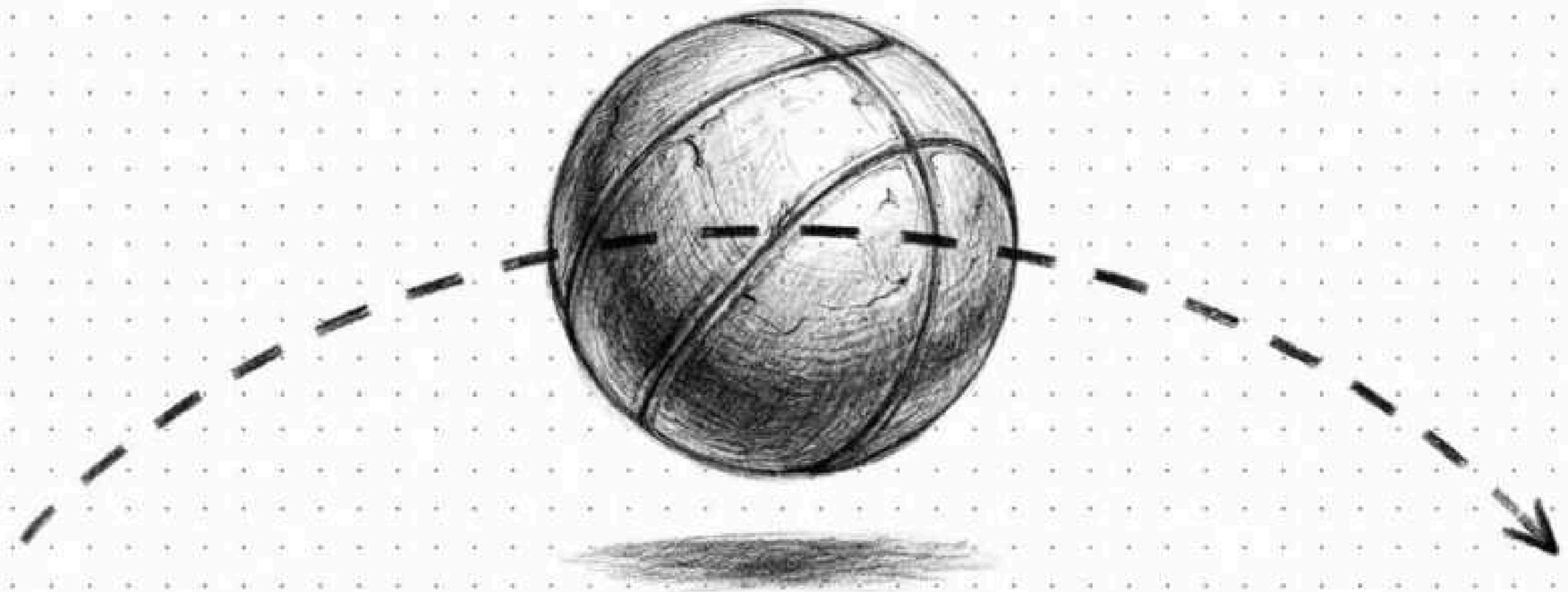
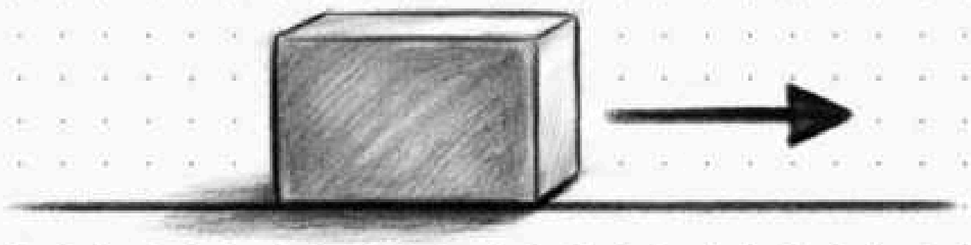
¶ 3.1



Vectors and Projectile Motion

Vectors & Components • Projectile Motion & Range • Relative Motion

1D Motion



2D Motion



Real motion rises and travels forward simultaneously. We need a new tool: Vectors.

Scalars

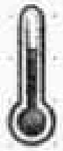
Only Size.



Mass



Time



Temperature

Vectors

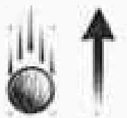
Size + Direction.



