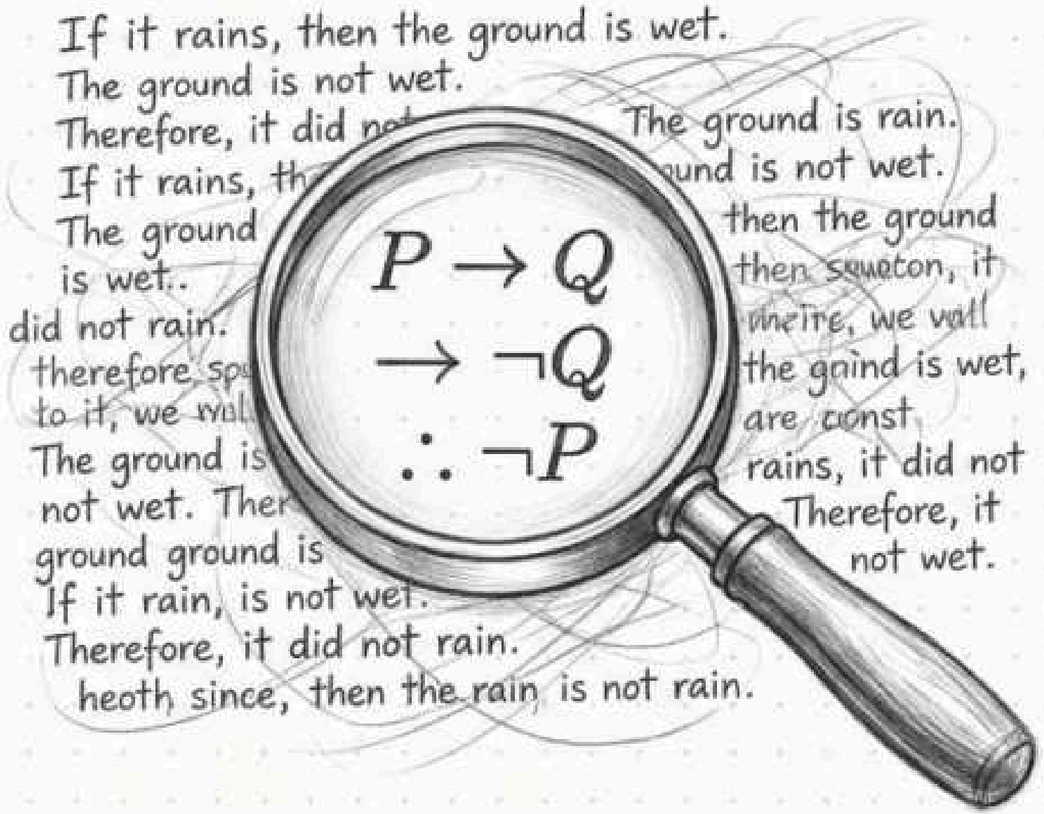
Propositional Logic & Truth Tables (Lecture 4)

¶ 4.1



Core Focus:

✓ - Symbolizing statements with connectives.

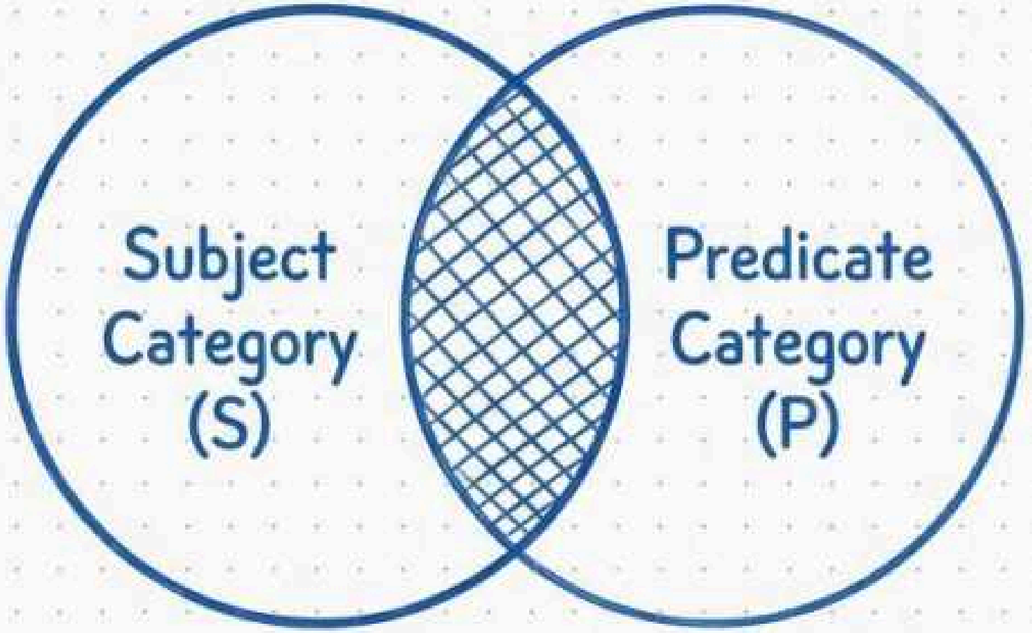
✓ - Testing arguments with truth tables.

