addition Rule

For events that cannot happen together (mutually exclusive).

$$P(A \text{ or } B) = P(A) + P(B)$$



The Multiplication Rule

For independent events (one does not affect the other).

$$P(A \text{ and } B) = P(A) \times P(B)$$

Chance of <u>two</u> independent tosses both landing heads = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

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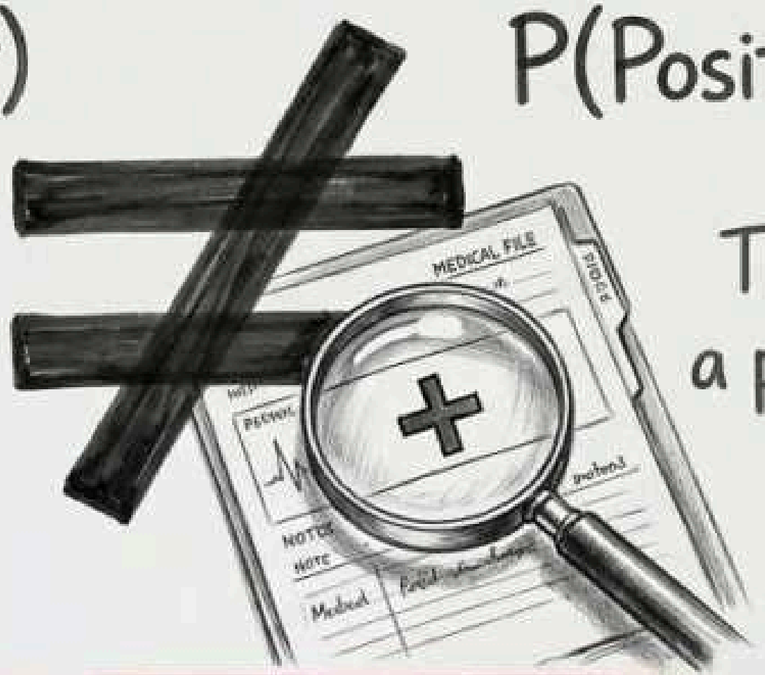
The Trap of Conditional Probability

¶2.3

$P(\text{Disease} \mid \text{Positive Test})$

The probability that a patient has a disease given a positive test.

$\neq$



$P(\text{Positive Test} \mid \text{Disease})$

The probability of a positive test given the disease.

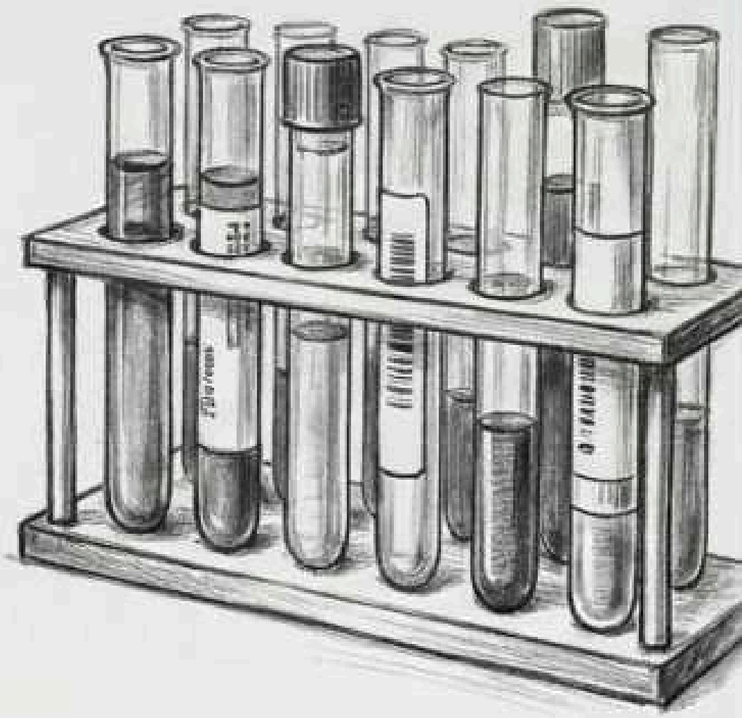
Confusing these two is the most common error in clinical reasoning about tests.