Displacement



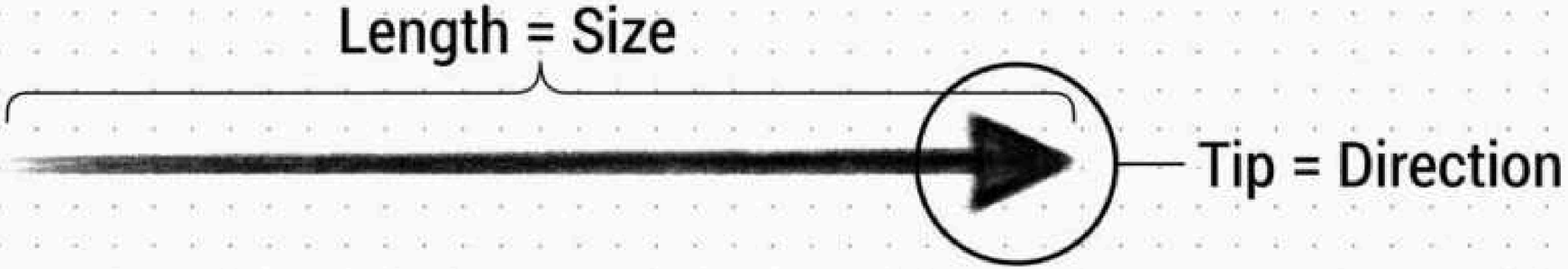
Velocity



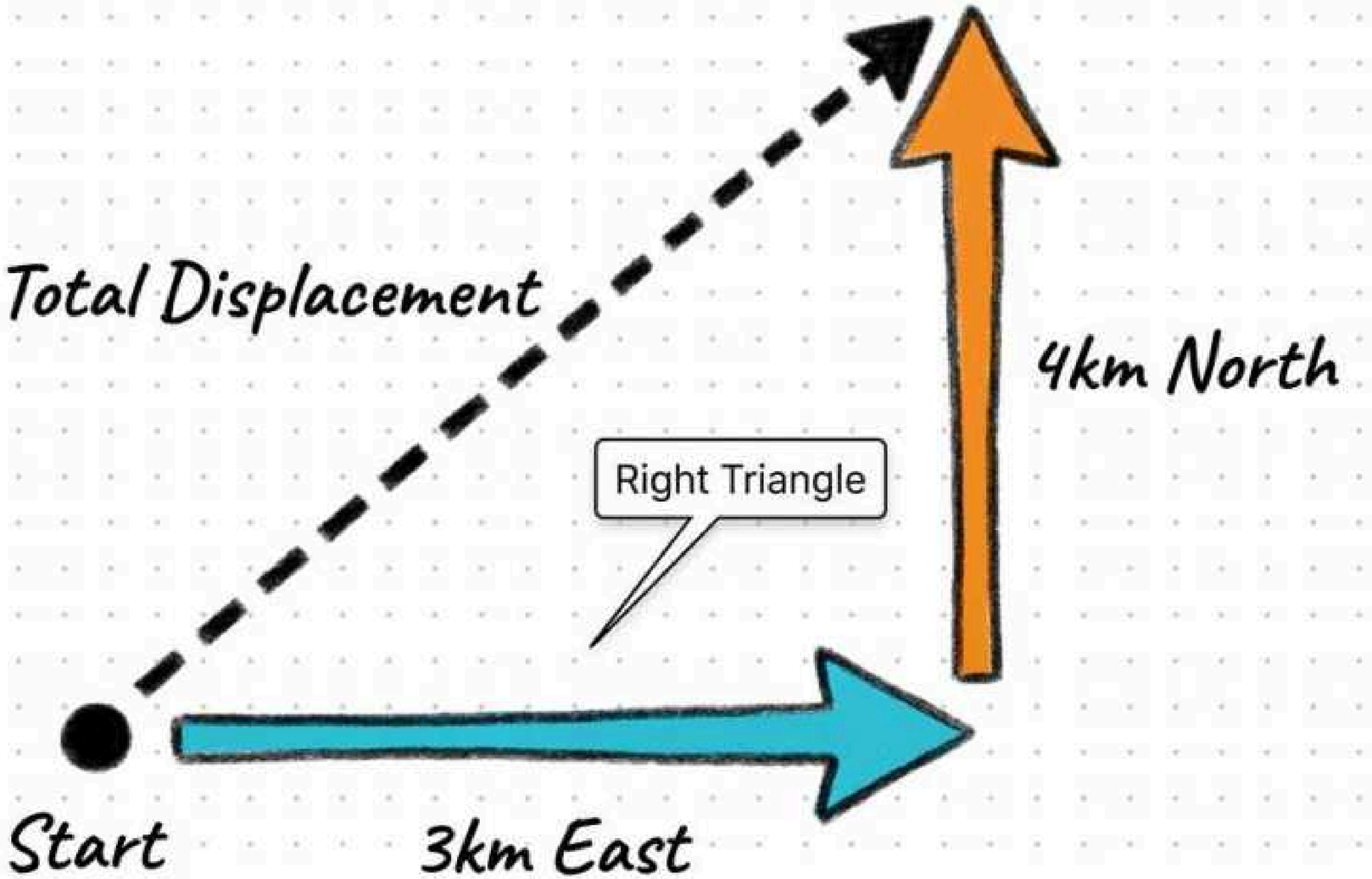
Acceleration

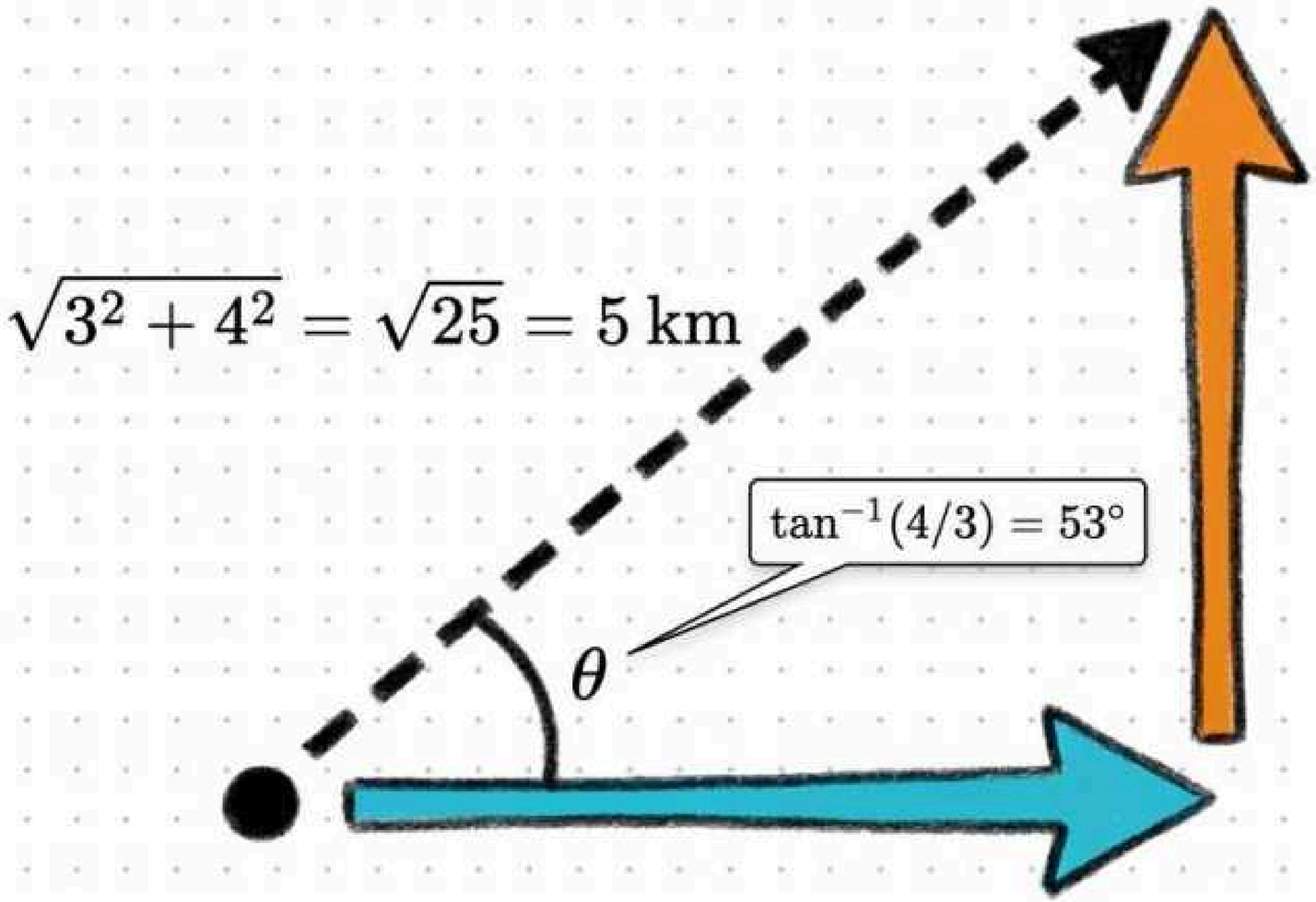


Force



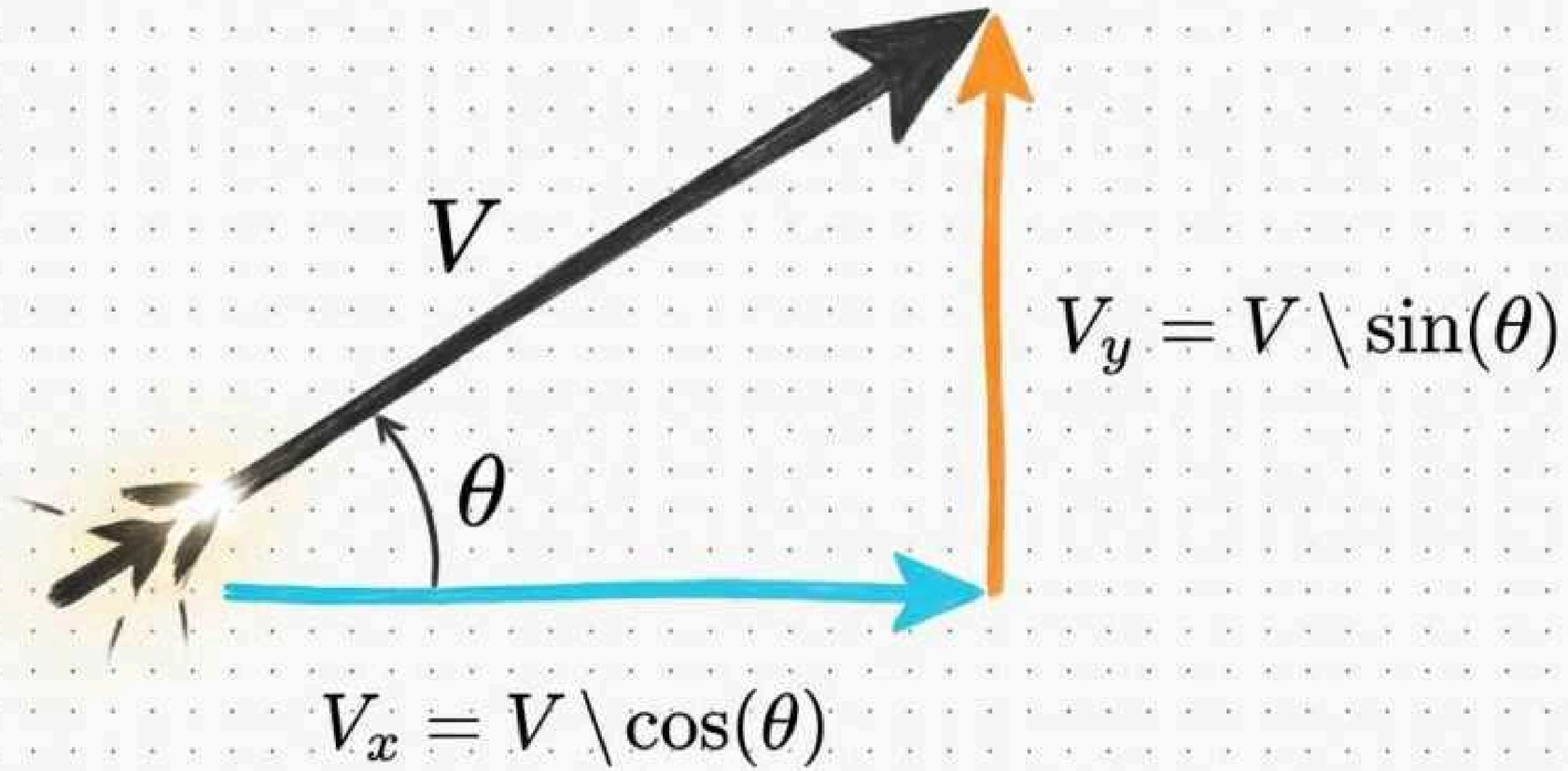
¶ 3.4


Total Displacement
4km North
Right Triangle
Start
3km East

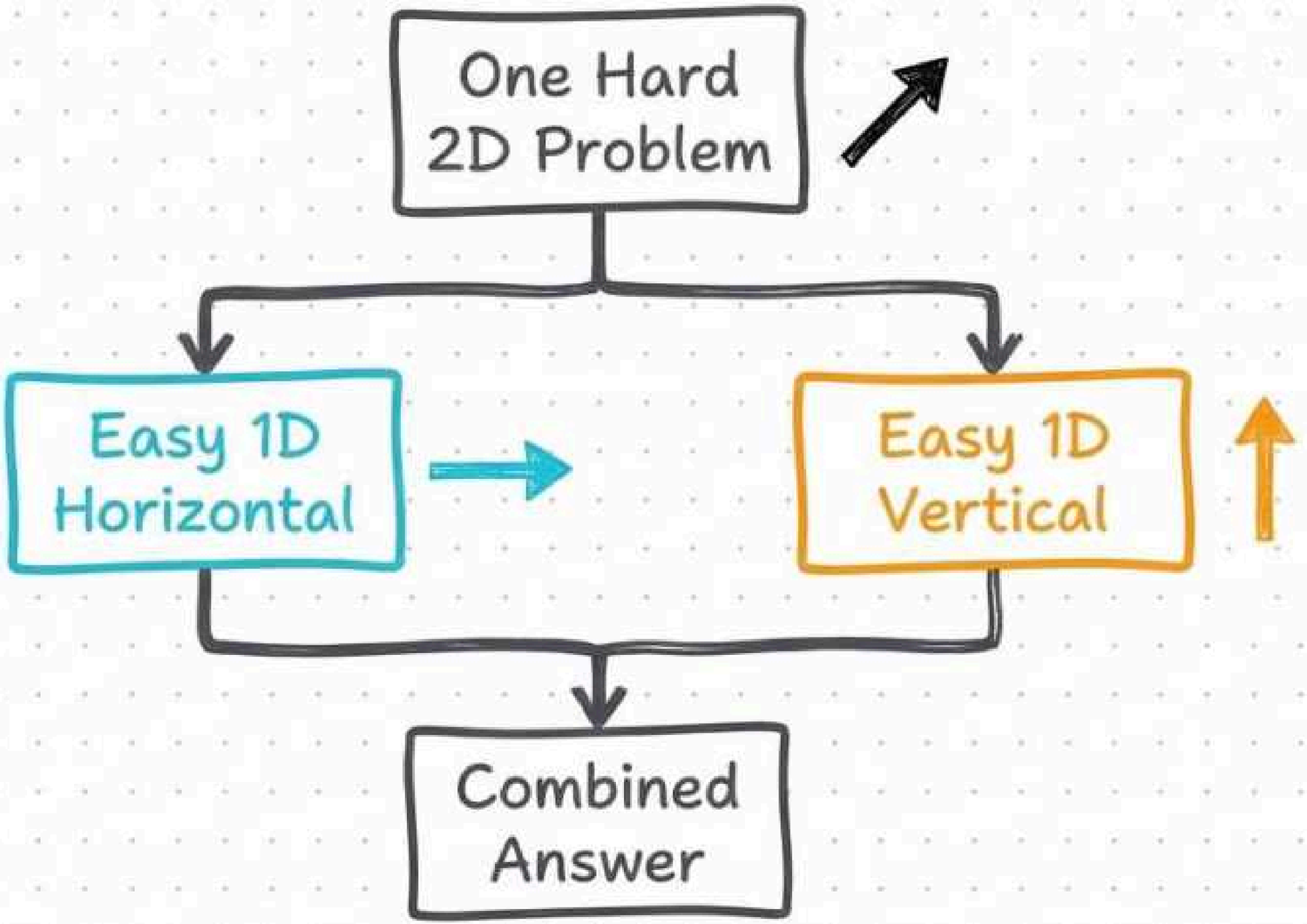