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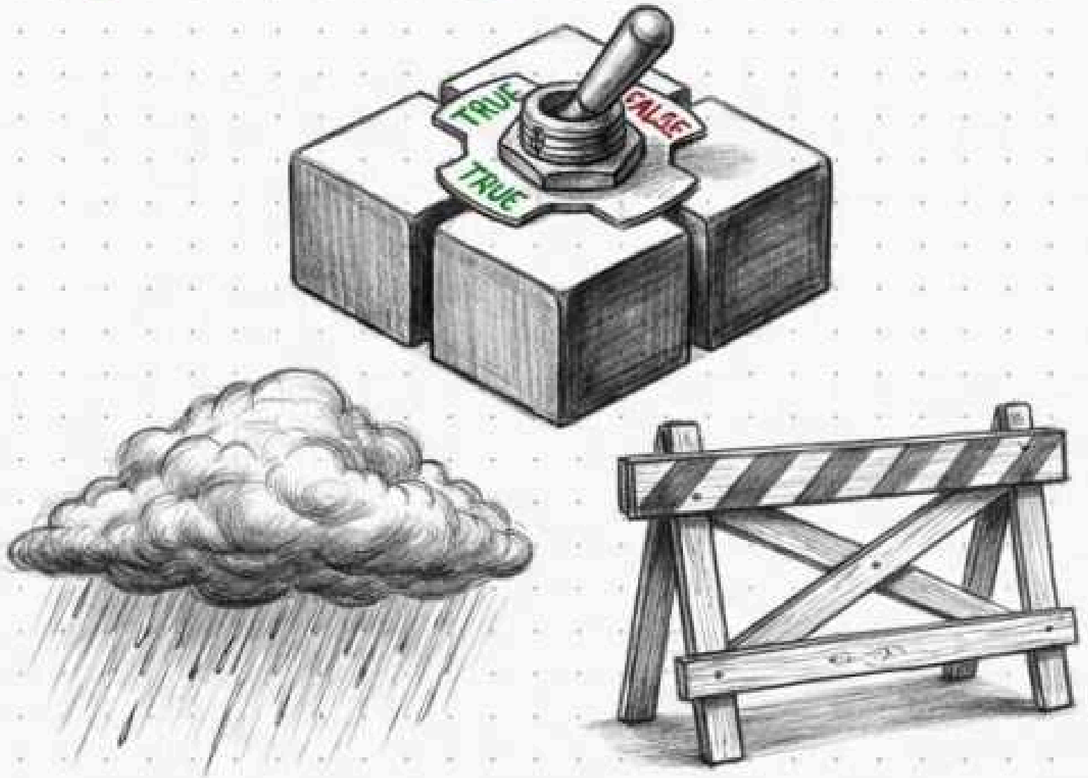
Categories vs. Propositions

¶ 4.2

Syllogisms: Categories of Things



Propositional Logic: Whole Statements

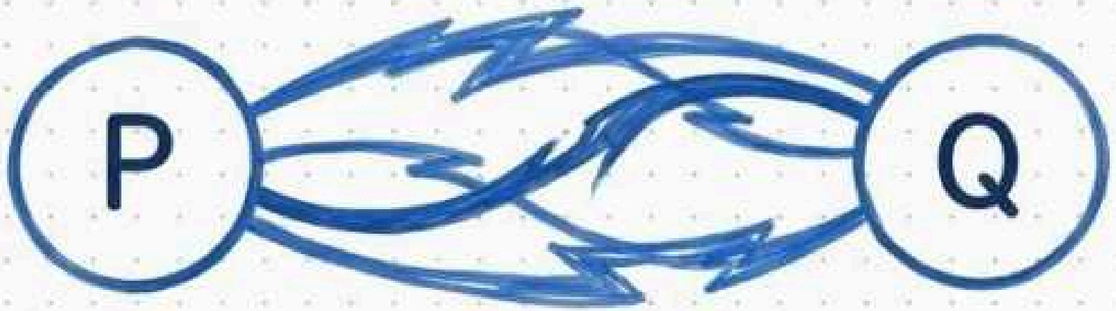


A proposition is a statement that is definitively True or False.
Logic studies how truth flows through combinations of these statements.

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Connectives: The Glue of Logic

¶ 4.3



Variables P, Q: Stand for simple propositions.