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Diagnostic Test Metrics: Sensitivity vs. Specificity

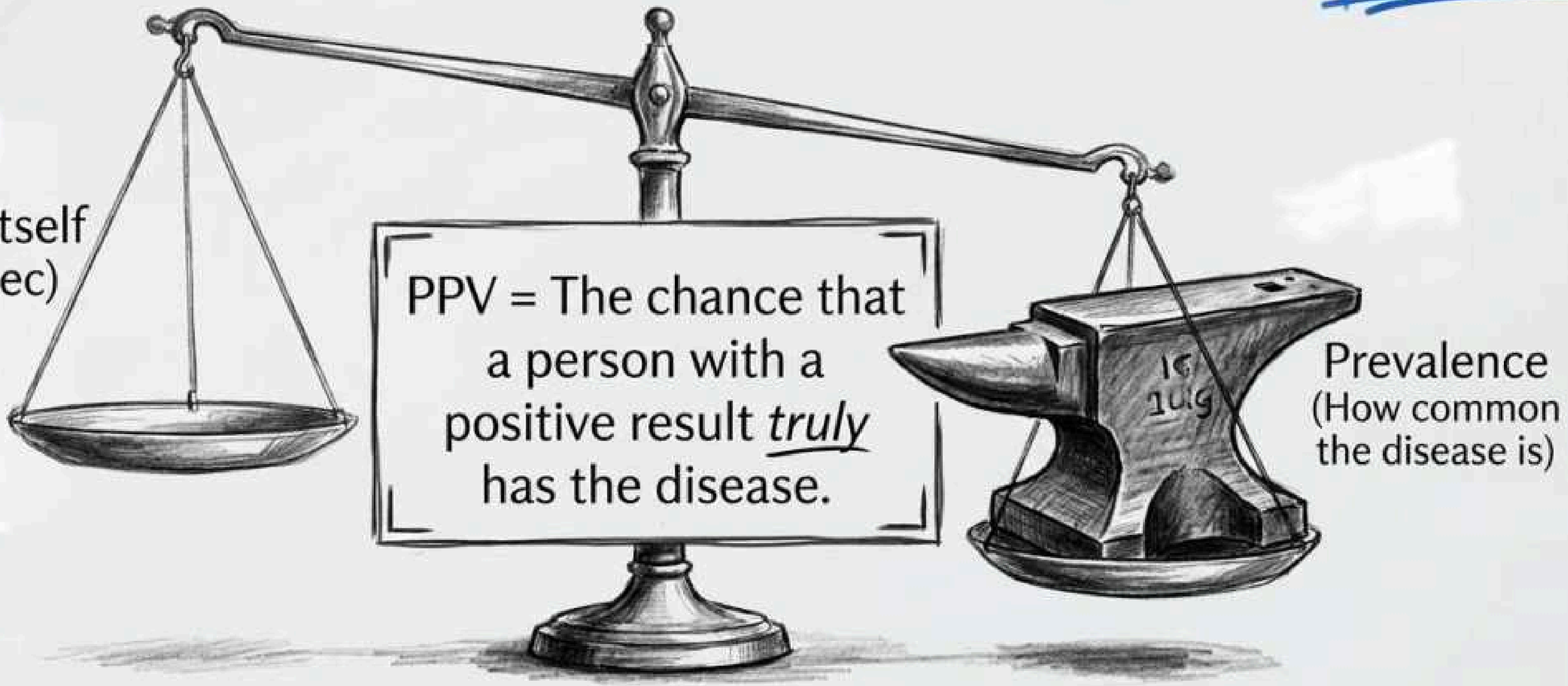
Sensitivity	Proportion of people WITH the disease who test POSITIVE.	Rarely misses the disease. A negative result helps RULE IT OUT.
Specificity	Proportion of people WITHOUT the disease who test NEGATIVE.	Rarely raises false alarms. A positive result helps RULE IT IN.