Size via Pythagoras, Direction via Trigonometry.

The Core Trick: Splitting Vectors



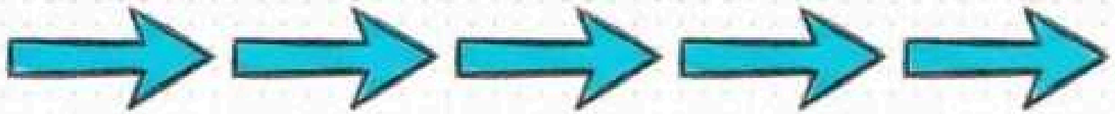
Any vector can be replaced by two perpendicular parts.



Perpendicular directions do not interfere.
This is the core method of mechanics.

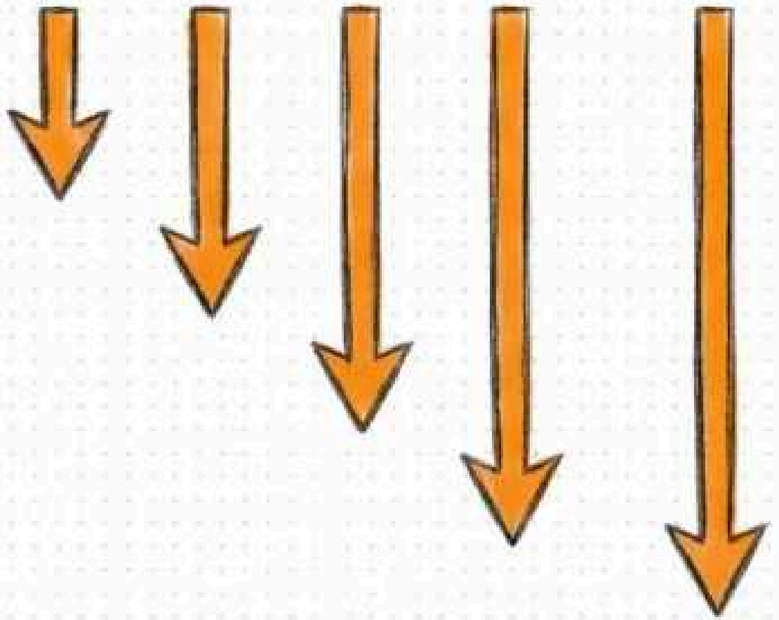
Galileo's Insight: Independence of Motion