English Word	Mathematical Symbol
AND	$\wedge$
OR	$\vee$
NOT	$\neg$
IF THEN	$\rightarrow$
IF AND ONLY IF	$\leftrightarrow$

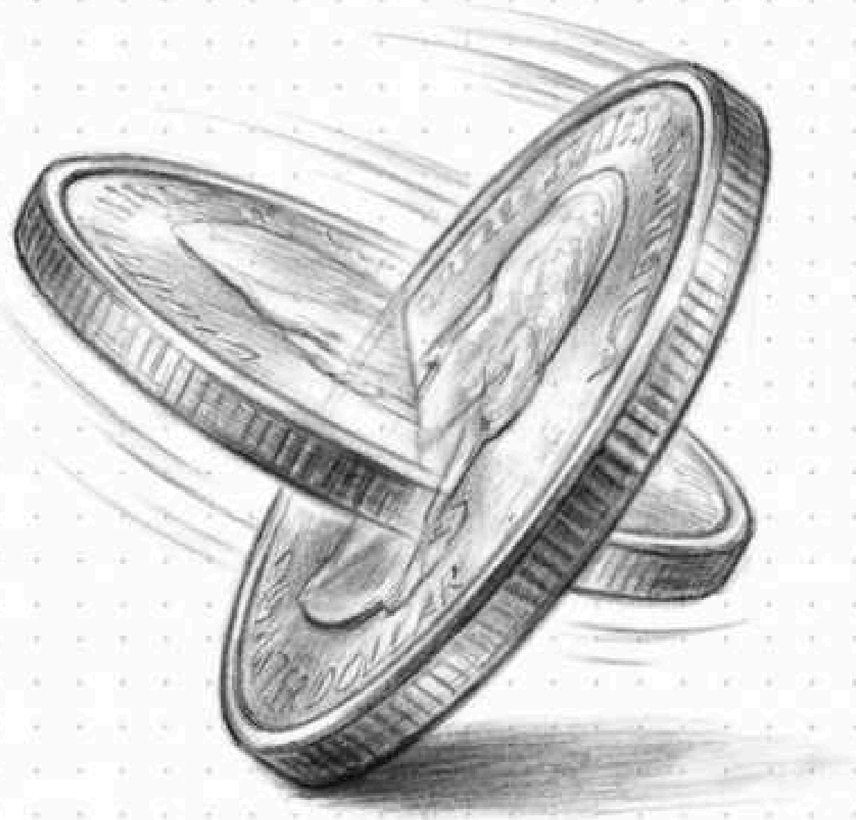
We will always say the English words, allowing you to follow without needing to memorize the symbols.

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NOT (¬): The Truth Flipper

¶ 4.4

P	Not P (¬P)
T	F
F	T



That is all there is to negation: it flips the truth value.

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AND (∧): The Strict Condition

¶ 4.5

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

If either one is false, or both are, the whole statement is infected.

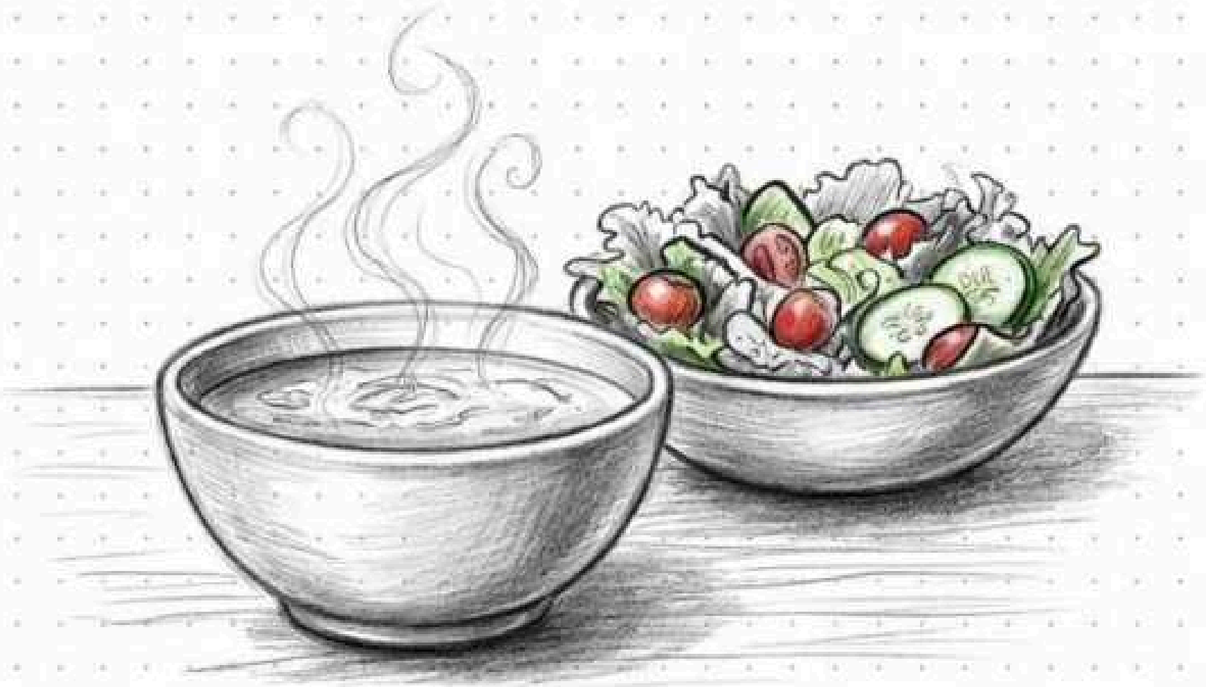
Rule: True only when both P and Q are true.