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The Positive Predictive Value (PPV)

¶ 2.5

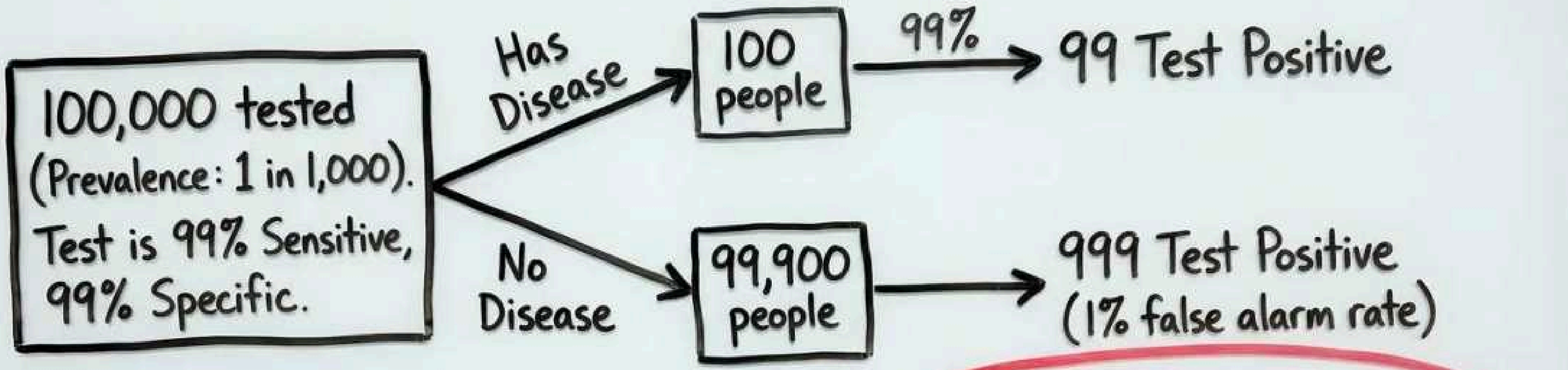


PPV depends not only on the test's *accuracy*, but heavily on the *baseline prevalence* of the disease in the tested population.

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¶2.6

The Prevalence Paradox:
Why Excellent Tests Yield False Positives



Total positive results = ~1,100.
True positives = 99.
Result: Only 9% of positive results are true. Most positives are false!

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Updating Beliefs with Bayes' Theorem

¶2.7



(Typical symptoms = higher pre-test probability)

Step 1:
Pre-test Probability
(Based on patient's story and setting)

Step 2:
New Test Result

Step 3:
Post-test Probability
(Updated belief)



(Random screening = lower pre-test probability)

A positive result means significantly more in a symptomatic patient than in someone screened at random. (Credit: 18th-century clergyman Thomas Bayes).

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The Binomial Distribution

¶2.8

$$X \sim B(n, p)$$



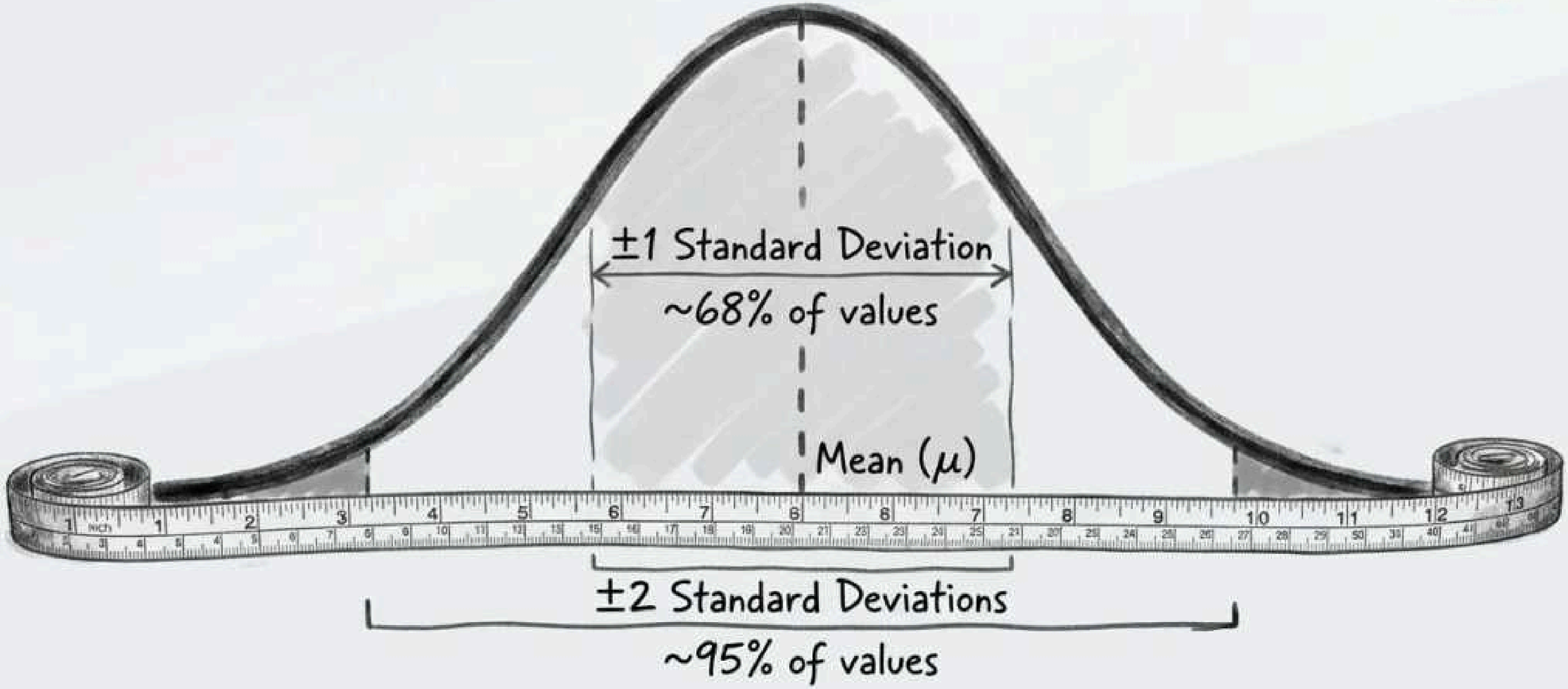
Describes the number of successes in a fixed number of independent trials.