Horizontal Panel



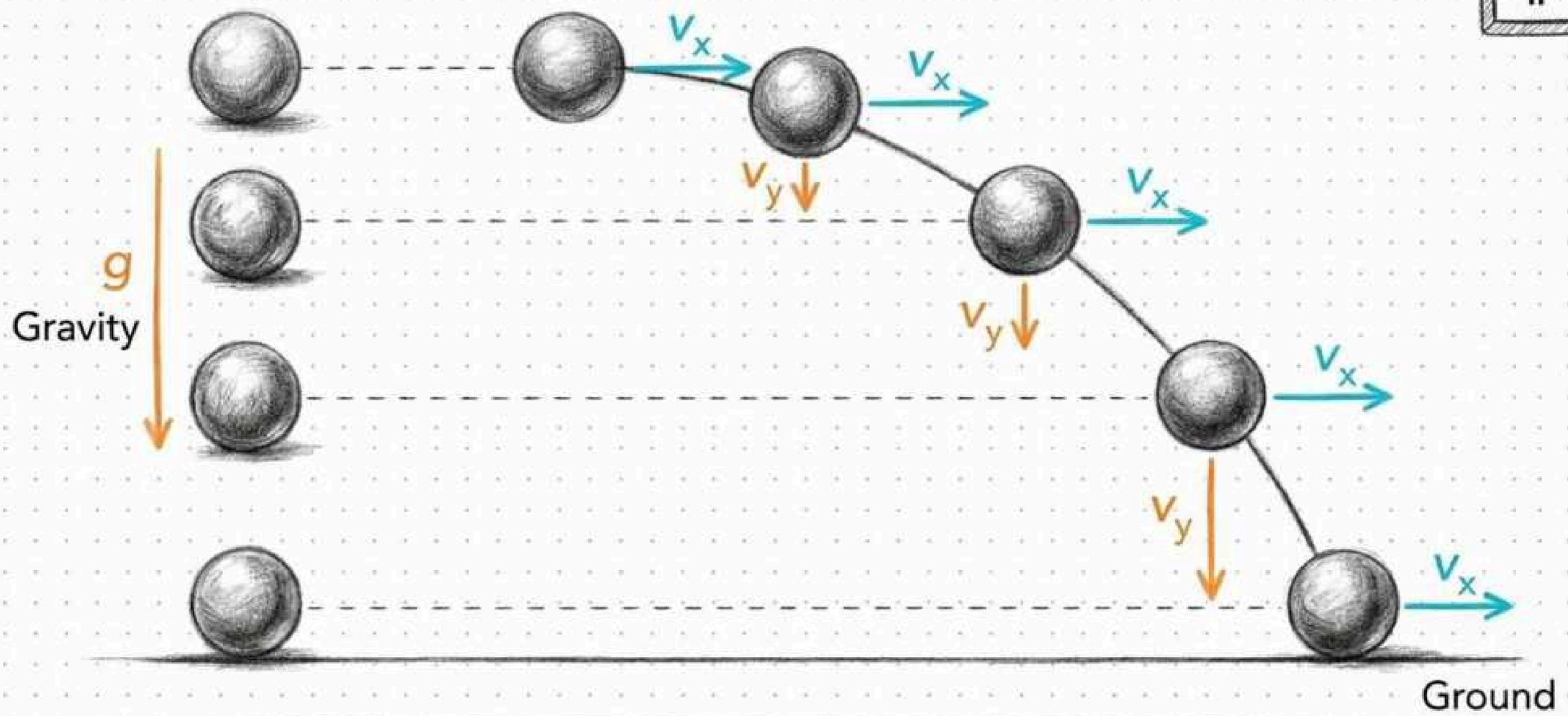
No Acceleration =
Constant Velocity

Vertical Panel



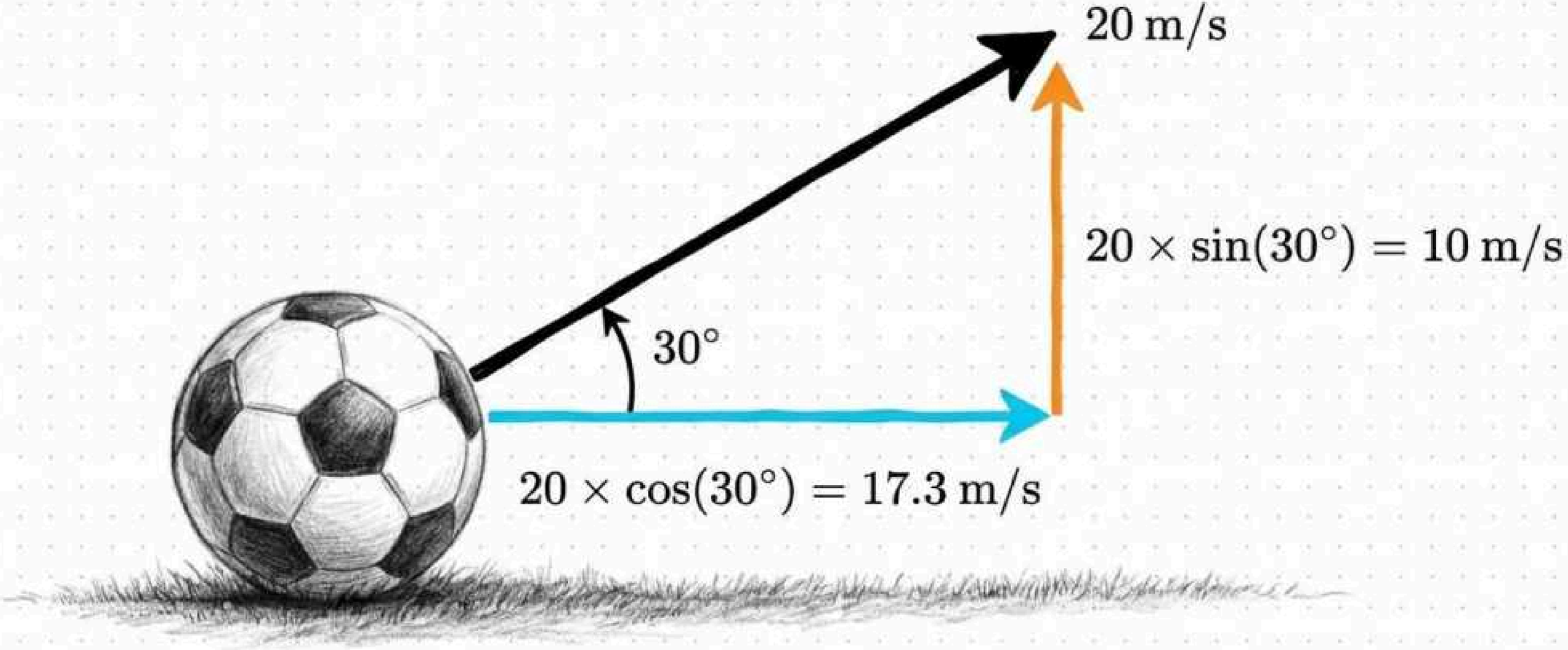
Gravity's Domain:
Acceleration of g = 9.8 m/s^2

¶ 3.9



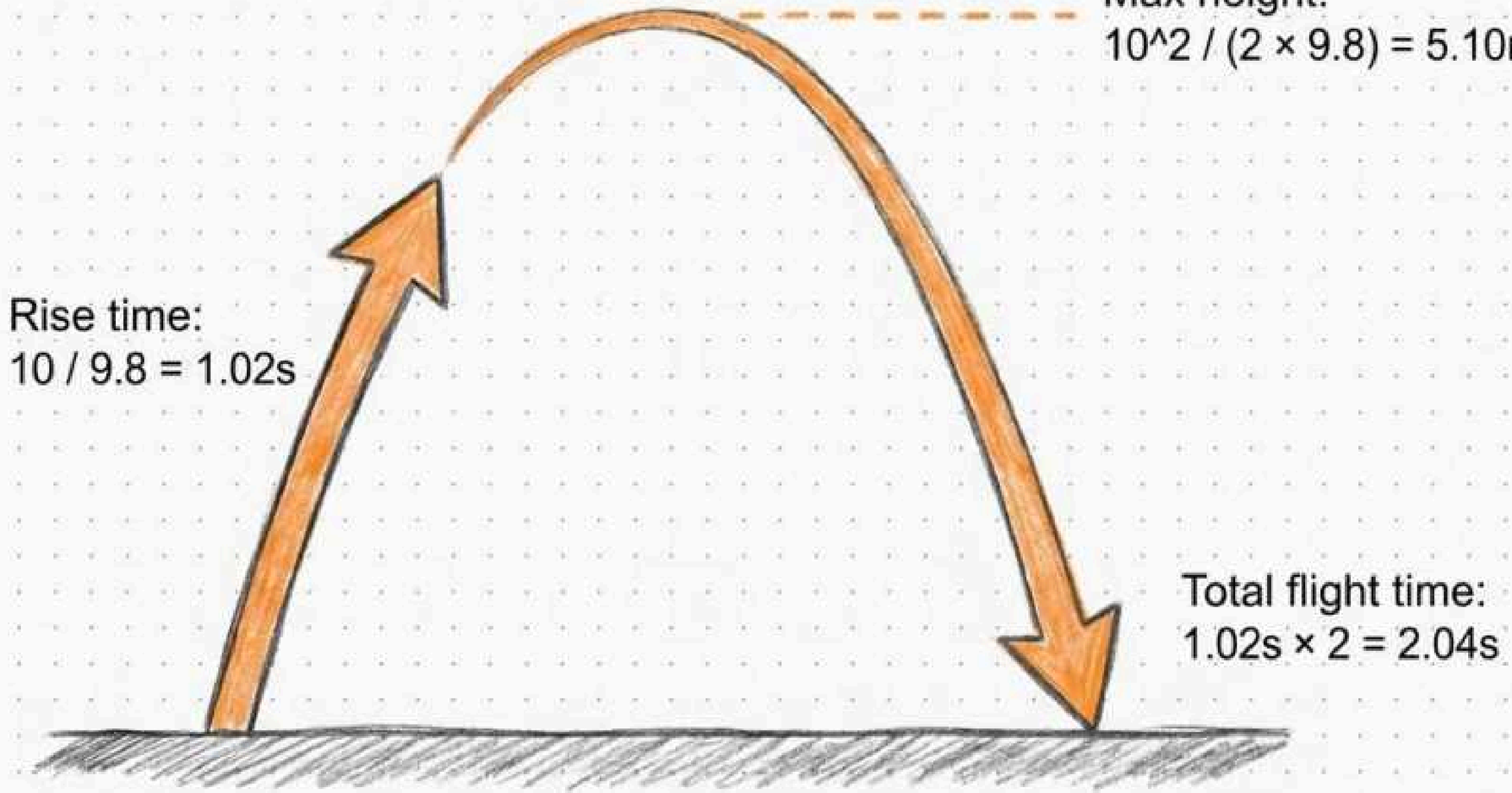
Sideways motion is simply a free extra.
Falling is controlled by gravity alone.