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OR (v): The Inclusive Choice

¶ 4.6

P	Q	P v Q
T	T	T
T	F	T
F	T	T
F	F	F



The Inclusive Rule:

- Logicians default to the inclusive OR.
- Having both the soup and the salad (T/T) does not make the offer false!

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IF THEN (→): The Conditional

¶ 4.7

[IF] P (Antecedent) → [THEN] Q (Consequent)

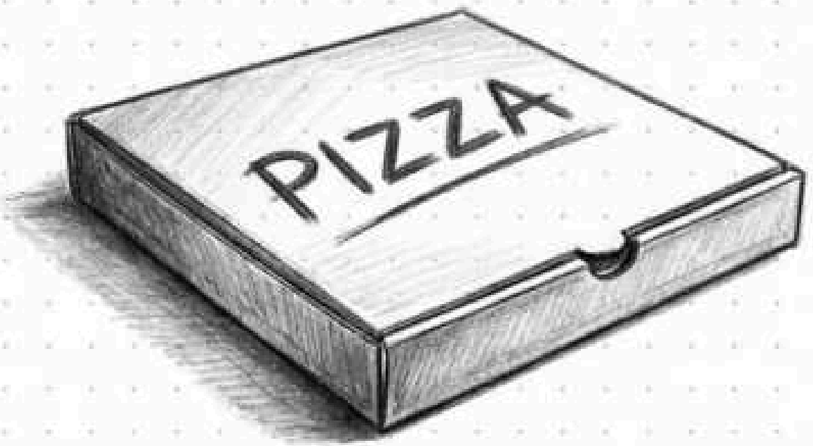
P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

The conditional is false in exactly <u>one</u> case: when P is true and Q is false.

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What if P is False? (The Broken Promise)

¶ 4.8





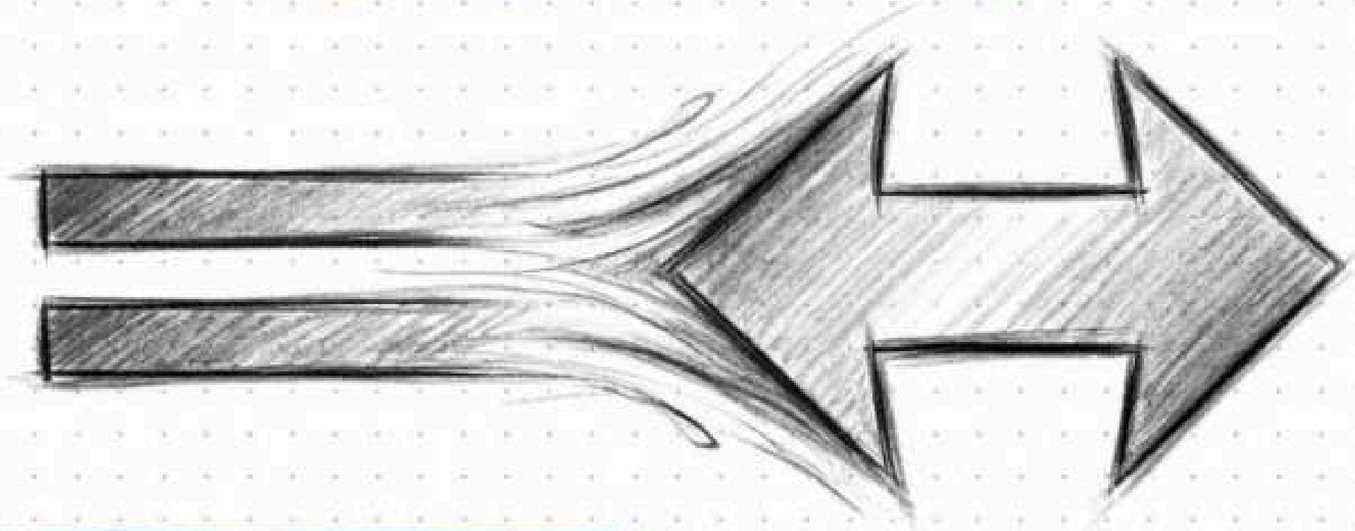
The Promise:	If you pass the test (P), I will buy you a pizza (Q).
The Reality:	You fail the test (P is False).
Question 1:	Have I broken my promise if I still buy you pizza? No.
Question 2:	Have I broken it if I don't? No.