Example: The number of patients who respond out of twenty treated, when each has the exact same chance of responding.

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The Normal Distribution

¶ 2.9

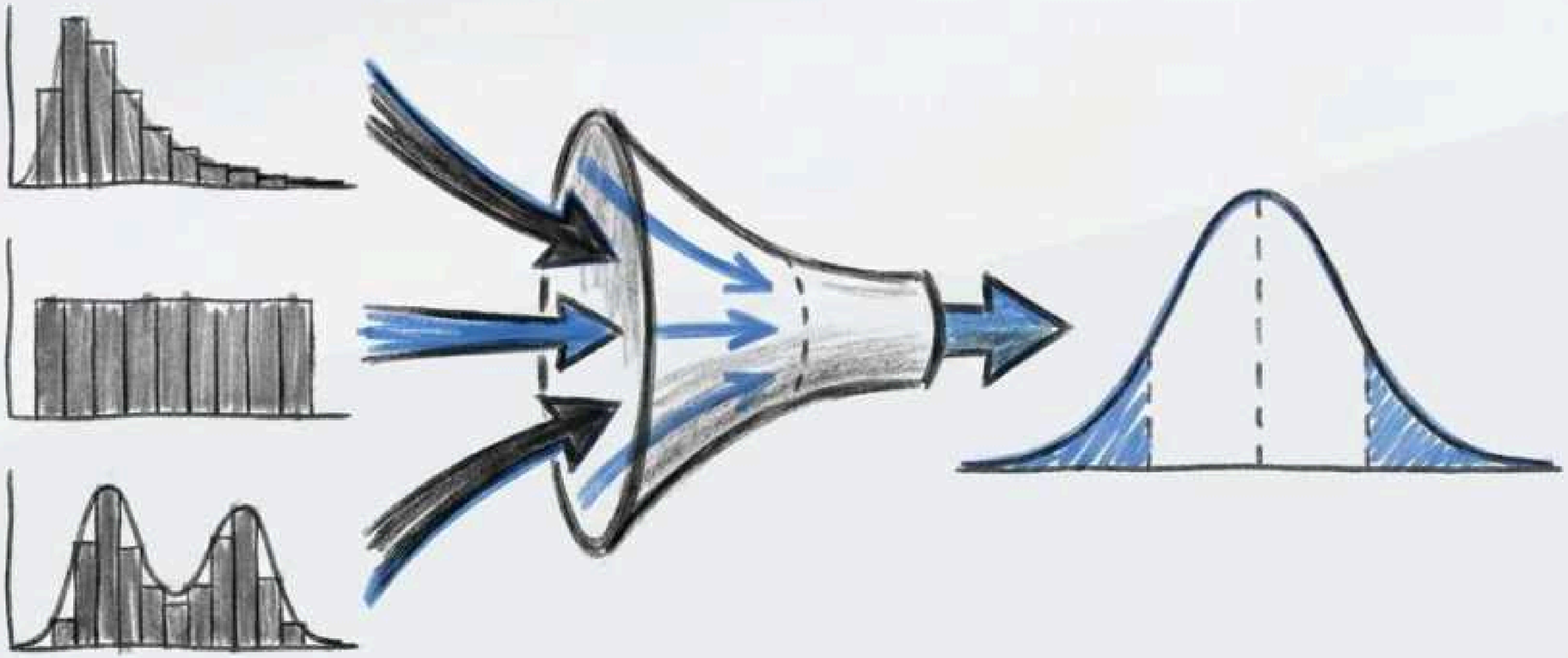


Fully described by its mean and standard deviation. Many biological measurements, such as human height, naturally follow this roughly normal pattern.

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The Central Limit Theorem

¶2.10



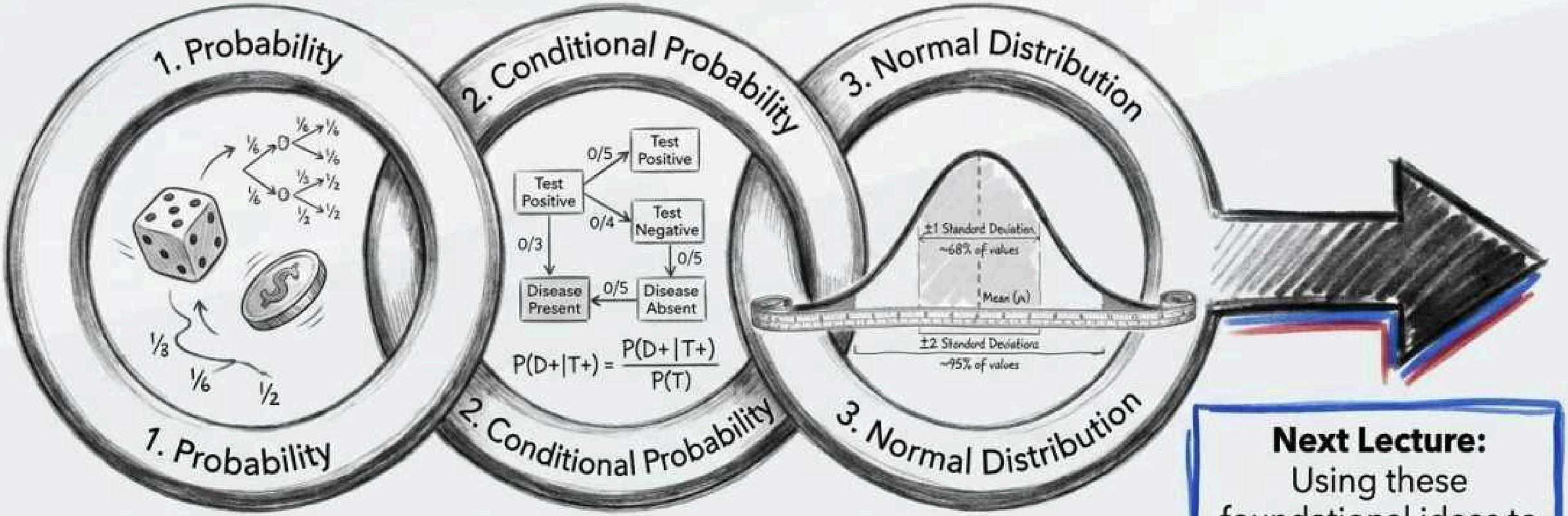
The Law: The averages of reasonably large samples follow a nearly normal distribution, whatever the shape of the original data.

The Impact: This theorem is exactly why so many statistical methods—which are built on the normal curve—work so well in practice.

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Lecture Synthesis & Next Steps

¶ 2.11



The fundamental language for clinical uncertainty.

The mechanic for updating medical beliefs based on diagnostic tests.

The mathematical foundation that underpins robust statistical methods.

Next Lecture:
Using these foundational ideas to estimate what is true in a whole population from a sample.

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