Example 1: The Setup. Treat each direction separately.



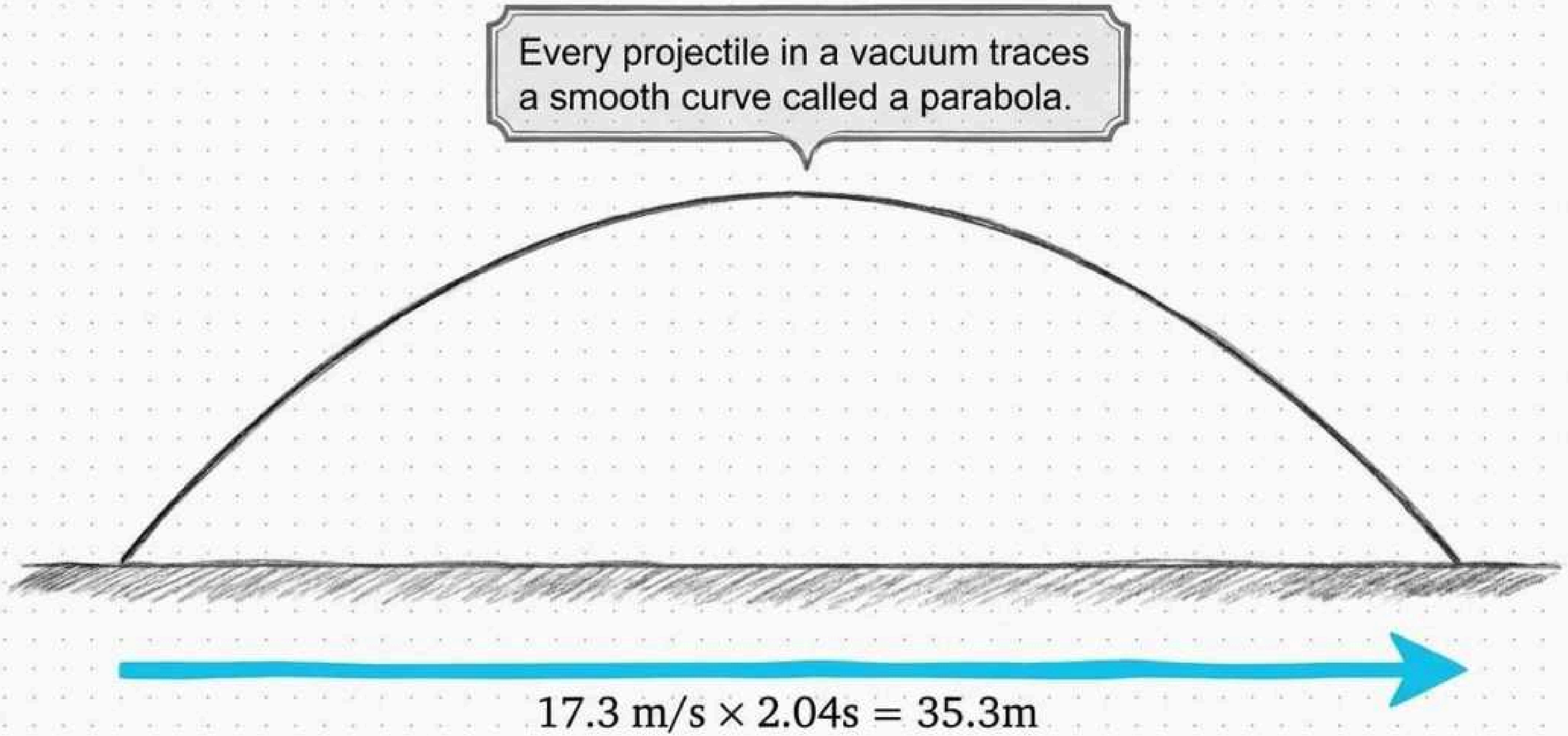
Vertical Analysis: Free fall dynamics dictate the clock.

¶ 3.11



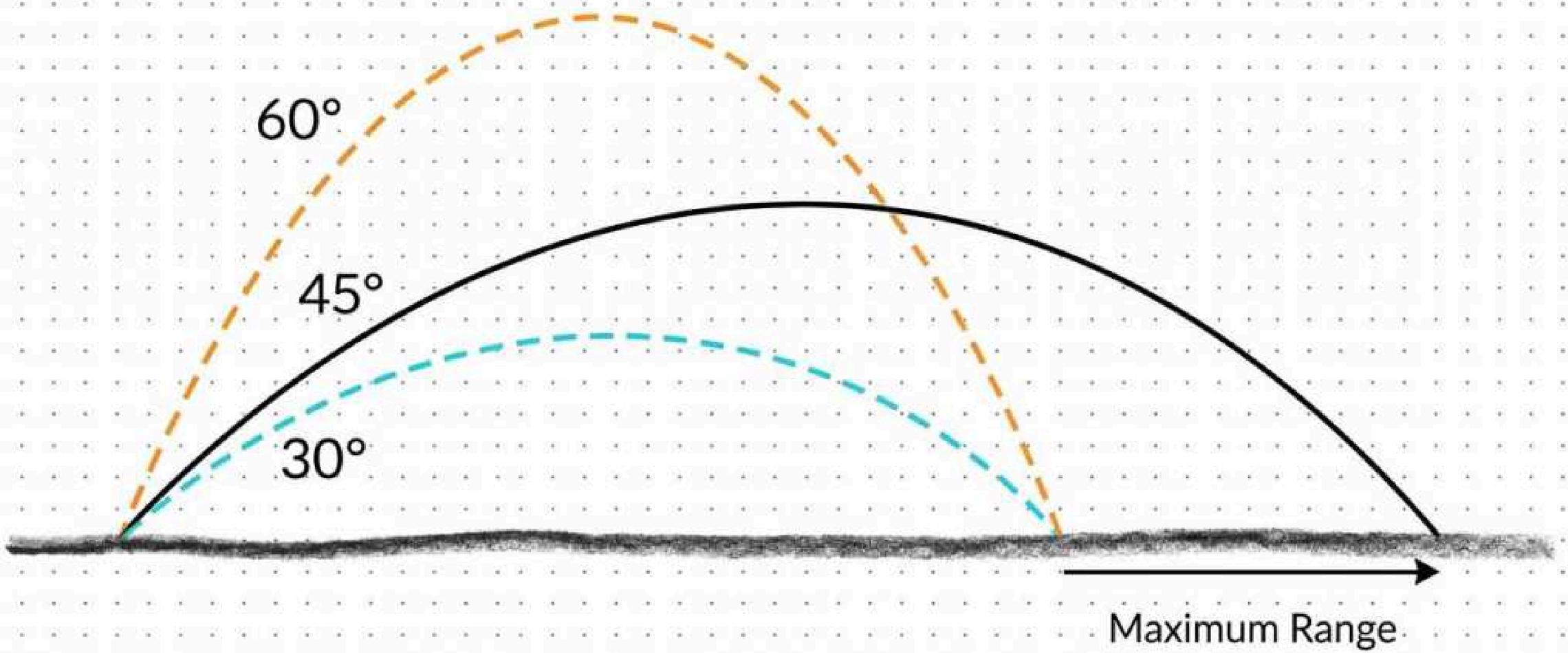
Horizontal Analysis: The x-velocity remains constant.