Conclusion: A promise is only broken if you pass (True) and I don't deliver (False). Therefore, when the antecedent (P) is false, the conditional counts as True.

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IF AND ONLY IF (↔): Total Equivalence

¶ 4.9



P	Q	P↔Q
T	T	T
T	F	F
F	T	F
F	F	T

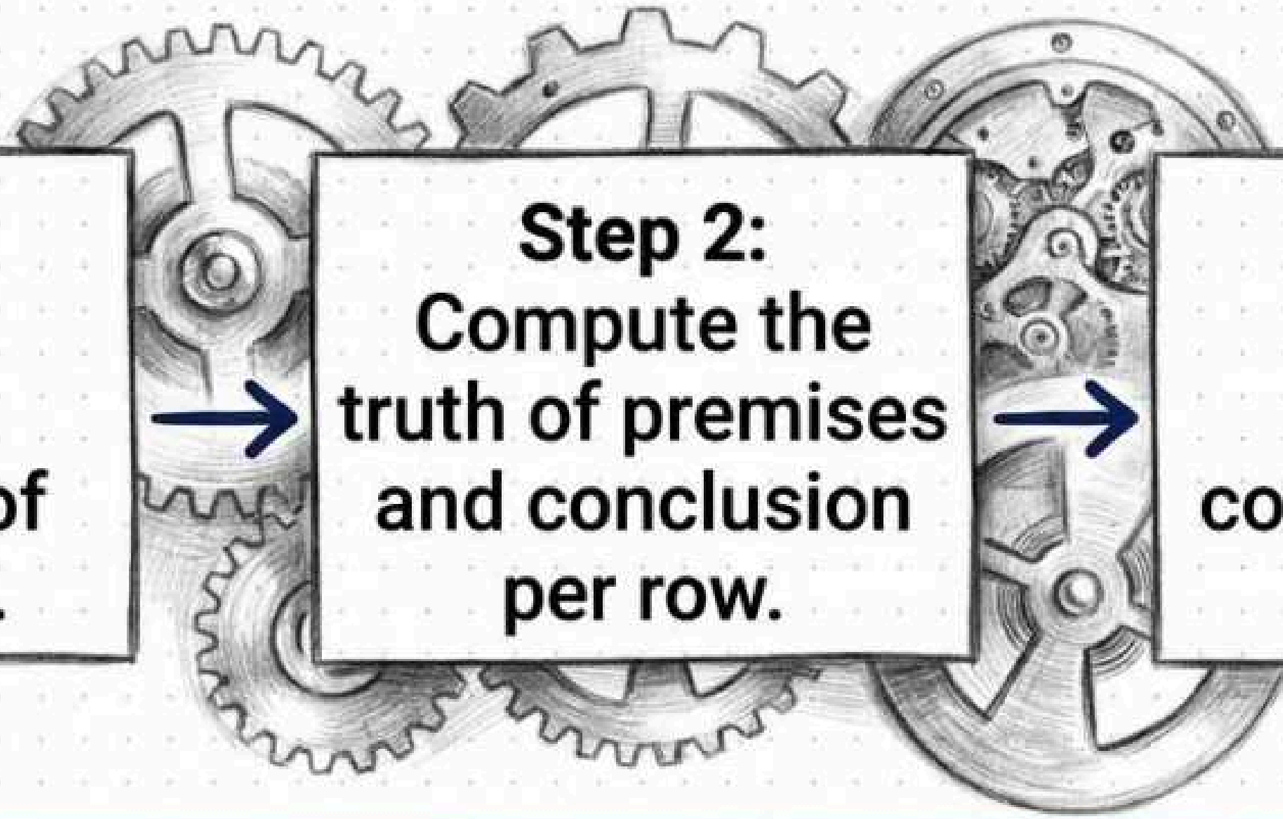
MATCH

Core Rule: **P and Q must have the same truth value.**
It is the **logician's** way of expressing a **necessary and sufficient condition** (often used for definitions).

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Testing Arguments: The Mechanical Method

¶ 4.10



Step 1:
List every possible combination of truth values.

→

Step 2:
Compute the truth of premises and conclusion per row.

→

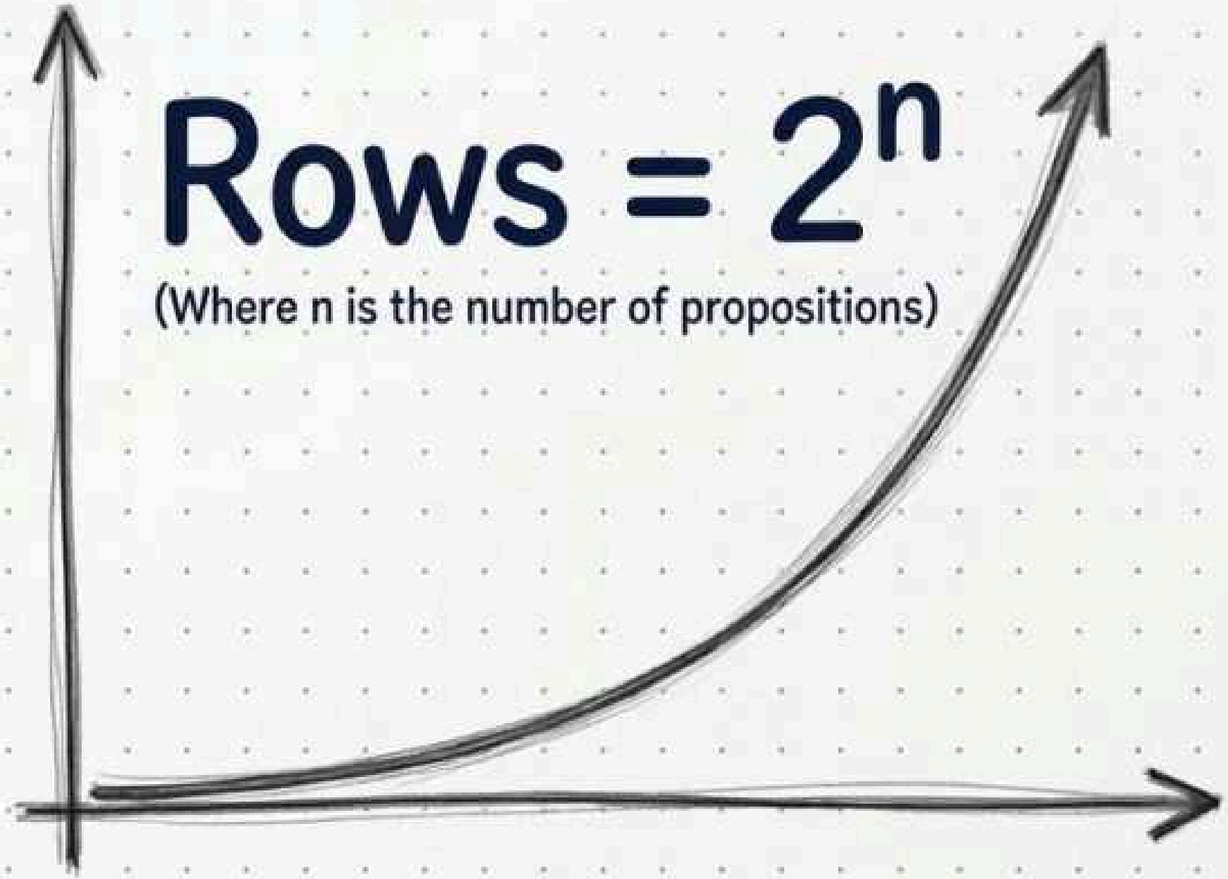
Step 3:
Hunt for a counterexample.

The **Rule of Validity:** An argument is valid if there is **NO** row where **all premises** are **True** and the conclusion is False.

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The Mathematics of Truth Tables

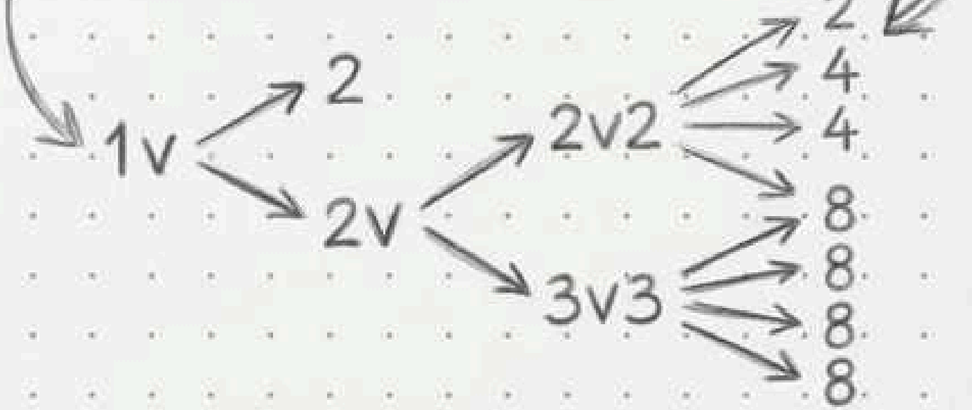
¶ 4.11



$$\text{Rows} = 2^n$$

(Where n is the number of propositions)

- 1 proposition = 2 rows
- 2 propositions = 4 rows
- 3 propositions = 8 rows



Insight: Truth tables are highly practical for small problems but double in size with every added proposition. Computers use cleverer methods for large cases, but the underlying idea remains the same.