¶ 3.12



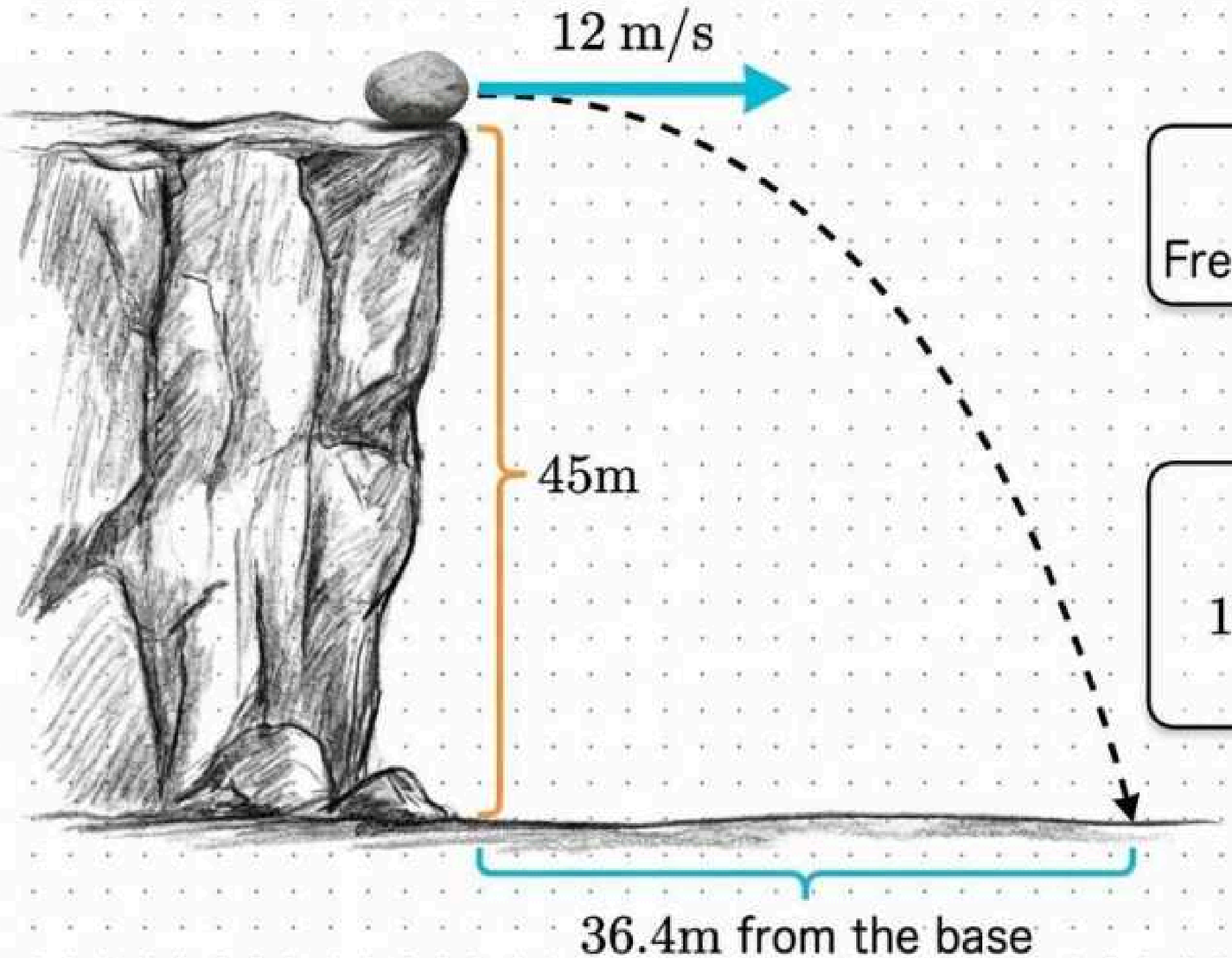
¶ 3.13

The Optimum Angle. 45° balances time in the air with horizontal speed. Angles that add to 90° give equal ranges.



Projectile Motion from a Cliff: Horizontal Launch

¶ 3.14



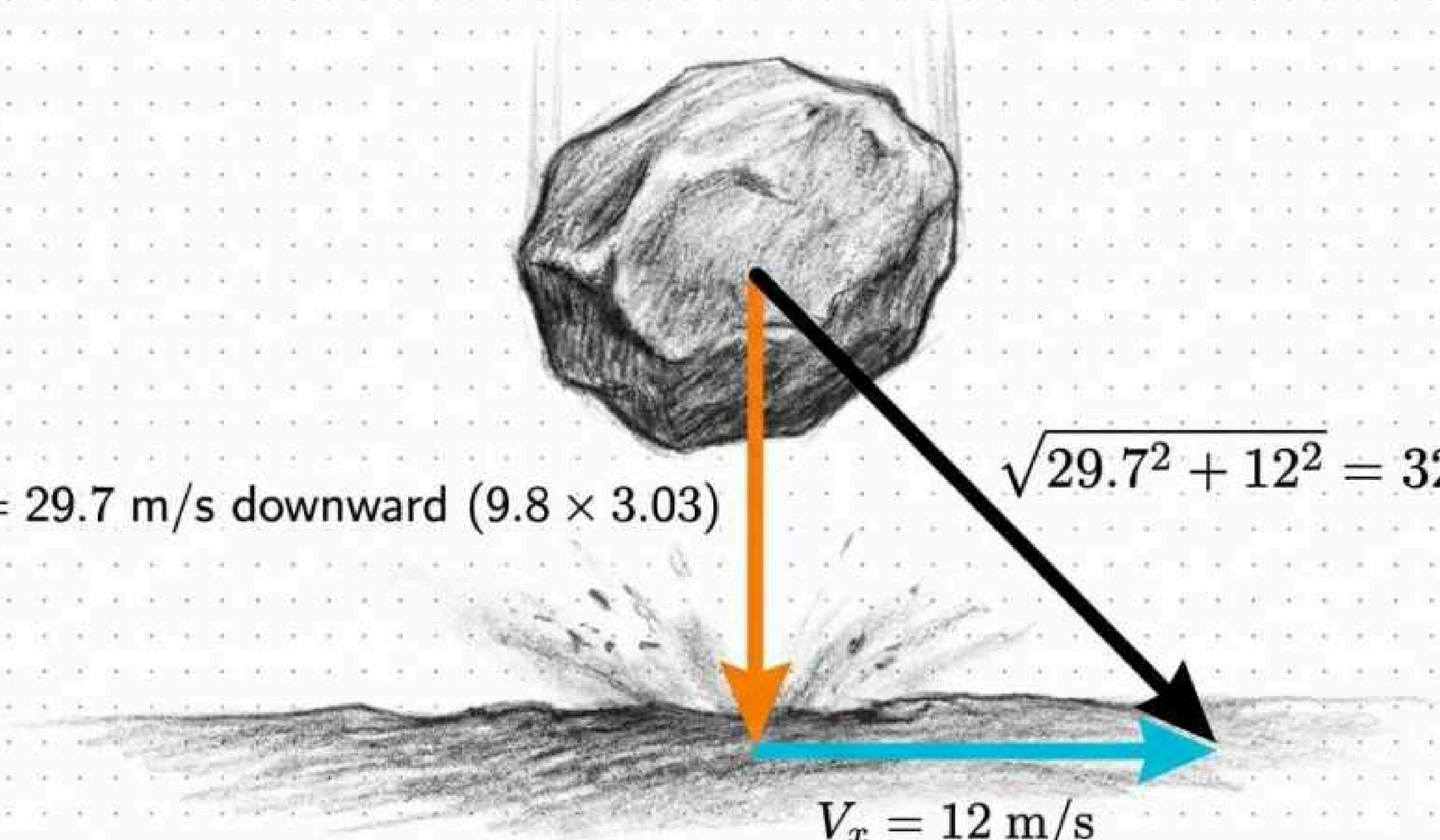
Step 1 (Vertical)

Free fall from rest → Time to fall = 3.03s

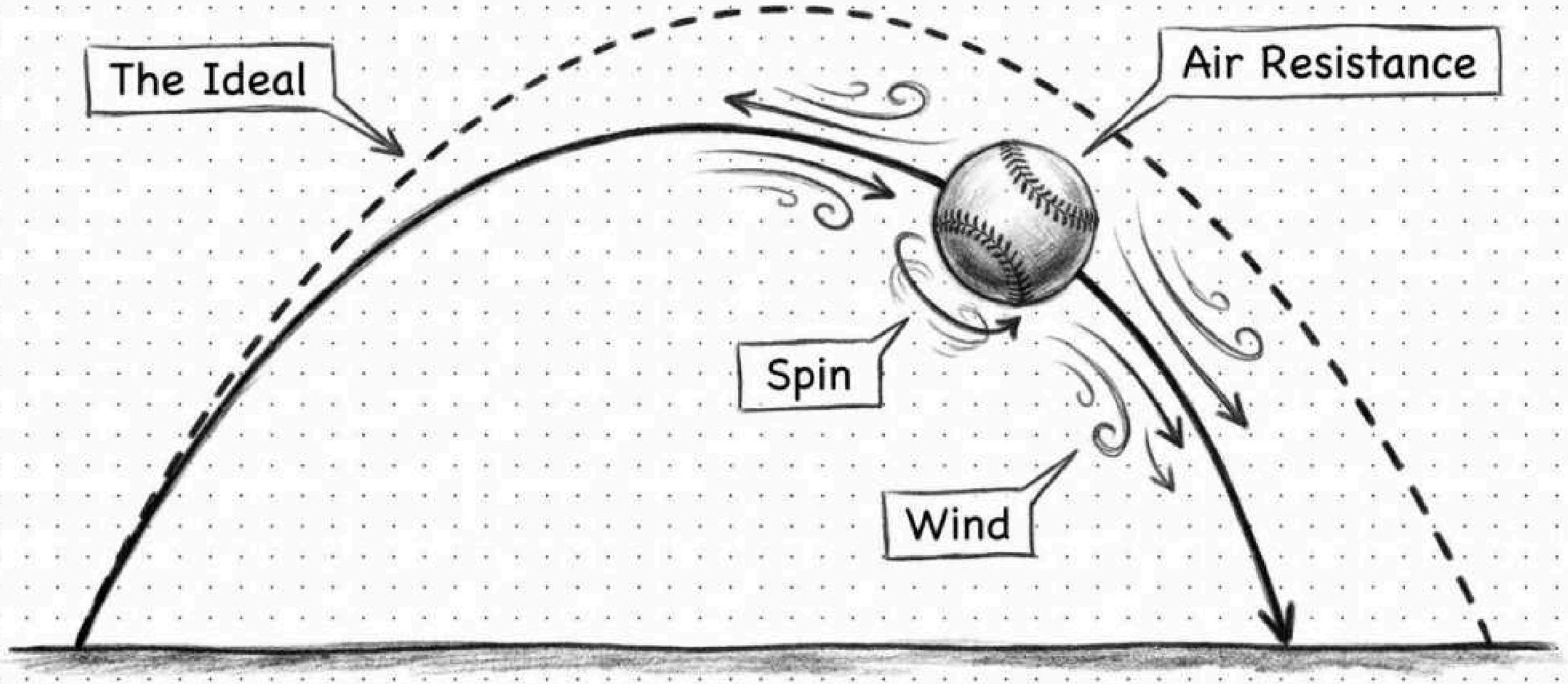
Step 2 (Horizontal)

$12\,\mathrm{m/s} \times 3.03\mathrm{s}$ → Range = 36.4m from the base

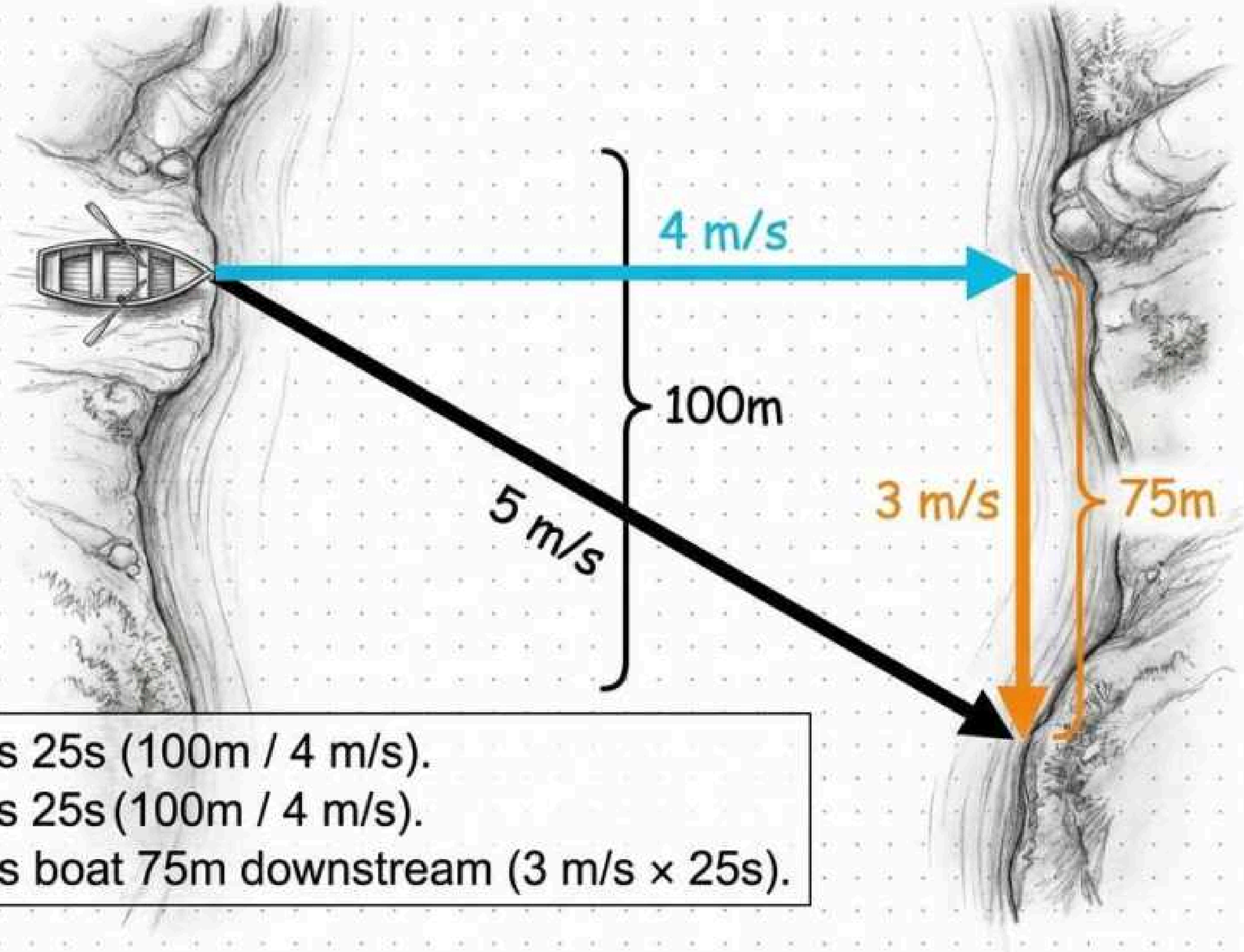
The Habit of Mechanics: Find each component, then put them back together.



The Limits of the Model. Ideal motion is an excellent first answer, but real engineers must add complex effects.



Vector Addition: Crossing the River



Crossing takes 25s (100m / 4 m/s).
Crossing takes 25s (100m / 4 m/s).
Current carries boat 75m downstream (3 m/s × 25s).