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Modus Ponens ("The Mode that Affirms")

¶ 4.12

Premise 1: If P then Q ($P \rightarrow Q$)

Premise 2: P

Conclusion: Therefore, Q ($\therefore Q$)



The Proof:

Check the truth table: the only row where both premises are true is the one where P and Q are both true. The conclusion must follow.

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Modus Tollens ("The Mode that Denies")

¶ 4.13

Premise 1: If P then Q ($P \rightarrow Q$)

Premise 2: Not Q ($\neg Q$)

Conclusion: Therefore, Not P ($\therefore \neg P$)



- The Street: If it rains, the street is wet. The street is not wet. Therefore, it did not rain.
- The Science: If a theory predicts an outcome and it does not occur, the theory is in trouble.

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Invalid Patterns: The Two Fallacies

¶ 4.14

Affirming the Consequent

Pattern: If P → Q, and Q, therefore P.

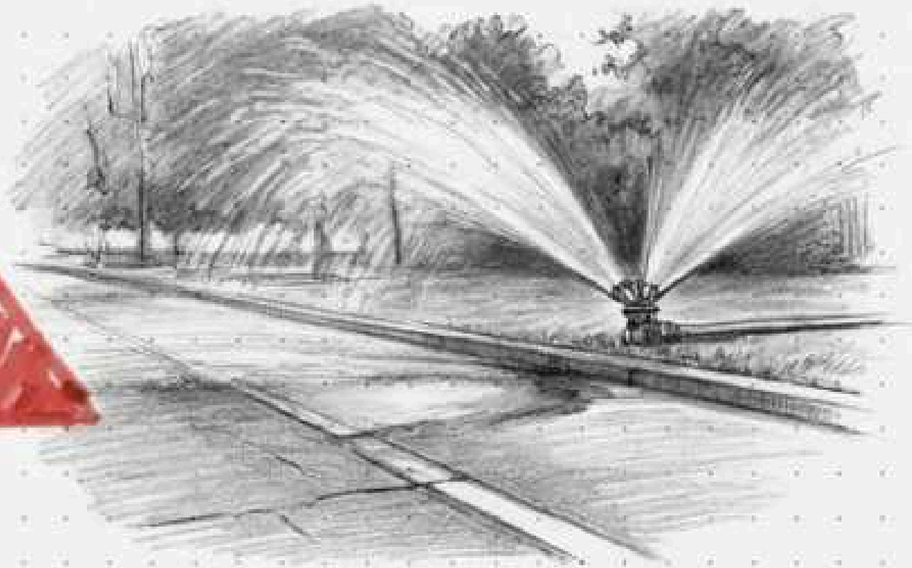
The Error: The Wet Street Fallacy.
(The street is wet, therefore it rained.
Correction: A sprinkler could have done it).



Denying the Antecedent

Pattern: If P → Q, and Not P, therefore Not Q.

The Error: (It didn't rain, so the street is dry.
Correction: Again, a sprinkler).



Core Note: Both are mathematically invalid; a truth table contains a specific row that refutes each pattern.

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Logical Equivalences: De Morgan's Laws

¶ 4.15

$$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$$

(Not both P and Q is equivalent to Not P OR Not Q)

$$\neg(P \vee Q) \equiv \neg P \wedge \neg Q$$

(Not either P or Q is equivalent to Not P AND Not Q)

Insight: Denying a conjunction requires subtle care. The mathematical equivalence symbol (≡) proves they will always share the exact same truth value.

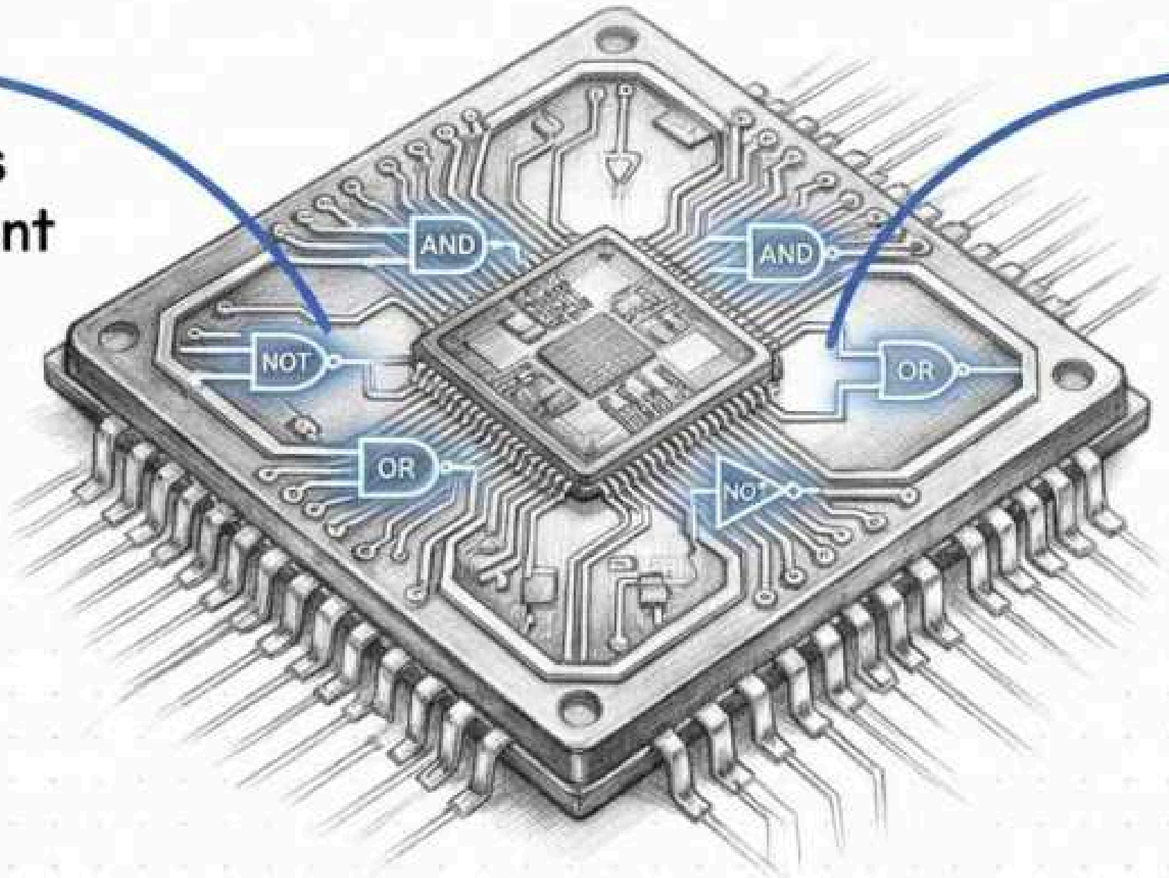
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Logic in the Real World

¶ 4.16

Hardware:

Tiny physical circuits (logic gates) implement exact AND, OR, and NOT truths.



Software:

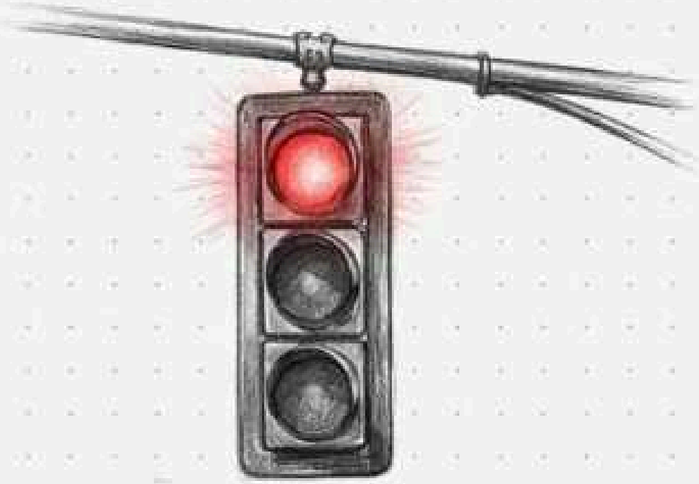
Every search engine query, every conditional "If" statement in code, and every complex legal contract relies on these connectives.

Propositional logic is more than a classroom game; it provides a precise language for careful, rigorous thinking in the real world.

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Question to Consider: The Converse

¶ 4.17



The Rule:
"If the light is red, then I stop."
$(P \rightarrow Q)$

The Converse:
"If I stop, then the light is red."
$(Q \rightarrow P)$

Is the converse logically true? Why is it crucial for survival not to mix up a conditional statement with its converse?

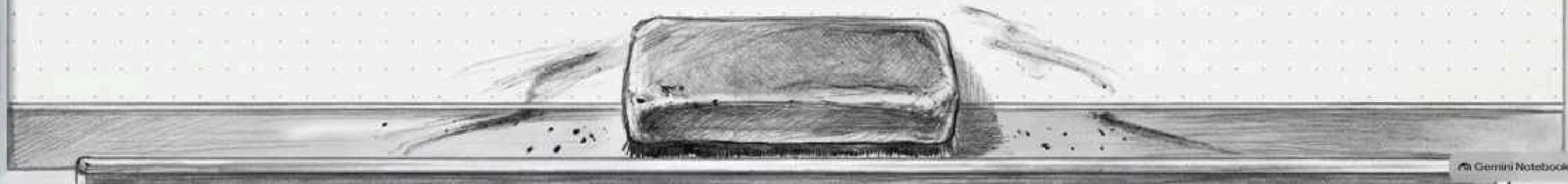
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Up Next: Lecture 5

¶ 4.18

FALLACIES

Next time, we study the common mistakes of reasoning.



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