¶ 3.18

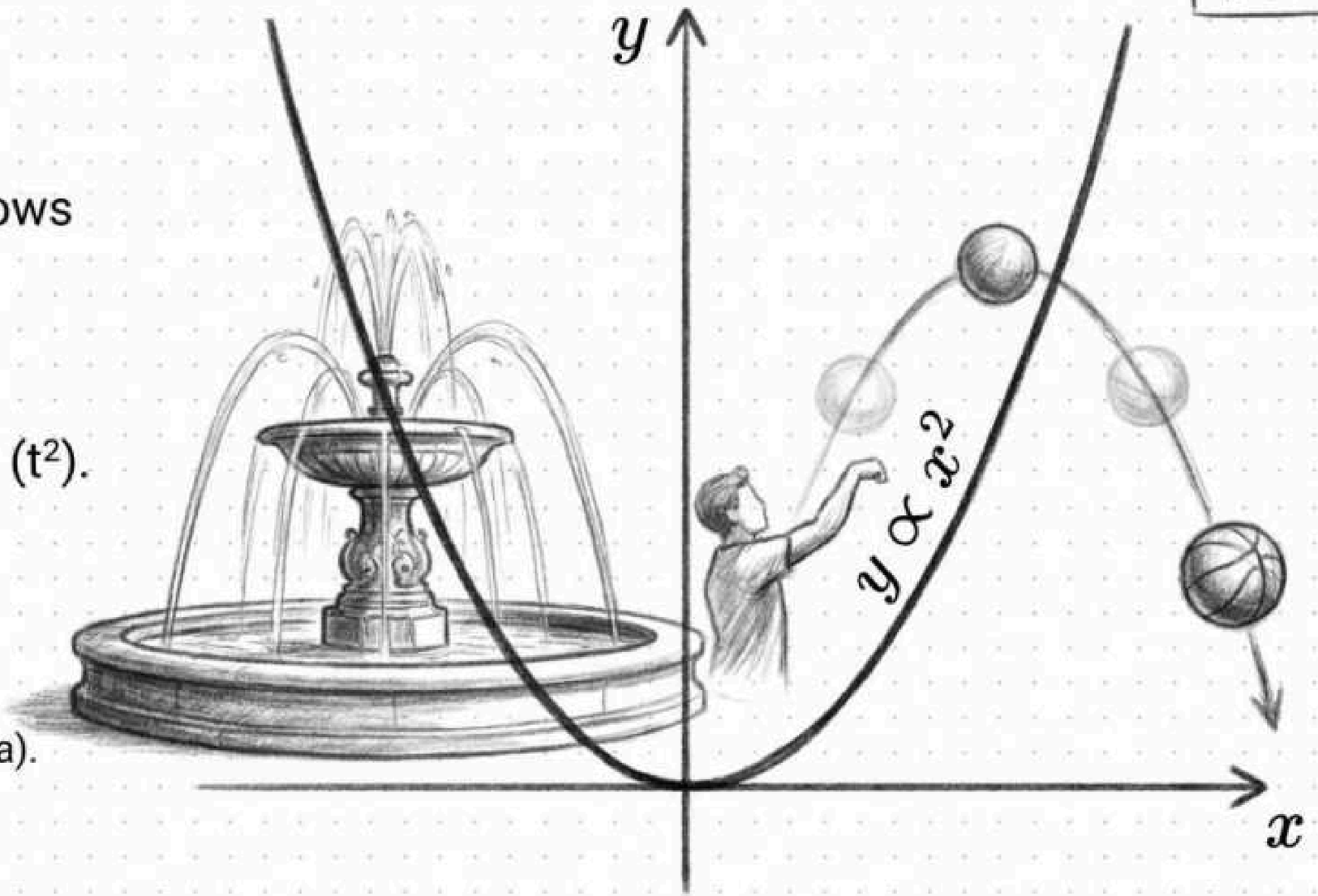
Horizontal position (x) grows proportional to time (t).

Vertical position (y) depends on time squared (t^2).

Eliminating t results in:

$$y \propto x^2$$

(The strict equation of a parabola).



The Master Formula. The absolute longest range possible at 20 m/s in a vacuum.

$$R = \frac{v^2 \sin(2\theta)}{g}$$

$$v = 20\,\mathrm{m/s},\ \theta = 45^\circ$$

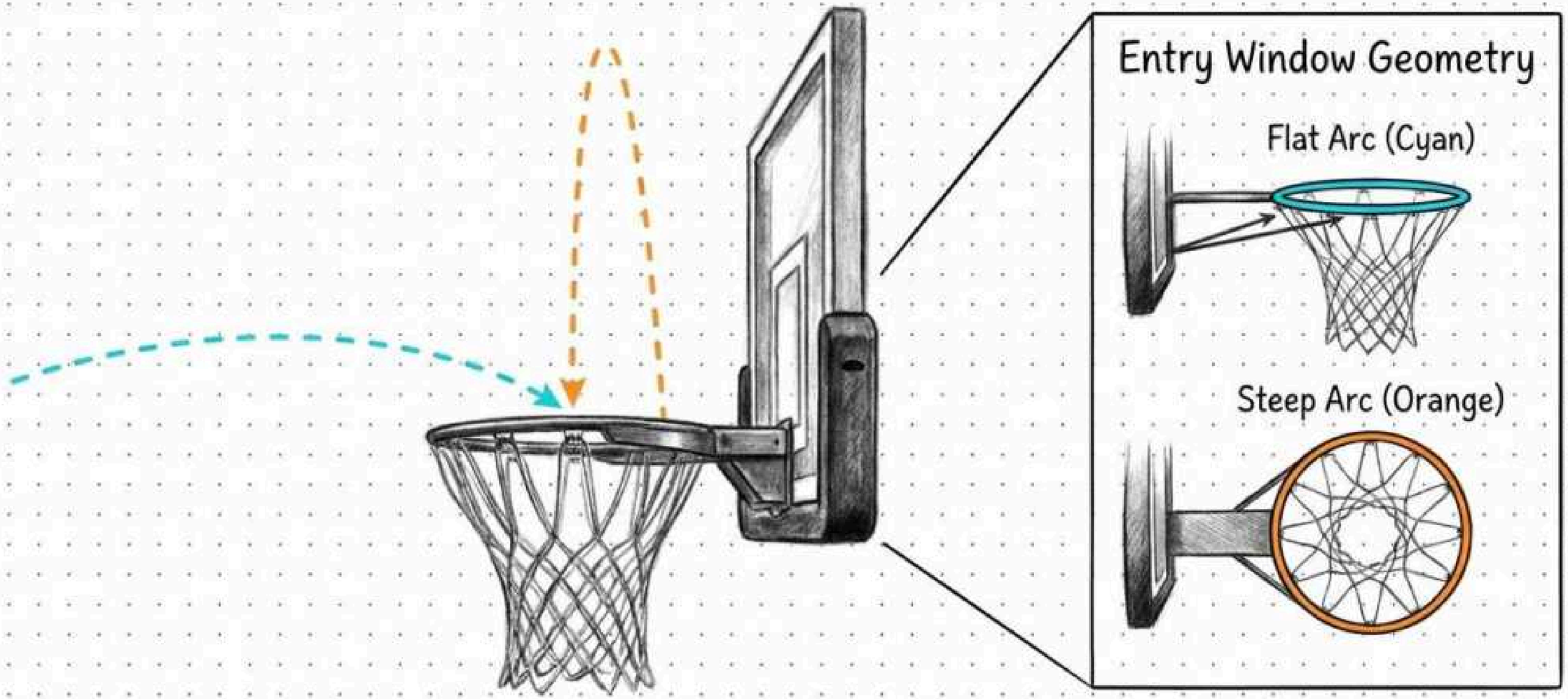
$$\sin(2 \times 45^\circ) = \sin(90^\circ) = 1$$

$$R = \frac{400}{9.8} = 40.8\mathrm{m}$$

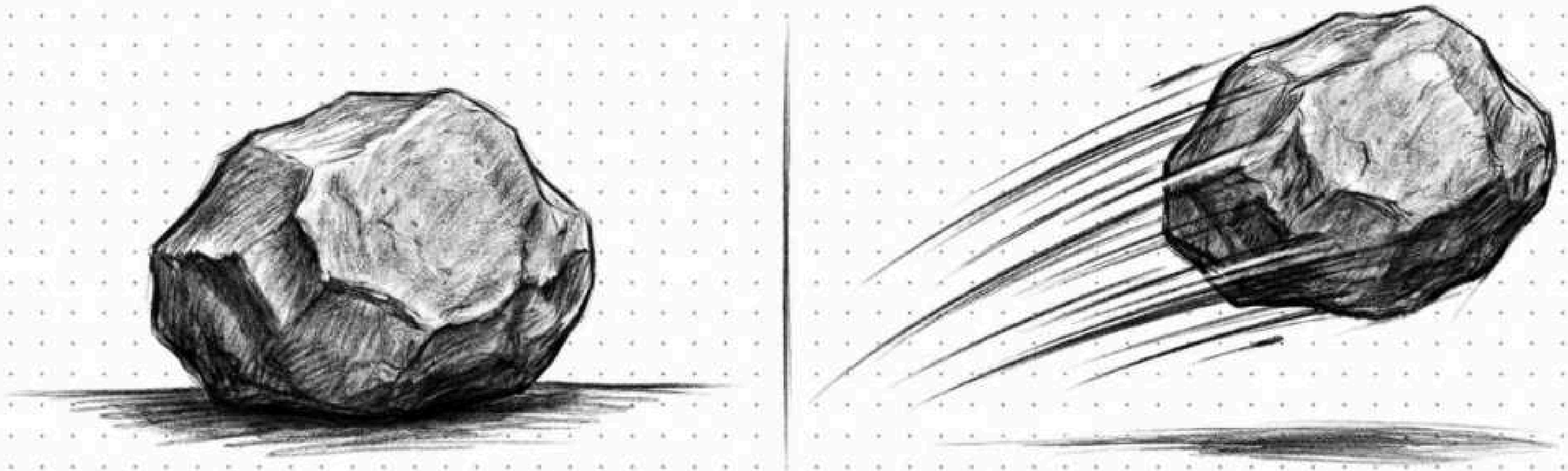
The Free Throw Challenge. Why add a high arc?

¶ 3.20

A steeper angle fundamentally changes the geometry of the entry window.



Why does anything accelerate in the first place?



Next: Newton’s Three Laws.